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ON THE SESAMOIDS OF THE KNEE-JOINT.

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PART I. MAN.

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(1) *Introductory.*

During the course of recent work* on the femur of the primates undertaken in the Biometric Laboratory, much material accumulated bearing on the sesamoids of the knee-joint. It was found impossible to incorporate this in that work itself, and as it was very largely due to a third investigator it is only fitting that the paper should be not only published separately, but with a different authorship. Some of those acquainted with the literature of the subject may consider that all that is necessary to be known on such an apparently small topic has been fully dealt with already in the voluminous memoirs of Gruber† and Pfitzner‡. Both of these memoirs have their individual excellences, but we still venture to think that they have no more exhausted the subject than we ourselves are likely to do. Indeed merely a résumé of their work in a form easily accessible to English readers would be of service; but we hope to have added at least something of value to it, having trusted to what we believe is the only safe method of research, namely the first-hand investigation of the phenomena themselves before starting any study of the observations and opinions of others§.

* Pearson and Bell, *A Study of the Longbones of the English Skeleton.* Part II, The Femur of Man, with special reference to the Primate Femora, see p. 512. *Drapers' Company Research Memoirs*, Cambridge University Press.

† For the full title of Gruber's classical memoir (hereinafter cited merely as Gruber) see our p. 156.

‡ For the full title of Pfitzner's memoir (hereinafter cited merely as Pfitzner) see our p. 163.

§ Some time may be lost by postponing the study of the literature of a topic until at least a late stage of one's own researches, but on the other hand a preliminary study will too often check one's own individual line of research either by emphasising others' methods of approach, as if they were the sole roads to truth, or by crowding the mind with a mass of other people's details, often best described as debris.

Before we can properly talk about our subject it is really needful to have some terminology. The sesamoid may be a true ossification of hyaline cartilage, or it may be a thickening or nodule of cartilage without ossification. In the young it is open to us to consider the latter as a stage of growth towards the former. In the adult this is not possible. We can, of course, assert, as many anatomists have done, that all sesamoids are produced by intensive stress acting on cartilage, but we shall then have to account for the absence of sesamoids in regions where such stress undoubtedly exists. Or we may look upon the non-ossified nodule as an attempted sesamoid, or better still as a vestige or trace in the species of where a sesamoid once has been. Whatever account of the matter we choose to accept, it is needful to distinguish between the two cases. Pfitzner proposed to call the bone a *sesamum* and the nodule a *sesamoid*. This upsets the historic origin of the word, for the Greeks most probably knew only the *sesamum* and called it a *sesamoid*, and the name sesamoid remained as a name for the bone through many centuries. We might speak of sesamoid bone and sesamoid nodule or cartilaginous sesamoid, but the terms are cumbersome. We shall accordingly reserve the term sesamoid or *orthosesamoid* for a true bone and adopt an independent term for the thickened but unossified element of cartilage, calling it a *hemisesamoid*. Further we shall term all pathological ossifications, exostoses, or again, broken off and not reunited portions of bone occurring in or near the positions of sesamoids *pseudosesamoids*.

In the knee-joint itself we have come across no less than ten sesamoids, some of which are, however, more frequent or better established than others. To these sesamoids we have given the following distinctive names:

We retain the names of lateral *fabella* and mesial *fabella* for the sesamoids found in the tendons of *M. gastrocnemius* above the lateral and mesial condyles respectively.

We introduce the term *cyamella** for the sesamoid which is to be found in the tendon of *M. popliteus*. It may be termed the *lateral cyamella* to distinguish it from a *mesial cyamella* which Pfitzner asserts he observed in two cats†. Gruber, notwithstanding his wide knowledge, seems to us to have confused the lateral *cyamella* in man with an unusual position of the lateral *fabella*. The existence in the cat of both lateral *fabella* and lateral *cyamella* ought to have warned him that both might appear in man.

Lastly a number of sesamoids occur in the borders of the semi-lunar cartilages. We term these *lunulae*‡. They may occur in six positions which we describe as follows: antero-lateral, antero-sagittal, antero-mesial, postero-lateral, postero-sagittal, and postero-mesial. Of these the sagittal are doubtful and may be anomalous in all species. Thus we have two *fabellae*, two *cyamellae* and possibly six *lunulae* as sesamoids of the knee-joint. In most cases some only of these will be present; in certain species some will always be present as true sesamoids, or will be present as

* *Cyamella*, a small bean, the bean-stone, an ornament worn by Roman women.

† Pfitzner, S. 584, without identification of an association with any particular muscle.

‡ *Lunula*, a little moon, a crescent, or crescent form of ornament worn by Roman women.

anomalies. In other species some will only appear as hemisesamoids; while in a third species, if they have been reported at all, it is probable that they were really pseudosesamoids.

Now to obtain a large number of knee-joints for dissection in the case of a particular species is practically impossible. In the case of dry material and skeletons the articulator may, perhaps generally does, preserve the *fabellae*, far more rarely the *cyamellae* or *lunulae**. Thus the actual presence of *cyamella* or *lunula* conveys definite information, their absence practically conveys nothing. It is extremely hard to demonstrate an invariable absence, especially in the case of an anomaly which may have a very small percentage of occurrences like the lateral *cyamella* in man. When one anatomist asserts the existence of a sesamoid which nine others deny, there is no grave difficulty in reconciling their statements, if the anomaly be present in only a small percentage of cases, and none of these anatomists have had the chance of examining more than a few individual cases of the species in question. This is, we think, the quite adequate explanation of the contradictions we find in the early history of the subject, and forms the chief difficulty in applying our analysis of the sesamoids of the knee-joint to the problems of evolution.

These problems are of the following character. Suppose we can classify those families in which certain sesamoids are (i) invariably present in the knee-joint, (ii) invariably absent, (iii) exist only as anomalies of a given order of rarity, will not such classifications throw light on evolutionary order? For example if we confine our attention to the primates, there are families in which the *fabellae* are both invariably present, others in which they are both invariably absent, others in which both are anomalous, and still another group in which the *fabella lateralis* is a true or orthosesamoid when it appears, and the *fabella mesialis* a hemisesamoid. Again the *cyamella lateralis* is invariable in certain monkeys, never appears in the gibbon, but is constant in the orang, and a rare anomaly in man. A similar series of statements can be made with regard to the presence or absence of *lunulae* in the primates. May it not be possible to throw light on primate evolution from such considerations?

The problems with regard to sesamoids are not only rendered difficult of solution by reason of the amount of material requisite for adequate classification, but by the longstanding existence of opinions and dogmas with regard to them.

The definition of the medical student that a sesamoid is any bone which it is unnecessary to study for examination purposes is, of course, an echo of the treatment of sesamoids in the current anatomical text-books, and that treatment has its historic evolution—broken only and for a short period by the discovery of the *fabellae* in man by Vesalius—from Galen to the present day; the text-book anatomists from his day through the centuries to our own have echoed his phrase as to sesamoids—that of those bones "which are termed sesamoids from their

* In 15 dog skeletons in the Biometric Laboratory, the *fabellae* are almost invariable, the *cyamella* has not been preserved in a single case.

likeness to sesame" *non est necessarium hoc opere pertractare**. If only Galen had told us which sesamoids he knew, what sesame was†, and how and why they were like the seeds of it, our task would have been simplified! The comparison may have been obvious to the Greek anatomists because the sesame seed that they were referring to was familiar to their readers and because they had special sesamoid bones in mind, it is far less obvious nineteen centuries later.

But we are not exact in stating that sesamoids have as a rule been dismissed in the language of Galen in the text-books. The opinion that it was unneedful to discuss them has been frequently accompanied by the dogma that sesamoids are the product of intensive stress acting upon cartilage. This dogma appeared to receive confirmation from the undoubted existence of hemisesamoids in regions where orthosesamoids are usually or on occasion discoverable. A hemisesamoid was an orthosesamoid in the making. This dogma appealed to that tendency in anatomy —largely the product of pathological study—which attributes innumerable characters of individual bones to specific use. The bone was not adapted by evolution to its use, but use was the source of its characters, and indeed if these were observed in the foetus or at birth, they were in themselves evidence of the inheritance of acquired characters. The *fossa hypotrochanterica* and the crural trough were the product of muscular action on the bone, even if they were to be found more frequently in female than male femora and were discoverable in infants and non-robust or even diseased bones. The "intensive stress" origin of sesamoids had indeed been questioned by Dr Robert Nesbit before the middle of the 18th century. He wrote as follows in his *Human Osteogeny*, London, 1736, p. 136:

"The only parts of a foetal skeleton, which remain to be taken notice of, are the *ossa sesamoidea* which all the writers, I have met with on this subject, have wholly passed over unobserved. The number of them in *foetuses* are, as in adults, very different in different subjects. Those, which are the most constantly found, are two in the bottom of the foot, fixt in the ligament of the articulation of the first bone of each great toe, with its *os metatarsi*. In all *foetuses*, from three months after conception to birth, the places of these *ossa sesamoidea* are always filled with cartilages of nearly the same shape those *sesamoide* bones usually have, when they are arrived at perfect maturity, *Plate* IV [actually Plate VI] *fig.* 18 *a. a. fig.* 19 *a. a.* In one subject at birth I found, in each of the *sesamoide* bones of one foot, a very small point of ossification.

"In like manner those *ossa sesamoidea* which are sometimes found at the beginning of the *musculi gastrocnemii*, are to be seen in *foetuses*.

"By the descriptions I have now given of all the different parts of foetal skeletons, it manifestly appears, that there is not one single bone, except the teeth, or one *epiphysis* in an adult skeleton, which is not to be found in a full grown *foetus*, or in its place a cartilage of nearly the same shape: consequently, the account cannot be right, which the ingenious professor *Monro* gives of the *ossa sesamoidea* being

* *Liber de ossibus Galeni Librorum Pars Quinta*, Basileae, 1538, p. 727.

† See, however, below, p. 141.

'nothing else than the ligaments of the articulations, or the firm tendons of strong 'muscles, or both, become bony by the violent compression they suffer in the 'situation they are.'" (*Anatomy of the Bones*, Edinburgh, 1726, p. 337.)

It is true that this statement of Nesbit's, notwithstanding its definiteness and perspicacity, has been questioned somewhat cavalierly by Gruber*. He says that for many years he examined the cadavers of embryos in last month, of new-born children in considerable number and of several children one to seven years old, and never found in the tendons of *M. gastrocnemius* a *cartilago hyalina* or *fibro-cartilago*. He does not state the exact numbers and appears to hold that Nesbit asserted the existence of orthosesamoids, whereas Nesbit as we read him is speaking of prefigurations. Yet Gruber did himself find the hyaline cartilage elements in the tendons of puppies at birth where the *fabellae* later develop, and notes that the lateral ossifies first (at about six months)†. He does not state how many embryos and children he examined, and it is clear that as at least 90 °/₀ of adults exhibit no *fabellae*, it is quite conceivable that Nesbit may have found the prefigurations and Gruber not. But the fact that prefigurations are found in puppies and kittens, and the additional fact that not all sesamoids are associated with intensive stress while many regions of intensive stress are not associated with sesamoids‡ should be enough to dismiss this hypothesis, were it not that its supporters can appeal, as has been done in the case of femoral facets, to the principle that acquired characters are inherited, a haven ready for the supporters of production by use, and one where the difficulty of demonstrating a negative—other than by Occam's Razor methods—leaves them in comparative security.

Perhaps the most effective ejectant is the appeal, such as we hope to make in this paper, to the facts of the evolution of the sesamoids. Pfitzner, who was a strong opponent of the intensive stress hypothesis, not only cites Nesbit's results, but cites the case of the *Marsupialia*, and figures the Wombat, of which he writes:

Wenn man annehmen müsste dass Gebilde von so reicher Formenentfaltung wie diese Epiperone (so möchte ich sie nennen) beim Wombat jederzeit als "Product entzündlicher Reizung" oder dergl. neu entstehen könnten, so würde man bald jede wissenschaftliche Bearbeitung des Skeletsystems als ein aussichtloses Unterfangen erklären müssen§.

The reader who wishes to appreciate the full force of Pfitzner's argument should examine our Plate V, Fig. 13. It seems unanswerable as far as the Marsupials are concerned. Can we look for any evolutionary links between Pfitzner's "Epiperone," the "parafibula" of the Marsupials, and the sesamoids of the knee-joint of higher types?

For Pfitzner the source of the sesamoids is the same as that of any other bone of the skeleton, i.e. the force of heredity‖. He distinguishes as we have seen between

* Gruber, p. 13.

† Gruber, p. 57. We have ourselves found them, as well as the *cyamella lateralis*, prefigured in puppies at birth, and the two *fabellae* and the *cyamella* in kittens at birth.

‡ Why should man and the greater anthropoids have lost their *fabellae* while they are universal in the lesser apes? The "stress" has hardly disappeared.

§ Pfitzner, S. 586.

‖ Pfitzner, S. 535 and S. 563.

orthosesamoids ("sesambeine") and hemisesamoids ("sesamoide"); the latter he considers develop in fibrous cartilage ("faseriges Gewebe") and do not ossify*. Hemisesamoids are abortive sesamoids; we may term them vestiges of past phases of evolutionary development. Pfitzner does not, however, attempt to follow up the view here suggested by the examination of sesamoids in a large number of types other than man. Man is of course the subject of his paper, and the sesamoids of the knee-joint with which we are at present interested form only one element in his monograph.

The keynote, however, of Pfitzner's suggestion that hemisesamoids are not necessarily orthosesamoids in the making, but may be the evolutionary debris of what have been orthosesamoids, vestiges of ossificatory centres where no ossification now sets in, is well worth bearing in mind. It is not incompatible with sesamoids appearing in regions of intensive stress, for it is possible that in such regions the sesamoid or its precursor would be of special value to the organism.

(2) *Origin of the term Sesamoid.*

The term sesamoid is not to be found in any author previous to Galen†, but he uses it in the sense of a word familiar to his readers. It is probable that Galen only knew the sesamoids of hand and foot. He does not directly refer to those of the knee-joint. Eustachius in 1564 argued that Galen must have dissected the human cadaver and not those of monkeys‡, because he does not refer to the *fabellae*, but as Galen clearly considered sesamoids of no great importance, he may simply have omitted reference to them. If he dissected carefully a considerable number of human bodies, it is singular that he should not have met with the *fabellae* occasionally, but the same criticism applies to all anatomical writers up to Vesalius (1555).

We have been able to discover four passages, three in the accepted, one in the spurious books of Galen, referring to sesamoids. We cite from the Latin translations.

* This does not seem in accordance with Gruber's view (see our pp. 157 and 175), that sesamoids are invariably prefigured by hyaline and not fibrous cartilage.

† Galen was born at Pergamum A.D. 131. He went to Rome and was known to Marcus Aurelius. He died aged 70.

‡ "Admiratione denique obstupui, cur anatomiae instauratores, in secunda libri de hominis fabrica editione, ubi administrationem musculorum pedis docent, duo ossicula magnitudine ciceris describant, quae supra utrumque femoris tuberculum reperiuntur, qua nimirum primi duo pedem moventes oriuntur musculi: nam & si ea in simijs & in canibus semper occurrant, in hominibus tamen adeo raro inueniuntur, ut non modo (quod posteriores addiderunt) saepius alterum, nunc internum, nunc externum deficiat, uerum etiam ut plurimum utrumq: ipsorum desit, miraculum enim existimabo si duo simul occurrent, & non mediocri diligentiæ ascribam, si post multorum cadaverum sectionem alterum illorum quis inveniat. ideo que tantum abest, ut aequum sit eiusmodi ossicula inter hominis partes numerare, ut etiam liceat suspicari ea a Galeno consulto fuisse pretermissa, indeque sumere argumentum tantum anatomicae artis praeceptorem homines non simias nobis descripsisse. Addo sine aliquo fructu laborare illos, qui commentitias quasdam utilitates horum ossiculorum confingunt: neque enim natura, quae provida est & omnium opportunitatum moderatrix, veretur ne in cruris directione principia paulo ante memoratum musculorum comprimantur & atterantur. Si enim hic esset eius scopus in illis conformandis multo magis in homine, quam in simia & cane reperirentur, quando horum uterque longe minus, quam homo tibiam ambulando extendit."

See Bartholomaeus Eustachius, *Opuscula Anatomica*, Venice, 1564, p. 180.

The first reference is in the *Liber de ossibus*, and contains the passage to which we have already referred:

Quod si quod ossiculum alibi utpote in corde, in naso, in gutture, itemque in aliquo ex digitis, ut quae a sesami similitudine sesamoide vocant, aliudque ejus generis deprehenditur, non est necessarium hoc opere pertractare*.

It will be seen that while the name sesamoid bone is of very early origin, the existence of the *fabellae*, the first sesamoids of the knee-joint to be talked about, if known was not recorded until 1555 and then not accurately, not indeed before Eustachius.

The second passage occurs in the *De usu partium corporis humani*† and runs as follows:

Nonum aliquis potuit hoc carpi os numerare, sed non est numeratum ab anatomicis, sicut nec aliquod eorum, quae σησαμοειδῆ, a sesami leguminis minuti specie, vocantur, quae multis manuum et pedum articulis natura ex abundanti velut auctarium circumponet securitatis gratia.

Another reference occurs in the *De usu partium*, Lib. III, Cap. viii, where Galen mentions the two metatarsial phalangeal sesamoids of the great toe. It runs as follows:

Quinetiam ei parti plantae, que magno digito praeposita est, duo [ossa *or* sesamoidea?] videntur subjecta esse inferiori regione, veluti sustentacula quaedam & stabilamenta, ut prius os magni digiti praedictae plantae parti, omnino jam in terram defixae conjugatur, undique, opinor, huic parti pedis natura suppeditante securitatem, ceu multum laboraturae propter praepositam sibi concavitatem & velut fornicem ossium. [Basel edition, 1561, *Prima Classis*, p. 259.]

Neither the Latin translation, nor the original Greek [Basel edition, 1538, p. 397], use the term sesamoid. But the word *ossa* seems omitted. It is supplied in the French translation (*De l'usage des parties du corps humain*, Paris, 1659, p. 120).

Now it is probable that these were the special sesamoid bones of the Greek anatomists, and they are bean-like rather than irregular or even lens-shaped. It is difficult to find a resemblance in them to the seed of the existing forms of sesame, as the reader will see by examining our Plate I, Fig. 2.

It will be noted that these passages from the real writings of Galen make the comparison with the sesame directly and not with its seed. It is otherwise in our third passage taken from the section *De anatomia oris* of the *Liber de anatomia virorum* ascribed to Galen‡:

Veniunt autem duo musculi ei ab additamentis sesaminis, id est quibusdam parvis ossiculis ad modum seminis sesami, et sunt supplementa spondylorum colli.

Thus in the earliest traces we have of sesamoids the name seems to have stood for any small bone of no recognised importance, which in some way was like sesame, a small species of pulse. The earlier references may, the later does indicate com-

* *Galeni Librorum Pars Quinta*, Basileae, 1538, p. 727. We can hardly expect that Galen would have anticipated the great evolutionary importance which the "sesamoidea gutturis" would attain to in later ages!

† Liber II, Cap. xii, *Galeni Librorum Prima Classis*, Basileae, 1561, p. 248.

‡ Galen's *Works*, Basileae, 1561, p. 49 c.

parison to the seed of this plant. These passages have been followed by later anatomists, and in each case we are told that certain bones are termed sesamoids because they resemble the seed of the sesame. What they themselves meant by "sesame," or what the Greeks understood by sesame in Galen's time, seems rarely to have entered the minds of these writers! Through the courtesy of the Director of Kew Gardens we have received samples of *Sesamum indicum* or *orientale*, the sesame as at present known in the East. It is almost impossible to believe that the Greek anatomists, who at least were familiar with the sesamoids "ex digitis," could have called these after their resemblance to the seed of sesame as now known*.

* The seed is crescent rather than lentil-shaped, and extremely small; only its coloration is in its favour. The most obvious sesamoids for the Greeks were the sesamoids of the big toe (see our Plate I, Fig. 2) and there is nothing really comparable between the seed and these bones. In size and shape, if less in coloration, they are remarkably like the seeds of *Ricinus*. We ventured to put our difficulties before the Director of the Royal Gardens, Kew, and he has most kindly sent us the following memorandum by Dr O. Stapf. It seems more than ever probable from this that Galen in his account of the plants was not speaking from personal experience, but following Theophrastus and Dioscorides. If the seed of the sesame was known to Galen he appears to have had rather a poor imagination for comparisons. Perhaps he was familiar with a variety having a much larger seed. The comparison would be excellent had the sesamoids first observed been the *lunulae* of the squirrel.

Dr Stapf's *Memorandum on Sesamum and Sesamoids.*

1. I have no doubt that the *SESAME* of Galen was *Sesamum indicum*. The Greeks knew it as far back as the days of Herodotus, and the Greeks of Asia Minor were probably quite familiar with it.

2. The history of the plant or drug mentioned by Greek writers under the name Σησαμοειδές has been shrouded in mystery from the earliest times. We meet with it first in one of the few fragments which have been preserved to us from the writings of Diocles of Karystos, the most prominent physician of the fourth century B.C., the "sectator Hippocratis quem Athenienses iuniorem Hippocratem vocarunt" (see Wellmann in *Festgabe f. Franz Susemihl*, 1898, S. 23). Διοκλῆς, says Erotianos in his Glossary of Hippocrates, *οὕτω φησὶ καλεῖσθαι τὸν ἐν Ἀντικύρᾳ ἑλλέβορον—ἕτεροι δὲ πόαν τινὰ ἑτέραν* (Wellmann, *l.c.* 26). Thus it seems the people of Anticyra, a town in Phokis reputed for its Hellebore cures, used the word as a synonym of ἑλλέβορος, whilst others applied it to another herb. Theophrastus (about 370—285 B.C.) somewhat later speaks of the fruit of the sesamoid Hellebore being administered by the people of Anticyra, adding in the way of explanation that the fruit is similar to sesame (*οἱ ἐν Ἀντικύρᾳ τοῦ σησαμώδους ἑλλεβόρου διδόασιν ὅτι ὁ καρπὸς ὁμοῖος σησάμῳ*, ed. Loeb, *Class. Libr.* with transl. by A. Hort, ii. 288); whilst in another place (*l.c.* 260) he says that the fruit of the Hellebore which at Anticyra is used as a purge contains the well-known *σησαμώδης*. Here then the conception of "sesamoides" as a part of Hellebore is quite unequivocal.

Pliny refers in two places to Sesamoides; but his account is confused. In the 22nd book, chap. 25, he introduces it after *Sesame* and contrasts it with it. Sesamoides, he says, has its name because it resembles Sesame, but its grain is bitter, its leaves are smaller and it grows in gravelly ground; but there is also another kind which grows in Anticyra and is called Anticyron by some and this has leaves like those of groundsel. The seeds of both are taken as a purgative and the latter kind has white Hellebore root added to enhance its action. He once more reverts to "Sesamoides" in the 25th book, chap. 5, where he says that the people of the island of Anticyra add Sesamoides to their Hellebore (the black Hellebore is meant) to render it safe to take. There are thus two kinds of Sesamoides indicated, just as in Diocles, the Anticyrian and another. The Anticyrian, however, is not any longer a Hellebore or part of a Hellebore, but a different plant with leaves like those of Groundsel, whilst the other is also a distinct plant but of a different kind. If we now turn to Dioscorides who has largely drawn on the same sources as Pliny, we find the two "Sesamoides" still more sharply divided as *μέγας* (lib. iv, chap. 149) and *μικρόν* (lib. iv, chap. 163; Wellmann, *Dioscor.* pp. 292 and 309). It is the greater Sesamoides that stands for Hellebore in Anticyra and is added to the white Hellebore in preparing the purgative. It is described as resembling Groundsel (*ἠριγέρων*) or Peganum, and as having large leaves, white flowers, a slender inefficacious root and bitter seeds similar to those of sesamum. The alternative Groundsel or Peganum is puzzling. It can clearly not be similar to both, but from Wellmann's annotations it appears

The natural thing appears to be to consult Galen himself as to what was the nature of the plant he termed sesame*. Galen in several passages refers to the sesame or to plants of a sesamoid nature. We will give here the references we have been able to discover. In his *De simplicum medicamentorum facultatibus*† we read:

Sesamon. Sesamon non panem in se continet viscosum et pingue; quare emplasticum est & emolliens ac modice calidum. Ejusdem facultatis est, quod ex eo conficitur oleum; et herbae quoque decoctum similem vim obtinet.

This is immediately followed by a description of the sesamoid plants‡.

Sesamoides. Sesamoides magnum, quod & anticyricos helleboros appellatum est, propterea videlicet quia semen eius ut helleborus purget; hoc & reliqua facultate helleboro simile est. Nam & in abstergendo, excalfaciendo, desiccandoque, similem illi vim obtinet.

Sesamoides album. Sesamoides albi semen nonnullum item acrem qualitatem continet; sed multum amarum est; excalfacit itaque, rumpit, extergit.

In Liber I, Cap. xxx, "De Sesamis et Erysimo" of the *De alimentorum facultatibus Libri tres*:

Sesami semen pingue est ideoque repositum celerrime fit oleosum; quamobrem eos, qui ipso

that the words ἠριγέρωντι ἢ are missing in Oribasius' manuscript of Dioscorides, whilst the codices of Constantinople, Naples and Munich lack the words ἢ πηγάνῳ. The latter reading is supported by the figure in the Constantinopolitan Codex, which represents faithfully a specimen of *Senecio lividus*. This figure is very like that of ἠριγέρων in the same code, the latter being unmistakably *Senecio vulgaris*. Compared with this *S. lividus* has actually larger leaves, whilst it shares with it the white pappus (flower) and slender inefficacious root; on the other hand it is difficult to see where the resemblance of the "seeds" (achenes) with Sesame seeds comes in. Nor are the achenes of either *Senecio* likely to be physiologically active.

The lesser Sesamoides (also called White Sesamoides or Wild Sesam) is said to have stems a span high, leaves like those of κορωνόπους, but rougher and smaller and to bear on the tips of the stems heads of purplish flowers with a white centre and in them bitter yellowish seeds like sesam; the root is slender and the plant grows in rough places. Of this too there is a fair figure in the Constantinopolitan Codex which can be identified with *Dorycnium hirsutum*, whilst κορωνόπους with which it is compared is from the table in the same Codex *Lotus arnithopodioides*. The representation of the leaves of the lesser *Sesamoides* and the *Coronopus* in the figures of the Code is indeed very similar. The white "centre" may mean the pale pods, whilst the seeds might very roughly be compared with those of sesame. As in the case of ἠριγέρων it is difficult to understand how small doses of the seeds, mixed with honey, as prescribed, could have been very effective. At the same time it is true that *Dorycnium hirsutum* was at one time in use as Herba Loti anti-haemorrhoidalis. Thus the plant was not unknown in the old herbalists' shops; but it long ago disappeared from the market.

To summarize briefly, the name *Sesamoides* designated early different plants or drugs. One of them was identical with the Hellebore of Anticyra or at least with its fruit. The others were entirely different species, in Pliny's and Dioscorides' times *Senecio lividus* and *Dorycnium hirsutum*. As there is no doubt that the (black) Hellebore of Anticyra was *Helleborus cyclophyllus*, the comparison of its fruit (follicles) with that of Sesame was not very far fetched, however little the two plants have in common in other respects.

To me the question of *Sesamoides*, whether magnum or parvum, seems to be irrelevant as far as the problem of the sesamoid bones is concerned. The sesamoid bones of Galen were no doubt in shape similar to the seeds of *Sesamum indicum*, hence "sesamoid," quâ forma, whilst the latter were similar to the seeds of *Ricinus* in yielding a fat oil, that is "sesamoid," quâ usu. Their relation to the sesamoid bones on one side and the plants called Sesamoides on the other is therefore merely that of "tertium comparationis."

* Pfitzner, S. 520, appears to confuse the sesame with the two sesamoides.

† Basileae, 1561, Liber VIII, p. 122.

‡ *Opera*, Secunda Classis, Basileae, 1561, p. 29, Greek Edition, Basileae, 1538, p. 317.

vescuntur celeriter implet; stomachumque subvertit, ac tarde concoquitur, pingueque corpori praebet alimentum. Liquet ergo, quod ventriculi partibus vigorem ac robur addere nequit, quemadmodum neque aliud quodvis pingue. Est autem crassi succi, ideoque non propere pervadit. Ipso aut solo non admodum vescuntur, sed cum melle crudo quas vulgo *σισμίδας* vocant effingentes. Panibus etiam inspergitur*. Porro, quemadmodum milio panicum (quod etiam *μελίνης* diximus appellari) adsimile quidem quodammodo est, verum undequaque deterius; ad eundem modum & sesamo erysimum corporis substantia quodam pacto est affine, sed in cibo est insuavius, corporique alimentum parcius exhibit, praedictoque omnino est deterius. Caeterum utrumque temperamento est calidum, ob eamque causam sitem etiam excitat.

A further reference to Sesamoides occurs in Galen's commentary on Hippocrates' book *De victus ratione in morbis acutis*†.

Sesamoides superiorem ventrem purgat sesquidrachmae pondere in oxymelete tritum et potui datum. Miscetur autem veratris ita ut tertiam efficiat partem. Nempe id hoc pacto minus suffocat.

Lastly in Galen's book *De substitutis medecinis*‡ we read that the substitute:

"pro sesamo" is *Linisemen* (λινόσπορος), i.e. linseed, "pro sesamoide" is *Amaranthi expressio* (*ἀμαράντου πίεσμα*), "pro Linisemine" is *Fabae medulla.*

There can be small doubt that for Galen sesame was an emollient, *Sesamoides magnum* a purgative, and *Sesamoides album* an emetic. The plants and their seeds have so little real resemblance that it is not impossible that the oil-giving property was the source of their common name§.

* Sesame cakes or puddings were known to Xenophon and Aristophanes.

† *Opera*, Basileae, 1561, Septima Classis, p. 296, com. III, cap. 123.

‡ *Opera*, Basileae, 1561, Quinta Classis, pp. 173—4. It is clear that Galen distinguished linseed definitely from sesame, although he notes the emollient properties of sesame. See also the *De alimentorum facultatibus*, Liber I, cap. xxxii.

§ On p. 295 of the *Rariorum Plantarum Historia* 1601 of Carolus Clusius are given cuts of *Sesamoides magnum Salmantic.* and *Sesamoides parvum Salmantic.*, which indicate what was the type of plant denoted by these names in the latter half of the sixteenth century. Dr Stapf has kindly examined Clusius' figures for us. He says they are excellent figures, that the *Sesamoides magnum* is *Silene Otites*, Sm. and that *Astrocarpus Clusii*, J. Gay, is drawn for *Sesamoides parvum*. He has further found figures of the two *Sesamoides* in the *Codex Vindobonensis* (± a. 500 A.D.). *Senecio lividus* is figured as *S. magnum* and *Dorycnium hirsutum* as *S. minus*. Thus in Galen's time, in 500 A.D. and at the end of the sixteenth century we find different plants named as Sesamoides, and neither of the sets of figures here referred to are in the least helpful in interpreting what Galen understood by these plants, nor do the seeds of these plants throw any additional light on the adoption of the word by the anatomists—they are not more like a sesamoid bone than sesame itself. Dr Stapf throws light on the matter in the following note: "I have also gathered much information with respect to the attempts made by the writers of the renaissance to explain the two Sesamoides of Dioscorides, and particularly their reference to species of *Reseda*. I thought there might have been some tradition linking up their interpretation with the classic writers. So far as the Arabic literature is available to me there does not seem to be any such connection, but I have not seen Ibn Baithar who I understand quotes from an Arab author. As far as I can see the interpretations of the herbalists were nothing but more or less ingenious guesses. The *Reseda* theory can be traced back to Luca Ghini who laid out and was in charge of the Botanic Garden at Pisa from 1544—1555. Amatus Lusitanicus (1554) also speaks of having been told by a nobleman of Padua that the *Sesamoides maius* of Dioscorides has been found in Italy, but does not say what it is. Amatus Lusitanicus was a Salamanca student, and it is just possible that he was instrumental in starting the *Reseda* theory in his University where Clusius seems to have picked it up subsequently. There was no prominent botanist at Salamanca at the time of Clusius' short visit (1565)—we know in fact that he had a very low opinion of the 'viri docti' of the Spain which he knew—and no importance can be attached to his interpretation of the two Sesamoides."

There appears to be some probability that *Sesamoides magnum* was *Ricinus communis**, the castor-oil plant, although Candolle identifies *Ricinus* with *Sesamoides parvum* or *album*. Sprengel states that *Sesamoides* was a kind of *Reseda*. The seeds of *Reseda*, which we have examined, are hardly comparable with sesamoid bones, but those of the castor-oil plant might well pass muster. This would suggest that the bones were called from a sesamoid plant and not from the sesame itself, did not Galen so clearly distinguish between *sesamum* and *sesamoides* and associate the bones with the former.

Both plants were largely grown in Egypt for the sake of their oils (Dioscorides, Sprengel, Lib. I, Cap. xxviii and Lib. II, Cap. cxxi), but we are informed that the root is Arabic and not Egyptian. The familiar words "Open Sesame" of the *Arabian Nights* might certainly suggest that the root SM SM was connected with a purgative. Cameron's *Arabic-English Vocabulary* gives *simsim* as sesame, millet. We have already seen that Galen associates its method of use with *panicum*. Owing to the courtesy of the Director of the Royal Gardens, Kew, we have been able to examine seeds of four kinds of millet†, of which three might pass for reasonable comparatives, but the fourth, great millet or guinea corn, is less plausible. We cannot discover, however, that *simsim* was used in early Arabian for millet. On the contrary, Socrates Spiro (*An Arabic-English Vocabulary*, London, 1895) renders it as *coriander* seed, probably following Freytag's *Lexicon Arabico-Latinum*, Halle, 1830, who gives *simsim* = *semen coriander*.

Thus far these references might only signify a modern use of sesamum for coriander, but on consulting Lane's great *Arabic Lexicon* (p. 1420) we find that the sense of coriander is very ancient indeed. It occurs in Fairuzabadi, 1329–1414, and in Ibn Sīda's *Al-Qāmūs* of 1066. The passage from Lane cited below does not throw very much light on the matter, but indicates that either the Arabian writers copied the Greeks or the Greeks the Arabians, which alternative considering the origin of the word is, perhaps, the more likely.

["Sesame; sesamum orientale" of Linn.; applied in the present day to the plant and its grain;] "a well-known grain"; (Mṣb;) it is called in Pers. كُنْجُدْ; (MA, KL;) i.q. جُلْجُلَانٌ, (M, Ḳ,) said by AḤn to be "abundant in the Saráh (السَّرَاة), and El-Yemen," and to be "white"; (M;) [by this is evidently here meant "sesame," or the "grain thereof," or "both"; though it also signifies the "fruit of the coriander"; for otherwise, the most commonly-known meaning of سِمْسِمٌ would be unmentioned in the M;] the "grain of the حَلٌّ"; [i.e. the "grain from which the oil called حَلٌّ is expressed";] (Ṣ, Ḳ; [by the author of the latter of which, this was evidently understood to be different from the جُلْجُلَان, which is mentioned by him after the description of properties here following;]) "it is glutinous, corruptive to the stomach and the mouth; but is rendered good by honey; and when it is digested, it fattens; and the washing of the hair with the water in which it has been cooked lengthens and improves it: the wild sort thereof is known by the name of جَلْجَبَهْنَك," (Ḳ, TA,) thus, with fet-ḥ to the ج and ب and

* *Sesamum silvestre*, of Pliny, which at any rate would seem to indicate that *sesamum* could be used for *sesamoides*.

† *Letaria italica* (Italian Millet), *Panicum miliaceum* (Indian Millet), *Pennisetum typhoideum* (African Millet), and *Sorghum vulgare* (Great Millet).

ـ, and sukoon to the ل and ن, [but written in the CḲ جَلْبَهْنَكْ,] a Pers. word, [originally جلْبَهْنَكْ,] arabicized; (TA;) "its action is nearly like that of the خَرْبَق [or "hellebore"]; and sometimes from half a drachm to a drachm is administered to him who is affected with palsy, and he is cured thereby, (Ḳ, TA,) speedily; (TA;) but a drachm thereof is dangerous, (Ḳ, TA,) in a great degree."

On the other hand the ribbed fruit of *Coriandrum sativum*, a sample of which we owe to the courtesy of the Director of the Royal Botanic Gardens, does not seem very suggestive of the sesamoid bones of the human digits*. We are thus thrown back on the following problems:

(*a*) Galen distinguishes *Sesamum* from *Sesamoides magnum*, and directly tells us that the sesamoid bones are called from the seeds of the former. But the seeds of the latter if identical with *Ricinus* are far more like sesamoid bones†.

(*b*) It is conceivable that the sesamoid bones were given their name long before Galen's day, and that like his successors he simply repeated a tag with regard to them—a commonplace of the Greek anatomists. The name being Arabic (or even possibly Persian) it is conceivable that it was applied to the sesamoid bones before it reached the Greeks.

Johannes Riolanus has a section *De sesamoideis ossiculis*‡ in which he refers to the sesamoids of hand and foot, but not to the *fabellae*. He especially notices the sesamoid bone of the great toe which he tells us the Magi called *Albadaran*, it was of the magnitude of a small pulse (*cicer*); it could not perish, and from it as from a seed the whole body would regenerate on the Day of Judgment. This tale also occurs in Cornelius Agrippa's *Liber de occulta Philosophia*, Cap. xx, with the Hebrews substituted for the Magi:

Est in humano corpore os quoddam minimum, quod Hebraei *Luz* vocant magnitudine ciceris mundati, quod nulli corruptioni obnoxium, ne igne quidem vincitur, sed semper conservatur illaesum, ex quo veluti planta ex semine, in resurrectione mortuorum nostrum corpus animale repullulabit§.

* Memorandum of Dr O. Stapf in reply to a letter from K. Pearson with regard to Arabic use of SM SM for coriander (Royal Botanic Gardens, Kew, 10 May 1920):

I cannot find anything in our books containing Arabic plant names that would suggest that SM SM (semsem) was ever used in Arabic for *Coriandrum sativum*. The latter is Kuzbarah in the Arabic of Egypt, Syria and India. The Greeks knew it well and the *Codex Vindobonensis* (c. 500 A.D.) of Dioscorides contains a very good figure of it. It is practically certain that the Greeks took the word sesame from the Semitic peoples of Asia, but they would hardly have mistaken it for *Coriandrum*, which has seeds (or rather fruits) of a very different appearance and taste and yields practically no fat oil. Dioscorides deals rather fully with its properties, but there is no direct allusion to a similarity with sesame, nor do his observations suggest it. Prof. Pearson may judge for himself whether there is any similarity between *Coriandrum* fruits and the sesamoid bones of man.

† While Galen uses *σησαμίς* as equal to sesame there is some evidence that it was also used for *σησαμοειδὲς μέγα*, see Liddell and Scott's *Greek-English Lexicon* who quote Dioscorides, Noth. 4, 152.

‡ *Osteologia*, Paris, 1614, p. 483. In this work Jacob Sylvius' commentary on Galen's *De ossibus* is given and on p. 477 we read: "In pollicibus et primis digitorum articulis & in simiarum poplite sesamoidea plura, pauciora extensionem immodicam, et que luxationem minaretur, prohibentia." Sylvius (1478–1555) antedates Vesalius (b. 1514); he clearly knew of the *fabellae* in apes, and may well have started Vesalius on the discovery of those in man.

§ See also Hyrtl, *Das Arabische und Hebräische in der Anatomie*, S. 165 *et seq.* Further Joannis Munnios, *De re anatomica*, Treves, 1697, p. 213.

Thus the seedlike character of the sesamoid bone is preserved in both the Arabian *Albadaran* and the Hebrew *Luz* with their mythical associations*. It would be of great interest to know the age of these myths. If the Semites before our epoch had discovered the sesamoid bones and attached these legends to them, there might be no weight at all in Galen's explanation of sesamoid as like to the sesame-seed—the whole idea and terminology may be Semite and not Greek, and Galen may have become acquainted with it in Asia Minor.

(3) *History of the Sesamoids of the Knee-Joint. First Period—that of isolated Records of Anatomical Writers.*

From the Greeks down through the Middle Ages the study of anatomy was essentially the study of human anatomy; references to dogs or apes are few and far between and then very superficial in their associations†. Probably Jacob Sylvius' reference to the *fabellae* in apes is the first note we have of the existence of sesamoids of the knee-joint (see our footnote ‡, p. 144). There is no evidence that the Greeks or Arabs were acquainted with them. Vesalius was the first to draw attention to their existence in man. In his *De corporis humani fabrica*, Lib. I, Cap. xxviii (Basel edition, 1555, p. 153, Leyden edition, 1725, Tom. I, p. 107) he writes:

Deinde bina recensebit in poplite occurrentia ossicula, quae duorum primorum pedem moventium musculorum innascuntur capitibus, mox in illorum ex femoris osse principio. Ossicula enim haec laevi sua et lubrica superficie, qua extra musculorum substantiam prominent, elatiorum spectant sedem posterioris regionis inferiorum femoris capitum, quorum impetum illa solvunt & sustinent, hoc privatim sibi vindicantia, quod musculorum exortibus, non vero aliorum fere omnium ossiculorum sesamo comparatorum modo tendinibus innectantur.

It is curious that in this passage Vesalius speaks as though the *fabellae* were constant in man, and this although as we shall see the mesial *fabella* is very rare indeed, and the lateral *fabella* does not exist in more than seven to ten per cent. of

* The word *Luz* signifies a tree producing a small nut or the nut itself. In Genesis xxx. 37 it occurs and is rendered in the English version by "almond." Rashi, prince of commentators, explains it as chestnut, but it has been interpreted of any tree bearing small nuts, or almonds. Its secondary meaning in Rabbinic literature is a bone or cartilage resembling the almond said to be in the vertebral column of man. It was quite possibly the word used by Jewish anatomists for the sesamoids.

Commenting on the words "And the almond tree shall blossom" we read in the *Midrash* (Ecclesiastes xii. 5: see also Genesis, Ralba § 28, and Leviticus, Ralba § 18) the following:

R. Levi says, This is the "Luz" (i.e. almond-shaped bone) of the spinal column. Hadrian once asked R. Joshua b. Hananiah from which part of the body of man shall he blossom forth in the life of the future? And he answered "From the 'Luz' of the spinal column." He then said: How so? They then brought such a "Luz," put it in water, and it did not dissolve finally; into fire, and it was not burnt; into the mill, and it was not ground; they put it on a block and beat upon it with a hammer, and the block was split and the hammer broke, and all was of no avail [to destroy it].

The *Midrash Ralba* is a very old collection of Hebrew lore, parts of which are probably antecedent to the Talmud recension. We have to thank our colleague Dr Hermann Gollancz for the above references. We think it probable, however, that Jewish anatomy was non-existent before the appearance of the Greek followed by the Arabian medical schools.

† The first *Simiae Osteologia* is due to the elder Riolanus and edited by the younger in the *Osteologia*, Paris, 1614. Caput xxvi is entitled: *De sesamoideis* and we read (p. 535), "Duo ossicula magnitudine ciceris supra utrumque tuberculum femoris in origine gemellorum reperiuntur."

cadavers. It is further noteworthy that he does not state that he has definitely in a given cadaver discovered sesamoids in the heads of *M. gastrocnemius*. He speaks as he might have spoken of the sesamoid of the great toe with fifteen centuries of history behind it. There is no statement of a new discovery, nor that the occurrence is not universal. He writes as he might write of the patella itself! We have not succeeded in finding any reference to the *fabellae* in Vesalius' work of 1543*.

The next reference we have been able to find to the *fabellae* occurs in Eustachius' *Opuscula Anatomica* issued in 1564, but the permission to print which dates from 1562. We have quoted the relevant passage in the footnote to our p. 138. We seem on reading Eustachius' words to be in the atmosphere of a man who had really examined the matter in many cases, and we are almost inclined to wonder whether, if Eustachius had been able to get his works printed when they were written†, Vesalius would have been the reputed discoverer of the *fabellae*. At any rate Eustachius' statements are correct, and here as elsewhere he not only criticises Vesalius, but does so legitimately‡.

Fallopius in his *Observationes Anatomicae* of 1561 (fol. 113) says that the *fabellae* are frequent in apes and that he has seen them, but that only Vesalius has seen them in man.

At the end of the 16th and beginning of the 17th centuries we have Caspar Bauhin. He has in his works a good deal about sesamoid bones. In his *De corporis humani fabrica* (Basel, 1590) there are references to the sesamoids on pp. 356, 372, 395, but we have failed to examine a copy, and cannot say whether he refers to the *fabellae*, but he probably does. In his *Anatome* (Basel, 1597), after enumerating (p. 259) *sesamina duodecim* of hand and foot, he continues "Hic addere possimus sesamina duo in poplite§." Again in his *Theatrum Anatomicum* (Frankfurt, 1603) he has considerable references to sesamoid bones (pp. 1169—1171, 1207, 1277—80), and for the *fabellae* (p. 1278). He gives, Tab. xv, Fig. 11, diagrams of the sesamoid bones of the hand and foot, even the very small ones, but not of the *fabellae*. This leads us to believe that he had never seen the latter. Confirmation for this view arises from the fact that what he says is principally paste-and-scissors-work from Galen and Vesalius. Speaking generally of sesamoids in his Cap. xxix, he does say, however, "solida sunt, rotunda, aliquantulum depressa, aliquando cartilaginosa, aliquando ossea." This may be original, but is most probably due to an earlier writer in a passage which has escaped us.

Lastly in Bauhin's *Institutiones anatomicae*, Basel, 1609, he refers to sesamoids in each head of origin of *M. gastrocnemius* (p. 247).

* The first edition of his anatomy which appeared at Basle in this year.

† As to Eustachius' difficulties in publication see *Biographie Universelle*, Paris, 1815, Tom xiii, p. 534.

‡ Eustachius' writings seem to indicate some deep ground for dislike of Vesalius. Can it be that unpublished discoveries of Eustachius were talked about and made use of by Vesalius in his great work? At any rate Eustachius clearly knew more about the *fabellae* in man than Vesalius did.

§ This statement of the *fabellae* being *in poplite* must not be taken as referring to the popliteal muscle's origin, but simply when met with in the early anatomists as indicating the neighbourhood of the popliteal surface.

It will be clear that Bauhin does little more than confirm the error of Vesalius by speaking as if the *fabellae* were constant in man and occurred equally in both heads of origin of *M. gastrocnemius*. To this nearly universal overlooking of Eustachius, and to the parrot repetition of the views of Vesalius we owe most of the obscurity about the *fabellae* which followed their discovery. In Riolanus, the Elder, *Osteologia*, Paris, 1614 (pp. 69, 121, 123, etc. of the *Isagogica de ossibus Tractatio*), we have not been able to find references to the *fabellae*. There is only the passage already cited (see our p. 144, footnote ‡) with regard to the "in simiarum poplite sesamoidea" in Sylvius' commentary on Galen's *De ossibus* (p. 475) and the reference to the *fabellae* in the apes in the *Simiae Osteologia*, p. 535. The younger Riolanus in his *Anthropographia*, Liber v, Paris, 1650, Cap. xliii, p. 333, writes:

Externus gastrocnimius (sic!) ab externo condylo nascitur...singulis eorum principiis primus Vesalius observavit singula ossiculis sesamena apposita, saepius alterum deficit, authore Fallopio. Ut laevi lubricaque sua superficie, dum ossi & musculo interjecta sunt, impediant ne in cruris directione musculi atterantur aut ab osse laedantur.

The direct attribution to Vesalius is here made and Fallopius is cited, not Eustachius, as casting some doubt on the constant appearance of at least *one* of the *fabellae*.

Gorraeus in 1601, Paaw, 1615, and Laurentius in the same year (also in 1628) confine their attention to the sesamoids of hand and foot*.

In 1632 we find published at Frankfurt Adrian Spiegel's *De humani corporis fabrica*, and on p. 78 under *Gastrocnemius externus* we read:

Huius musculi duobus capitibus, quemadmodum Vesalius notat non procul ab exortu sesamoidea ossicula tributa sunt.

In the *Tabulae anatomicae* of J. Casserius, edited by D. Bucretius (Frankfurt, 1632), which accompany Spiegel's work, Tabella XXXVIII, Fig. 11, professes to give a diagram of *gastrocnemius*. The figure represents the lower limb from knee downwards, the muscle appears to be cut at the two points of insertion and turned back from the knee, and S, S, two sesamoids, in it appear to rest on the ground. The figure is somewhat obscure, but it is the first professed representation of the *fabellae*, probably largely a product of the imagination (see our Plate I, Fig. 1). Dominicus de Marchetti returns to the healthy statement of his own personal experience. He rightly holds that, while he himself has not found the *fabellae* in the many bodies he has dissected, he cannot deny their existence. But that they are certainly not found in all bodies as Vesalius and Riolanus have asserted†.

Thomas Bartolinus in his *Anatomia reformata*, Hagae Com., 1655, returns very nearly to the Vesalius legend. In Liber IV, Cap. xxii, *De ossibus sesamoideis*, p. 528, we read:

Item bina ossicula in poplite juxta os femoris, musculorum (duorum priorum pedum moventium) non tendinibus, sed principiis uniata, quae in senibus reperiunter, & animalibus siccis, ut cervis, canibus et leporibus.

* There is also no reference that we have been able to discover in the Sylvius of 1635.

† *Compendium anatomicum*, Patavii, 1652, p. 167, and *Anatomia*, Padua, 1652, p. 153.

How these mediaeval anatomists used each others' very words! There is, however, the qualifying phrase soon to become a part of the myth, that the *fabellae* occur in the aged. Further the deer and the hare are added to the list, man, ape and dog, that own *fabellae*. This reference to the aged is emphasised again in the edition printed at Leyden, 1686, p. 756.

Diemerbroeck, *Anatome corporis humani*, Utrecht, 1672, p. 944, is content to return to the complete Vesalius legend, and finds two *fabellae* and these always occurring. Stephen Blancard's *Anatomia reformata*, Leyden, 1695, p. 729, and Gerhardus Blasius' *Anatome hominis*, Amsterdam, 1673, p. 62, give also the complete Vesalian story. But the 17th century is not to close without one or two cases of men who report what they have actually seen. Thus Schrader in 1674 reports that he had found a *fabella* in the anatomy of a *cadaver virile**, and that sturdy old surgeon-anatomist William Cowper, in his *Myotomia reformata*, London, 1694, p. 206, states first the opinion of Vesalius and Riolanus:

> that in the two beginnings of this muscle there are two *Ossicula Sesamoidea*, which we must acknowledge with Marchetti have hitherto escaped our Observation though it is likely it may be so in Aged Bodies, as it appeared in a subject I lately dissected on one side only.

We might almost say that from the reported discovery of Vesalius, only Eustachius, Schrader and Cowper had really seen a *fabella* up to the end of the 16th century.

Even in the case of Cowper we are to find later that the dogmas as to the appearance of *fabellae* in the aged, and as to their object, are accepted. Thus in his *The Anatomy of Human Bodies*, 2nd edition, Leyden, 1737, T. 103, F. 2, under *Ossa sesamoidea*, we read:

> In some Bodies especially Aged, we find Two Ossa Sesamoidea on the Superior Parts of the Two Lower Heads of the Thighbone D, E: The Office of which is to Defend the Bending Tendons of the Tibia from too great a Collision on those Heads of the Bone which they would else be subject to.

At the very end of the century Joannis Munnics, in his *De re anatomica*, Treves, 1697, p. 213, sums up the pure Vesalian doctrine:

> In censu quoque ossiculorum sesamoidum poni debent duo, quae in poplite inferioribus femoris appendicibus apponuntur, duorum priorum pedem moventium musculorum principiis inhaerent.

For him all sesamoids also start as cartilaginous and become solid with the progress of age.

Thus far the history of the *fabellae* has been fully representative of an oft-occurring scientific tragedy. Authority dominates inquiry and dogma fills up the space with its story of how a non-existing phenomenon (the constant appearance of both *fabellae* in man) arises and what purpose it serves. The time was therefore ripe for a little comedy, and very early in the 18th century we find this in a double fashion, the Trew-Heister incident. In 1715 C. J. Trew wrote an Altorf Dissertation entitled *De chylosi foetus in utero*. Tabella II contains a large but crude drawing of a lateral *fabella* resting in a cavity on the condyle. There is a separate drawing

* *Observationes anatomico-medicae*, Amsterdam, 1674, p. 193.

of the sesamoid itself (FVI) (see our Plate II, Fig. 4). Trew's thesis was republished by Haller, *Disputationum anatomicarum selectarum*, Volumen v, Göttingen, 1750, p. 475. Figs. v and vi should give the *fabella*, but the plate is missing in the two copies of Haller that we have examined, possibly the plate never existed in Haller. The description to Fig. v, C is "C est os sesamoideum insidens cavitati condyli exterioris in omnibus femoribus conspicuae." This sesamoid (if only a lateral *fabella*), and the cavity conspicuous in all femora, were something extraordinary, and Trew seems to have thought so, for he writes as follows:

Observatio II. Ossicula sesamoidea in femore ac minimo manus digito.

Cum aliquando occupatus eram in ossibus humanis a carne sordibusque mundandis, atque accurate in praeparatione artuum in numerum ossiculorum inquisiverim, cum primis deprehendi numerum sesamoideorum auctum. Primum mihi antea nondum cognitum se offerebat in femoris parte inferiori, ubi cum tibia articulatur, in condylo exteriori: ubi in fovea quadam deprehendi ossiculum rotundum figura & magnitudine Fig. VI. quod instar parvae patellae loco C. Fig. v. insidebat in utroque femore. Putabam tunc temporis, id extraordinarium fuisse; dum vero illud postmodum inquisivi in aliis femoribus, luculenter semper deprehendi foveam in loco, cui insidet, ita ut deinceps ulterius illius existentiam perscrutari excitarer; quod & mihi nunquam non in tot, quotquot deinceps cadavera inspicere poteram, reperire contigit; siquidem brevi in quatuor illud inveni, atque etiam forsan cuilibet inquirenti patebit.

It will be observed that Trew does not say that he has ever discovered a mesial *fabella*. But such a big discovery was too much for Trew's anatomical instructor, Lawrence Heister. In the first edition of his *Compendium anatomicum*, Nürnberg, 1719, he magnified Trew's lateral *fabella* into two *fabellae*, he forgot to mention Trew's name and he proclaimed the discovery of two new bones! One might at least have supposed an acquaintance with the text-books of his day. The English translation of Heister's work, *The Compendium of Anatomy*, London, 1752, appears to have been made from the earliest edition of Heister, and does not contain the long apologetic note of the Nürnberg edition of 1732 (Vol. II, p. 47) or the Amsterdam edition of 1748 (Vol. II, p. 48). We read on p. 65:

There remain now to be considered, the *ossa sesamoidea*. These are small bones, and are most conspicuous in old subjects; they somewhat resemble the seeds of the sesamum, whence they have their name. Their most usual situation is...[details of sesamoids of hand and foot] one frequently in each external condyle of the *os femoris* (Tab. I. figs. 2, 3, 4)..... These are usually found in adults, or in elderly people: sometimes though more rarely there is one also in the internal condyle of the *os femoris*..... Upon the whole there are very rarely found more than sixteen of them often fewer. Those anatomists, therefore err greatly who reckon more than forty of them.....too many, however, have given in to this error.

All these bones, except those that are found in the condyles of the femur, adhere to the tendons of the muscles: those excepted, are connected to the origin of the muscles. Their size and shape are various and irregular: they are cartilaginous in young subjects, but they grow hard and boney by age: and it is therefore that they are so much the more easily found in old people. They serve as a kind of trochleae to the muscles and increase their power.

On p. 463 Heister gives an "Explanation of the Figures" (see our Plate II, Fig. 5):

Fig. 2. Represents the lower part of the *os femoris* with two sesamoid bones not described by other authors. A, The *os femoris*. B, The internal condyle. C, The external condyle. D, The larger of the two new sesamoide bones, situated in a considerable cavity in the external condyle. This, however, in some subjects, is much larger than it is represented here. E, The lesser of the

two new sesamoide bones, situated in the internal condyle. This is but very seldom met with in dissection, most subjects wanting it: but the other is seldom wanting.

Fig. 3. Represents the larger of these two sesamoide bones, separated from the cavity in the external condyle, in which it is generally lodged.

Fig. 4. Shews the lesser sesamoide bone, of the internal condyle, separate.

Apparently Heister was not left long to enjoy his triumph at the discovery of "two new sesamoide bones." In the long and somewhat abject note of 1732, he admits that the "rediscovery" was due to Trew; he cites a few authors of the past who did not refer to the *fabellae*, but he does not adequately explain why he had apparently overlooked Vesalius, Eustachius, Riolanus, Cowper, etc. Indeed "rediscovery" is hardly the right description for the *fabellae* in 1715, when W. Cheselden* could write in *The Anatomy of the Human Body*, 1713, pp. 22—23:

Ossa Sesamoidea...and sometimes one in the lower end of each Thigh-bone at the beginning of the *Plantaris* muscle. Their use is the same with the Patella.

A brief and pregnant sentence almost the most suggestive since that of Eustachius. Another remarkable point is that Heister should have "rediscovered" the suppositious influence of old age, one of the points the 17th century writers had much emphasised.

Gruber† throws complete discredit on Heister's statements and especially on his plate. He says that Heister has placed the *fabellae* where they do not usually exist, and apparently Gruber considers that if they occurred at all they must have been pathological osteomata. Yet Heister says that he has found them in four corpses, one after another, and that they can always be found except in youthful cadavers, where they are cartilaginous‡! We are by no means certain that Heister's drawing, as far as the lateral *fabella* is concerned, is not a rough copy of the crude diagram in Trew's Dissertation of 1715; both exhibit the lateral *fabella* resting in a hollow, not of the articular surface, but of the lateral condylar eminence in its proximal part. As for the mesial *fabella*, which Trew does not provide, Heister rests it, the femur being vertical, on the commencement of the popliteal area as it springs from the articular surface of the mesial condyle: see our Plate II, Fig. 5. We believe the whole drawing to be imaginary copied from no actual subject, and totally out of accordance with all skiagrams of the lateral *fabella* in man with which we are acquainted. Not one of the first three drawings, Casserius (1632), Trew (1715) and Heister (1717), appears to us to represent in any way the *fabella* as it exists in man. The comedy of Heister on the *fabella* might well be preserved in the history of science as an illustration of the process of concocting a text-book.

Saltzmann§ (*Decas observationum illustrium anatomicarum*, p. 6), Strassburg, 1725, reports that he found no *fabellae* on the condyles of a given cadaver. Bass (*Observationes anat.-chirurg.-med.*, p. 220), Halle, 1731, and Albinus (*Historia*

* He did not apparently return to the subject in his *Osteographia or the Anatomy of Bones*, London, 1733.

† Gruber, p. 11, footnote.

‡ *Academiae Cassareo-Leopoldinae Naturae Curiosorum Ephemerides Centuria* VI. Obs. XXIX. pp. 245—6, Ossa sesamoidea in femore...Francofurti et Lipsiae, 1717. See also *Centuria* VII. Obs. XXIII. pp. 49—51, 1719.

§ For fuller details of the following eight references: see Pfitzner's bibliography.

musculorum hominis and other works), 1734—1757, have no reference to the *fabellae*. Palfin (*Anatomie chirurgicale...du corps humain*, Vol. II, p. 159), 1726—1734, says they are not always to be found as Vesalius believed, while Winslow (*An Anatomical Exposition of the Structure of the Human Body*, London, 1749, and *Exposition anatomique de la structure du corps humain*, Paris, 1732) apparently considers they are always to be found, and in cartilage-lined hollows. George Thomson (*Anatomy of Human Bones*, London, 1734, p. 104) asserted the existence of both, which was the view taken by Kulmus in his *Tabulae anatomiae* (p. 62) of 1732 and supported by the authority of Haller in 1745. In such a sea of wearisome positive and negative assertions unaccompanied by personal examination, it is almost a comfort to meet such a bare statement as that of Drake in his *Anthropologia nova*, 3rd edition, 1750, p. 434:

> There are sometimes found two *ossa sesamoidea* in the two beginnings of the *Gastrocnemius externus* muscle, but they are rarely met with and only in aged Bodies.

He at least knew that they did not always occur or fail to occur, even if he supposed no difference in frequency of the pair. Still more comforting is Morgagni's statement (*Adversaria anatomica*, T. II, Leyden, 1723, p. 64*) that he had found the lateral *fabella* several times, but the mesial *fabella* only once. It goes at any rate a little way to balance the statements of Lieutaud (1742) that he had found *fabellae*, more often mesial than lateral, of Disdier (1745) that one usually exists in the groove behind the lateral condyle, of Lauth (1798) that he had never found any *fabellae*, and of Bertin (1783) that he had found two on each femur!

How strikingly barren these 18th century anatomists seem to have been, at least in this matter of the knee-joint! It is not till the very end of the century that with Sömmering and Peter Camper we get some progress in our problem, and we begin again to grasp the difference between big and little naturalists. We will take the former first because his contribution is less important than those of Camper.

Sömmering (*Vom Bau des menschlichen Körpers*, Leipzig, 1791, Bd. III, S. 295) recognised that a *fabella* is not infrequent in the lateral head of *M. gastrocnemius*. He continued to study the subject, however, and in his *Lehre von den Muskeln* of 1841, S. 347—351, he recognised that the lateral *fabella* is often only a hemisesamoid, i.e. cartilaginous, and that the mesial *fabella* is very rare, and when it does occur is far more likely to be a hemisesamoid than an orthosesamoid. Further like Cheselden (see our p. 150) he recognises a relation of the lateral *fabella* to *M. plantaris*:

> Der Ursprung des *M. plantaris* hängt mit der Ursprungsehne des lateralen *Gastrocnemius*-Kopfes, namentlich auch mit dessen Sehnenbeinchen eng zusammen.

We are clearly in touch with a man, who is observing and thinking on his own.

Peter Camper's first contribution occurs in his *Dissertatio de fractura patellae et olecrani*, Haag, 1789, translated as *Abhandlung von Bruch der Kniescheibe und*

* His words are: "Nos certe quod alterum eorum in nonnullis cadaveribus et in uno internum nominatim invenisse meminimus." They do not occur in the edition of 1714 but will be found in the Venice edition of 1782.

des Olecranon, reprinted in the German translation in the *Vermischte Schriften,* Lengen, 1801, S. 68, and Plate I, Fig. i. K. We read (see our Plate III, Fig. 6):

A, K, I, L die Sehne des äusseren Bauchs des Wadenmuskels (*Gemelli seu gastrocnemii externi*) worin sich ein Knöchelgen (*ossiculum sesamoideum*) f. g. findet welches man in den inneren Bauch derselben nie sieht. Es wird bei den Affen, den Orang ausgenommen, bei dem Hunde allzeit in beiden Portionen, auch beim Fuchse, bei der Katze, dem Igel, in dem Agu und allen Thieren gefunden.

The statements are too sweeping, but they are those of a man who is stating what he knows of his own observation, and are not the mere reiterations of his predecessors. We are beginning also to get a wider range of comparative observation—comparative anatomy is starting its career.

We have reproduced a sketch of Camper's diagram of the "Knöchelgen" near K on Plate III, it is far from an ideal representation, but it is a great advance on those of Casserius, Trew and Heister. We next turn to Camper's *Naturgeschichte des Orang-utangs,* Düsseldorf, 1791, S. 126, and note that Camper, after citing Eustachius' view about Galen and the *fabellae* in man, continues:

Hier muss ich aber Eustach aus eigener Erfahrung widersprechen, weil ich dieselben in sehr vielen Körpern sowohl von Männern, als Weibern, doch allein in äussern Kopfe des Wadenmuskels oder Gastrocnemius, gefunden habe. In meiner Sammlung habe ich eine grosse Menge derselben aufbewahrt. Am meisten wundere ich mich, dass Albin derselben gar nicht erwähnt, da Eustach sehr deutlich sagt, dass er diese Beinchen nicht selten bei Menschen, in Affen und Hunden aber immer gefunden habe. Nicht allein Eustach hat dieses wahrgenommen, sondern auch Coiter (59) und Sylvius dem Blasius (60) zufolge, kannten den Sitz derselben in den Affen sehr wohl, dass sie nämlich in beiden Köpfen der Gastrocnemii in den Affen, bisweilen in der Sehne des Peroneus Longus und Tibialis zu finden sein: dieses kann ich in verschiedenen Affengerippen, und selbst ein Sesambeinchen in der Tibialis eines weiblichen Gerippes zeigen.

Im Orang, auch im Rumpfe des Orangs, der im Thiergarten des Prinzen von Oranien gelebt hat, habe ich keine gefunden—ich bin auch gewiss, dass sie nicht da waren, weil ich sonst den Knorpel müsste angetroffen haben, so wie ich das deutlich in der Sehne des Muskels Popliteus fand, und noch aufbewahre. Tyson spricht auch nicht davon, und bei dem allen findet man sie immer in den geschwänzten Affen, im Pithecus oder Aegyptischen Affen, in den Hunden, Katzen, Füchsen, Stachelschweinen und dergleichen, und zwar doppelt.

Further on S. 187 we read:

Von dem Knie werden wir nichts mehr sagen, als dass ich in der Sehne Kniescheibenmuskels (popliteus) ein sehr grosses, doch knorplichtes Sesambeinchen gefunden habe. Nicht allein im Rumpfe, sondern auch in dem Orang des Hoffmann, das ist, in meinem ersten traf ich ein solches Bein an.

Dieses Sesambeinchen konnte meiner Aufmerksamkeit destoweniger entwischen, weil es nie beim Menschen, sondern immer in den Pavianen, Hunden, Fuchsen, Katzen u. s. w. angetroffen wird.

Ich sahe also gleich das Werk des Tyson nach, weil Cowper, der sich durch die Zergliederung der Muskeln so berühmt gemacht hat, auch die des Pigmy beschrieben hatte.—Doch anstatt hier einiges Licht anzutreffen, fand ich diese offenherzige Erklärung, dass ihm dieser Muskel entgangen sei (70). Auch suchte ich vergebens in der uns vom Daubenton mitgetheilten Zergliederung des Gibbon nach demselben.

According to Peter Camper we have accordingly:

(*a*) Both *fabellae,* the lateral and mesial, in a variety of apes, in dogs, cats, hedgehogs, etc.

(*b*) In man only a single *fabella* occasionally, and this on the lateral condyle.

(*c*) A sesamoid in the tendon of the popliteus (which we term the *cyamella lateralis*) in the orang, but no *fabellae*.

(*d*) No *cyamella lateralis* in man, but it does occur in baboons, dogs, foxes, cats, etc.

Most of the above statements of Camper's can be confirmed, but there are certain exceptions, notably the *cyamella lateralis*, the sesamoid of the popliteal tendon, has been found in man, but it is very rare. Thus while it required 14 centuries after Galen to establish the existence of the first two sesamoids of the knee-joint, the *fabellae*, in man and nearly 250 years after Vesalius to reach the third sesamoid, the *cyamella lateralis*, in the orang, it took 80 years after Camper to establish the *cyamella* as an occasional anomaly in man. Such is the slow growth of scientific knowledge as evidenced in a single anatomical detail like the present. It must not, however, be supposed that with this rapid increase of knowledge in the last years of the 18th century, the 19th century would see rapid progress in our anatomical knowledge of the knee-joint. On the contrary the inertia and follow-the-other-fellow policy of the first half of the 18th century reappears in the 19th.

Cloquet in 1816 (*Traité d'anatomie descriptive*, Paris, I, p. 205) assures us that the *fabellae* appear fairly persistently on both condyles of the femur in man. Bourgery (*Traité complet de l'anatomie*, Paris, 1831, II, p. 103) after describing the *M. gastrocnemius* continues:

> Parfois chez les vieillards des os sésamoïdes se développent dans leurs tendons fémoraux, surtout dans celui du jumeau interne (!).

Indeed no treatise of anatomy could be complete without these "vieillards"; we have followed them through the centuries. The doubting Thomas, however, is always with us. Sappey as late as 1876 (*Traité d'anatomie descriptive*, Paris, II, p. 430) denies the very existence of *fabellae* in man! Even the modern English text-books of anatomy often do not exhibit a knowledge of the sesamoid of the knee-joint comparable with that of Eustachius, or as suggestive as the remark in Humphry (*A Treatise on the Human Skeleton*, 1858, p. 537):

> Where this band (posterior ligament of the knee-joint = *lig. popliteum auct. ab semi-membranoso*) joins the *gastrocnemius* upon the outer condyle of the femur is commonly a thick mass of fibrous tissue and sometimes a sesamoide bone (Pl. 51, Fig. 2D*) is developed.

Humphry (see our Plate IV, Fig. 7) is clearly adding to the accumulating knowledge of the mass of muscular attachments associated with the lateral *fabella*;

* This is we think the *fifth* representation of a *fabella*. Gruber (Gruber, p. 7) suggests that Weitbrecht (*Syndesmologia*, 1742, Fig. 57, *h*) shows a *fabella*: "hat es vom Ursprunge bedeutend entferut (etwas 2 cent.) im *Gastrocnemius externus* abgebildet, aber un Texte nicht bezeichnet." Pfitzner (Bibl. Pfitzner, S. 758) denies this. If is possible that Weitbrecht or his artist indicated something they did not understand, but the plate is too obscure to allow us to come to any conclusion whatever, and as Weitbrecht does not refer to the point in his text, the matter is not of the slightest real importance. There could not be another rediscovery of the *fabella* in 1742!

we have already cited in this direction Cheselden and Sömmering (see our pp. 150 and 151). Quain, however, in 1867 (*Elements of Anatomy*, 7th edition, London, Vol. I, p. 284) can tell us nothing more than:

A sesamoid fibro-cartilage is sometimes met with over the outer condyle and occasionally over the inner, it is rarely ossified—

a statement which is apparently incorrect in at least two points;—while Gray as late as 1905 (*Anatomy, Descriptive and Surgical*) is content with the assertion that there is a sesamoid fibro-cartilage (rarely) osseus in the tendon of the outer head and one occasionally in the tendon of the inner head—another statement too brief to be in any way accurate.

The German anatomists have been somewhat more precise. Hyrtl in 1838 ("Physiologische anatomische Bemerkungen über die Kniegelenksknorpel," *Medicinisches Jahrbuch d. K. K. Oesterr. States.* Bd. XX, Wien, S. 31—32) tells us that the *fabella lateralis* is more frequent than the *fabella mesialis*, that they may be hemisesamoids, and that he has found them in both heads of *M. gastrocnemius*. Again in his *Lehrbuch der Anatomie der Menschen*, 12 Aufl., Wien, 1873, S. 420, after stating that Camper had only seen *fabellae* in the external head, he continues: "Nach meinen Beobachtungen kommt es in beiden Köpfen vor, obwohl in äusseren ungleich häufiger." Lastly we may note that Gegenbauer in 1890 (*Lehrbuch der Anatomie der Menschen*, Leipzig, S. 457) has for a text-book a fairly complete and adequate statement.

Before we come to the important memoirs of the last quarter of the 19th century which have made the sesamoids a special study we ought to note an important memoir by Macalister of 1872 (*Muscular Anomalies in Human Anatomy*, Dublin, 1872*). It is important for our purposes for three reasons:

(*a*) Because it provides evidence for the existence of the mesial *fabella* in man:

Gastrocnemius—1. May have a sesamoid bone in its inner head: I have found it: 2. Or in its outer—this is more common. (p. 118.)

Macalister clearly speaks here of an orthosesamoid, not a hemisesamoid.

(*b*) Because it gives the first statement of the existence of the *cyamella lateralis* as an anomaly in man.

(*c*) Because it records anomalies of the popliteus muscle which have considerable bearing on both the *fabella lateralis* and the *cyamella lateralis*.

Popliteus muscle has been described as double (Fabricus ab Aquapendente: "De Motu locali Animalium" in *Opera Anatomica et Physiologica*, Lipsiae, 1687, p. 359). The same has been seen by Professor Bevan, of the Royal College of Surgeons, Ireland....A small superior popliteus, arising from the outer condyle, and above the popliteus muscle, to which it was united, and inserted into the upper part of the tibia, was seen by Calori†. A sesamoid bone has been found in its tendon. Slips from the semi-membranosus have been found inserted into the fascia over this muscle.

* Also as *Transactions Royal Irish Academy*, Vol. XXV, 1872. J. F. Knott, "Muscular Anomalies," *Proc. R. Irish Academy*, Second Series, Vol. III, p. 639, 1882, notes a case of bilateral orthosesamoid in tendon of *Popliteus*.

† *Mem. della Accad. delle Scienze dell' Istituto di Bologna*, 2 series, Vol. VI, p. 143, refers only to the supernumerary popliteal muscle not to sesamoids.

The bearing of the whole of this paragraph on Gruber's later work is important, and it is regrettable that he did not give more consideration to its statements.

The exact position of affairs with regard to the sesamoids of the knee-joint in man in 1872 was as follows:

(i) The erroneous statement of Vesalius, at once corrected by Eustachius, had been more or less discarded, even in the text-books.

(ii) The existence of the lateral *fabella* in man was recognised as a not uncommon occurrence and it might be an orthosesamoid or a hemisesamoid. If the latter it was described as "fibro-cartilaginous."

(iii) The existence of the mesial *fabella* was realised to be something much rarer, and less often an orthosesamoid. It was, however, vouched for by such credible observers as Eustachius, Morgagni, Sömmering, Hyrtl and Macalister, and by the less credible Heister.

(iv) There was a growing realisation that in man the *fabella lateralis* had relationship not only to the tendon of the outer head of *gastrocnemius*, but to *M. plantaris* (Cheselden and Sömmering) and to the posterior ligament of the knee-joint (Humphry).

(v) Camper had discovered the *cyamella lateralis* in the orang as a persistent feature, although the *fabellae* are absent. He had found it accompanied by the *fabella lateralis* in certain of the lesser apes, and such association is invariable in the cat*. There was really no excuse in 1872 for confusing the lateral *cyamella* with the lateral *fabella*. While Camper denied the existence of the *cyamella* in man, Macalister had discovered it as a rare anomaly.

(4) *History of the Sesamoids of the Knee-Joint. Second Period—that of Monographic Literature.*

The three chief memoirs to be now considered are those of Gillette (1872), Gruber (1875) and Pfitzner (1892). Of these authors Gillette and Pfitzner consider all the sesamoids of the human skeleton, Gruber only those of the knee-joint, and of these he realises only the *fabellae*.

(*a*) Gillette's paper is entitled "Des os sesamoides chez l'homme," *Journal de l'Anatomie et de la Physiologie*, pp. 506—538, Plate XX, Paris, 1872. Gillette commences by dividing sesamoids into *peri-articulaires* and *intra-tendineux*—this distinction is not without its uses. He says, however, that the first are true bones and the second are not (p. 535). He admits, however, when he comes to the *fabellae*, which clearly are intratendinous, that they, apparently because they are more voluminous, have greater resemblance to the periarticular sesamoids. He gives in his Fig. 16 the first really good illustration of the *fabella lateralis in situ*. We have reproduced it in our Plate I, Fig. 3.

* See *Mammalian Anatomy* of Horace Jayne, London and Philadelphia 1898. Part I. The Skeleton of the Cat, pp. 704—5.

Gillette (p. 533) asserts that in the foetus and the child there are no traces of the *fabellae*. This is, of course, opposed to Nesbit's conclusion, but as in perhaps nine cases out of ten no trace of the *fabellae* can be found in the adult, his assertion is like Gruber's by no means conclusive*. Gillette is a strong supporter of the intensive stress theory of the origin of sesamoids. He asserts that little by little:

par le frottement le tendon s'élargit, s'aplatit, devient plus dense. Il se produit à ce niveau et dans son épaisseur un épaissement d'abord fibro-cartilagineux, c'est *l'os sesamoide intra-tendineux cartilagineux*, mais sous certaines influences toujours de nature mécanique, il se dépose des molecules osseuses dans l'intérieur de ce noyau qui se constitue *os sesamoide osseux* (p. 534).

The argument seems to us of precisely the same character as the argument that the environment produces the plant. In a certain sense it does, but the seed contains the potentiality of the plant. We might argue that nourishment and exercise produce the human skeleton, and it is only in this sense, that the *fabella* is no more the product of mechanical causes than the femur itself, that we can accept Gillette's statement. We believe, however, that both he and other supporters of the theory of intensive stress mean a great deal more by it than we are willing to admit. Some decision might be made by the examination of the puppy in various stages of growth, we should be much surprised, if the intense activity and restlessness of a young puppy could be subdued, to find it growing up without or even with much diminished *fabellae*!

The classification of sesamoids into intratendinous and periarticular is convenient, but to identify these with hemisesamoids and orthosesamoids seems to us extremely doubtful, and to state that the former pass into the latter owing to mechanical causes we do not find at all helpful.

(*b*) *Gruber's Monograph*†: This memoir exhibits all the surpassing merits and some of the small but disturbing faults of this eminent anatomist. It presents Gruber's usual wealth of new material and his extensive knowledge of earlier literature. We have at once a sweeping clearance out of all the old anatomical myths, and the forcible statement of new and sounder ideas based on wider ranges of observation. For the first time in the history of our subject we have adequate material directly examined and recorded. Again for the first time we have a real attempt made to record the appearance or non-appearance of the *fabellae* in various living forms. We begin to realise at once that as in the case of man, so in the case of many other living animals the contradictions of earlier writers chiefly arise from want of adequate material; the problem is not whether *fabellae* do or do not exist, but in what proportion of cases they will be found. In

* It is needful to take a case where a *fabella* always occurs, e.g. the dog, and when Gruber dissected puppies he did find the *fabellae* prefigured. We have repeated this observation, and we have also found that the kitten at birth prefigures the lateral *fabella* and *cyamella* by nodules of *hyaline*, not fibrous cartilage.

† "Monographie über die aus wahren (hyalinschen) cartilagines praeformirten *Ossicula sesamoidea* in den Ursprungssehnen der Köpfe des *M. gastrocnemius* bei den Menschen und bei den Säugethieren." *Mémoires de l'Académie impériale des Sciences de St Pétersbourg*, VII^e^ Série, Tome XXII, 1875, No. 4.

other words it is not the "always-present" nor" always-absent" categories that are the difficult ones, but the "sometimes present" category—needing statistical treatment—which is the hardest to ascertain and yet is, perhaps, the most valuable for evolutionary suggestion.

Wenzel Gruber starts by telling us (1875) that the *fabellae* in man have been known for 319 years, which gives us closely the date of the 1555 edition of Vesalius' work, and that those of the mammals have been known for 313 years, which is a close approximation to the date of Eustachius' work (permission to print 1562, issued 1564). There is no reference to Sylvius' commentary on Galen's *De ossibus* (see our p. 144). Gruber at once sweeps away the statements that *fabellae* occur always in man, that they occur only or occur occasionally on the mesial side, that they are produced by stress or by friction, or that they occur most frequently in the aged or in males*. While an osteoma may and does occur in the head of both *M. gastrocnemii* it is far rarer than the true sesamoid and does not occur in the same situation. The origin of the *fabellae* is normal and perfectly analogous in the mammals and in man. Gruber then gives the statistics of his observations, to these we shall refer later; they are rather obscurely expressed, Gruber not having a keen sense for figures. We shall refer here only to a few points which bear on his general conclusions. He says that for many years he examined the cadavers of human embryos in last month (total *not* stated), new-born children "in considerable number," "several" children aged one to seven and never in any of these nor in two five year old boys or in one seven year old girl did he find in the tendons of *M. gastrocnemius*, a *cartilago hyalina*, or *fibro-cartilago*. The wording does not strike one as referring to very copious material and possibly the material was inadequate for a *negative* conclusion. In 44 cadavers 10 to 17 years Gruber found only the *hyalina cartilago* as hemisesamoid; in 426 cadavers 18 to 83 years he found only ossified *fabellae*, the orthosesamoids. In the external head he found only *ossicula sesamoidea* and hyaline *cartilagines sesamoideae* (our orthosesamoids and hemisesamoids), never *fibro-cartilagines* or pathological ossifications (pseudosesamoids†). In the internal head none of these occurred except a single pathological ossification, which he figures in Table III, Fig. 3*g* (see our Plate IV, Fig. 8) and this would never be confused with a true *fabella*. From these data he concludes that (i) neither hemisesamoid nor orthosesamoid occur in the mesial head of *M. gastrocnemius* (p. 67), (ii) there is no trace of a sesamoid in the lateral head in the case of man before 10 years of age (p. 70), and (iii) orthosesamoids originate in a hyaline *cartilago* never a *fibro-cartilago* (p. 68). In other words our hemisesamoid according to Gruber is always hyaline and not fibro-cartilaginous. Here we think Gruber's lesser faults appear. He dogmatically asserts the non-appearance of hemisesamoid or orthosesamoid on the mesial side. He simply discards the

* The existence of *fabellae* in man is wholly independent of age and occupation and does not arise from friction (p. 69).

† Pathological ossifications in *M. gastrocnemius* are very rare in man and when they do occur are never in the neighbourhood of articular condylar surfaces, but in head of tendon close to femur and so removed far from site of *fabella* in man (p. 68).

records of other anatomists because his own large but necessarily limited experience has not confirmed theirs*.

Again in new-born mammals he admits that hyaline cartilaginous *fabellae* are to be seen (pp. 69—70) "as small faint points or streaks macroscopically visible," and yet he asserts that no traces of the *fabellae* are to be found in man before 10 years of age, although elsewhere he emphasises the point that the *fabellae* in man and the lower animals are "völlig analog." We believe that sparsity of material and the comparative ease with which when they do occur (as anomaly) they would escape notice probably account for the discrepancy. The statement that orthosesamoids are always prefigured by hyaline cartilaginous hemisesamoids is very definite and emphasised in the very title of Gruber's paper. All statements as to sesamoids arising from ossification of fibro-cartilage are Gruber tells us incorrect (p. 68). If Gruber be right then we have another and striking instance of how erroneous statements are still perpetuated in the history of sesamoids. For Pfitzner writing only 17 years later, and referring to Gruber's memoir† as a classic of the subject still speaks of our hemisesamoids as developing from "faserigen Gewebe" (S. 563), and even as late as 1897 in the *Seventh Report of the Committee of Collective Investigation of the Anatomical Society of Great Britain* we find sesamoids divided into two classes: "firstly those in which the bodies are osseous, and secondly those in which they were of some other material (fibrous, fibro-cartilaginous, or cartilaginous)‡. Thus if Gruber's statement be correct, it has not got current in the course of 20 years in English anatomy; if incorrect, not even as accurate a German as Pfitzner thought it desirable apparently to issue a formal contradiction! Yet Gruber wrote in 1875 that true sesamoids are always prefigured by the existence of hyaline cartilage, never by fibro-cartilage (p. 65). This prefiguration, he tells us, had not been previously noted in man, and had not been adequately emphasised in mammals generally. All statements as to sesamoids arising from ossification of fibro-cartilage are incorrect (p. 68). Surely such a statement from such an authority deserved at least a contradiction if incorrect?

Yet the anatomists still talk glibly of the origin of sesamoids by intensive stress in fibro-cartilage, and it is left to biometricians to confirm or confute Gruber! We were not able to make an intensive investigation, but we pursued two lines of research.

(i) We had sections taken of the *fabellae* and the *cyamella*, or rather of the nodules where we should suppose them to be, of a kitten at birth. The lateral *fabella* and the *cyamella* exhibited a kernel of hyaline cartilage. We were less successful in our sections of the mesial *fabella*, either it was absent, or else the very

* All statements during 319 years of the existence of a mesial *fabella* in man are "durchaus irrige" (p. 67), Gruber even goes so far as to assert that owing to the *Bursa mucosa supracondyloidea interna* (discovered by him) the internal *fabella* in man would be rather harmful than valuable if it occurred in the neighbourhood of this *bursa*, and if some 2 cms. higher than the external *fabella* would be useless (p. 68). This appeal to "use" is odd.

† Pfitzner gives the wrong volume T. XXIV instead of T. XXII of the St Petersburg memoirs as the locus of Gruber's monograph.

‡ See also Humphry, Quain and other English anatomists for the fibro-cartilaginous sesamoid.

small prefiguration of it had been ejected in the sectioning as one or two cavities were visible in the section. Either alternative is possible as the nodule is much harder than its environment and the knife may slip on it, and again, not only is the mesial *fabella* not infrequently absent in the cat, but it develops far less rapidly than the lateral.

(ii) We asked Professor Elliot Smith if he could provide any information as to the origin of sesamoids in general from a human foetus. The *fabella* and *cyamella* not being universal in man the patella and a metatarsial sesamoid were selected. He placed the matter in the hands of Mr R. B. Green, who has most kindly sent us the accompanying report.

Note on the Histology of Sesamoids in the full-term human Foetus.

Longitudinal sections through the developing sesamoid bone in the tendon of the *Flexor brevis Hallucis* show the cartilage to be essentially of the hyaline variety. At the periphery of the cartilaginous nodule a narrow transition zone between the cartilage and the surrounding young connective tissue may be seen. In this zone fine fibres are interspersed in and gradually fade away into the hyaline matrix. Other fibres pass round the periphery of the cartilage from one extremity to the other.

Longitudinal sections of the patella in the full-term foetus show a similar arrangement in which those fibres which are a continuation of the proximal to the distal tendinous attachment on the surface of the cartilage are more marked.

It will thus be seen that independent lines of investigation confirm Gruber's statement and there is little doubt that if it holds from cat to man, we are fairly safe in asserting that sesamoids are always prefigured by hyaline cartilage*.

The importance of this conclusion is very great. If sesamoids are always prefigured by hyaline cartilage, and do not appear without this origin, and further if this origin can be detected in the foetal state, it is clear that the presence or absence of hyaline cartilage in the foetal state is the condition for the presence or absence of either hemisesamoid or orthosesamoid; and the theory that they are produced by intensive stress or friction falls to the ground, unless it, indeed, be asserted that such factors have produced antenatally the hyaline cartilage nodule! Unless fibrous cartilage could first be converted by stress into hyaline, it is hopeless to maintain that such stress could produce sesamoids. But a study of the regions intermediate between fibrous and hyaline cartilaginous areas appears to indicate that, while there is a form of cartilage which is neither and yet may become either, the one is not an antecedent of the other. In our Plates XVII, XVIII are shown (i) the section of the kitten's *cyamella* and the enlargement of the hyaline cartilage, (ii) the section of the kitten's lateral *fabella*, (iii) the section of the metatarsial sesamoid in the human foetus and (iv) the section of the foetal human *patella*. A more complete study of such material would we believe be of great interest. At present we are only concerned with the fact that the master-craftsman was correct and the text-book writers, who still cling to mediaeval traditions and hypotheses, are wrong.

* Since the above words were written, we have been able to confirm Gruber's statement in the case of the dog, the rabbit, and other mammals. The results will be discussed in the next instalment of this memoir.

While Gruber asserts the universal absence of the mesial *fabella* in man, he draws attention to the fact that in other mammals the nature of the mesial is differentiated from that of the lateral. Lateral *fabellae* may always occur without mesial; or the mesial may occur occasionally; but there is no type of life in which the mesial always occurs without a lateral *fabella*. This differentiation both in man and other mammals between mesial and lateral *fabellae* is very significant and should find its expression in any theory of origin*. Among other points dealt with by Gruber are the shapes and sizes of the *fabellae*. He divides them into 20 shapes to which he gives mathematical terminology, the most common being tetrahedra, cones, half ovals, four-sided pyramids, oval or circular plates or discs. There is no mathematical accuracy in these descriptions, see for example his Table IV, Figs. 9 and 10 (which he calls a three sided pyramid form!), pp. 30–31. All that we can say is that the *fabellae* are very variable in shape, and that they may be pyramidal as in the marsupial type, or lens-form even to spherical as in the apes, and this only in the roughest resemblance. The "base" is usually smooth. It seems to us that the shape largely depends on the number of muscle-attachments of which the *fabella* is in any case the node, but this point is not emphasised by Gruber. On pp. 35–38 certain measurements are given. The greatest weight of any human *fabella* was 0·847 gr. and the smallest weight 0·009 gr. In the greatest tetrahedral form the height was 13·5 to 14 mm. and the base is given as 10 mm. diameter.

We now come to a most interesting consideration of the *locus* of the external *fabella* in man. Gruber gives three situations for the gastrocnemic ossicles in man. The first of these (p. 38) is the *sulcus popliteus externus*! Now if the reader will examine the case described by Gruber on his p. 38 and illustrated in Fig. 1 of his Table I† (see our Plate V, Fig. 10) I think he will agree that it is no *fabella* at all, but a true *cyamella* and probably most of the pyramidal *fabellae* described by Gruber are really *cyamellae*. Now both *fabella* and *cyamella* occur simultaneously in the knee-joints of many genera and are frequent in the apes. Hence it is remarkable that Gruber should have overlooked this important distinction, and confused a sesamoid occurring in the *depth* of the *sulcus popliteus externus* (p. 71) with a *fabella* of the tendon of the *M. gastrocnemius*. Gruber even tells us that the hump due to this sesamoid can be so large that it can be mistaken for an exostosis; in the extended leg the hump can be so great as to be felt or even *seen*‡ (p. 71). Now there is, perhaps, more reason for this confusion than

* Confirmation of this differentiation of origin of the *fabellae* is to be found (i) in the fact that the two *fabellae* are not equal in size. It is possible that in the apes the mesial may occasionally be larger, but in all other species it is smaller and frequently much smaller, (ii) the position of the *fabellae* is often different, (iii) ossification of the external *fabella* begins and ends sooner, (iv) the mesial more often than the lateral fails to ossify, and (v) the microscopic structure may be different.

† This figure represents what is almost certainly a *cyamella* embedded in *M. gastrocnemius*, *Ligamentum popliteum* and short external lateral ligament.

‡ "Der Höcker von dem Ossiculum oder der Cartilago wird 2·6—4·2 cms. (in medium 3·325) über der Spitze der mittleren, höchsten Zacke als dem höchsten Punkte des *capitulum fibulae* gefühlt oder sichtbar" (p. 39).

appears if we loosely speak of the *fabella* as lying in the tendon of *M. gastrocnemius* and the *cyamella* as in the tendon of *M. popliteus*.

We have already prepared the reader by citing Cheselden and Humphry (see our pp. 150, 153) for such not being the case; both *cyamella* and *fabella* are or may be the node, "Knotenpunkt," or junction of a whole network of fibrous structures, tendons and ligaments. Thus Gruber himself tells us (p. 69) that the lateral *fabella* is at the junction of *M. gastrocnemius externus* and the *Ligamentum popliteum* at the point before *M. gastrocnemius* is free of the knee-capsule and where the short lateral ligament takes its departure. For this reason Gruber asserts that the external *fabella* serves (i) to strengthen this junction, (ii) to keep the "Knotenpunkt" on the articular surface of the condyle, and (iii) to prevent *M. gastrocnemius externus* slipping on to the lateral face of the external condyle. This explanation does not seem to us of great validity for if the *fabella* were of such great service, how would nine-tenths of the human population get on without it? But this conception of the *fabella* as a node to which there are many muscular attachments not only accounts largely for its variety of form, but also leads us to seek for its origin in something to which such muscular attachments were peculiar.

Another most suggestive point here is that Gruber noticed occasionally a supernumerary head* of *M. popliteus* (see our p. 154 and our Fig. 9, Plate IV) springing directly from the *fabella*. Thus the two heads of *popliteus* are brought into touch with the two lateral sesamoids and this division of the head is not inconsistent with the division of an original structure. Gruber's confusion of lateral *fabella* and *cyamella* has in itself a slight contributory significance.

More than once Gruber speaks of the controlling power of the *fabella* whether orthosesamoidal or hemisesamoidal over the ligaments and tendons. Thus on p. 72 he notes that in man its movements control: *M. gastrocnemius externus, M. plantaris, M. semimembranosus,* and *M. popliteus biceps* by means of its supernumerary head supposing this abnormal muscle present†. Whenever both *M. plantaris* and the *fabella lateralis* exist in man, a portion of the former, or the whole of it, originates in this sesamoid (p. 45). In short in the case of man in the extended position of

* On seven occasions this supernumerary head sprang from the *fabella*; on four occasions without a sesamoid this anomalous head originated from the head of the oblique popliteal ligament, or and also from the knee-capsule, pp. 45—6. We think, but it is not quite clear, that Macalister (see our p. 154) also had associated the supernumerary head of *M. popliteus* with a sesamoid. See also J. F. Knott, *Proc. R. Irish Academy*, Second Series, Vol. III, p. 639, 1882.

† Again p. 48 we read: "Da nun das Ossiculum und die Cartilago gerade im Knotenpunkte der Vereinigung der Kniekapsel, der Sehne des *Gastrocnemius externus*, des *Lig. popliteum* und des *Lig. laterale externum breve genu* sitzen, da von denselben der *Gastrocnemius externus* mit Fleischbündeln abgehen kann, immer der *Plantaris* davon entspringt, oder falls er fehlt, durch von da kommende Fleischbündel des *Gastrocnemius externus* ersetzt wird, bisweilen von denselben der *Popliteus* mit einem supernumerären Kopfe entspringt; so werden auch diese Muskeln die Verschiebung des *Ossiculum* und der *Cartilago* dirigiren, und es werden durch denselben Apparat, durch den die im Bereiche des *Condylus externus* laxere Kniekapsel vor Einklemmung geschützt wird, auch das *Ossiculum* und die *Cartilago* vor Einklemmung bewahrt. Beide werden bei diesem Manöver, und bevor es zur grössten Spannung kömmt, schräg ein—und aufwärts an dem *Condylus externus*, wenn auch nur eine mässige Strecke, gleiten müssen, um der Einklemmung zu entgehen, und bei der Streckung des Kniegelenkes wieder auf ihren Platz auf demselben Wege zurückkehren."

the limb tendons (*gastrocnemius, popliteus, plantaris*) and ligaments (oblique popliteal, and short external lateral) all united to the sesamoid hold it *in situ* on the posterior surface of the external condyle, near, or on, or in rare cases projecting over the outer condylar border (pp. 46—71).

The reader will recognise what an enormous stride the master-craftsman made in thus investigating the muscular attachments of the *fabella lateralis* in man! Yet even to-day the anatomical text-books content themselves by describing this sesamoid as occasionally occurring in the external head of *M. gastrocnemius*,—a rough description as old as the mid 16th century—and suggesting as Galen did that, as a sesamoid, it is scarcely worthy of study. If we disregard, as we think we must, Gruber's theory of the purpose of the *fabella lateralis* and his identification of the *cyamella* with this sesamoid, we have still to trace the origin of these muscular attachments and the undoubtedly close relations of these two sesamoids. If we discard the first *locus*, i.e. the popliteal sulcus, which Gruber gives to the *fabella lateralis* in man, we find him recording two alternatives*.

(i) Embedded in fibrous structures. The place of the embedment is the posterior wall of the knee-capsule where the tendon of *M. gastrocnemius externus* unites with the oblique popliteal ligament and with the origin of the short external lateral ligament or the partial union of some of these and the knee-capsule itself (pp. 39—40).

(ii) In a facet or groove on the condylar articular surface generally near the border. This facet is either flat or slightly concave, sometimes saddle-shaped† (p. 40).

In the case of the *fabella* lying in a facet the base in contact with the facet has according to Gruber no cartilaginous covering. In the case of the *fabellae* of all mammals—*Gulo vittālus* excepted—the base where they articulate has a cartilaginous covering (p. 71).

Accordingly it will be seen that it is hardly accurate to say of the *fabella* and the *cyamella* that the former is found in the tendon of *M. gastrocnemius* and the latter in the tendon of *M. popliteus*. It is more accurate to assert that the latter will be found in the popliteal sulcus, and the former on the posterior articular surface of the condyle, either somewhat removed from it, or in the close contact of a facet. The muscular attachments of both are found to be in part closely similar. Thus Gruber's error in failing to distinguish between the *fabella* and *cyamella* in man,

* It is singular how very closely Gruber's description of the sesamoid of the popliteal sulcus (our *cyamella*) agrees in its muscular attachments with those of the *fabella lateralis* in its embedment locus. Thus he writes: referring to Fig. No. 4 of his Tafel (see our Plate V, Fig. 10) "Das *ossiculum* und die *Cartilago* liegen im *Sulcus popliteus externus* von fibrösen Gebilden (a, b, d [i.e. Knee capsule, oblique popliteal ligament and short external lateral ligament]) mit welchen sie verwachsen sind, und von Muskelbündeln (k, l, [i.e. *M. gastrocnemius externus, M. plantaris*]) welche von ihnen entspringen umlagert." It will be seen that when both sesamoids are not simultaneously present—as they are in the case of the cat which has both lateral *fabella* and lateral *cyamella*,—there is some reason for confusing the two.

† A flat facet occurs in ⅔ of such cases, a really deep groove is very infrequent (p. 41).

while noteworthy, if he had started from other primates where both coexist, is not so remarkable when he started from the musculature in man, where both rarely coexist.

The bulk of the remainder of Gruber's monograph is taken up with a discussion of the *fabellae* in other living forms than man. He made a long study of the literature of comparative anatomy and added many observations of his own. To these matters we shall return in the following section of this memoir.

Before we leave the topic of the *locus* of the *fabella lateralis* in man we think some study of our Plates I and II is desirable. In Plates I and II, Figs. 1, 4, 5 represent the strange positions of the *fabella* provided by Trew, Heister, and Casserius. Fig. 4 gives the position as envisaged by Camper, while Fig. 5 first shows some daylight as cast on the subject by Humphry*. In Plate I, Fig. 3 we provide a copy of Gillette's representation. It is clearly a case of *Locus II*, the *fabella* resting in a flat facet on the condyle. Plate VI, Figs. 14 and 15 give a knee-joint from the Dissecting-Room at University College, London, to which our attention was drawn by Dr D. E. Derry. It is again a case like Gillette's of a facet *fabella*. The facet lies somewhat high on the articular surface of the condyle. Plate V, Fig. 10 gives Gruber's representation of a sesamoid in the popliteal sulcus. Fig. 12 is a good representation from Gruber of a "Knotenpunkt" *fabella*. Plate IV, Fig. 9 is an illustration of the *M. popliteus biceps* with the supernumerary head attached to the sesamoid; it also shows the short lateral ligament likewise attached and the relation to *M. gastrocnemius externus*. Plate V, Fig. 11 shows the relationship of the *fabella* to *M. plantaris* as pointed out by Cheselden and Humphry, while Plate IV, Fig. 8 gives the only mesial ossicle observed by Gruber, an obviously pathological ossification as he states.

We may say that both *locus* and muscular attachments had been fully studied as far as the lateral *fabella* is concerned by 1876.

(c) *Pfitzner's Monograph.* "Die Sesambeine des Menschen." Schwalbe's *Morphologische Arbeiten*, Bd. I. S. 517—762, Jena, 1892. We have already referred to many points of Pfitzner's work (see our pp. 134, 137, 158). He rejected the intensive stress hypothesis and carried the subject a step beyond Gruber by asserting a vestigial rather than a "use" origin. Sesamoid bodies (hemisesamoids) which he considers develop in "faserigen Gewebe" are "Abortivzustande" of sesamoid bones (orthosesamoids), p. 571. He places the *fabella lateralis* on the top and edge of condyle (pp. 568 and 578)—a position in accord with some of Gruber's observations, but differing widely from our own skiagram series. Only in one case (p. 581) did Pfitzner find the *fabella* in a facet on the condyle, which he speaks of as being its original place. While Gruber uses the word "gleiten" = glide, for the action of *fabella* on articular surface of condyle, Pfitzner states that the sesamoid is incapable of moving on the condyle (p. 548), although it is never united (verschmolzen) to the femur.

* Gruber considers that Weitbrecht and Humphry first placed the lateral *fabella* in its true position. We have already (p. 153, footnote) referred to the grave doubts which arise about what the former intended.

Pfitzner is less dogmatic than Gruber as to the existence of a mesial fabella in man allowing a certain amount of weight to the records of Heister, Morgagni, Hyrtl and Macalister (see our p. 155), but he states that he has never seen it himself (p. 580). On the other hand he has always found it in dog, fox and hare, while in 52 cats he found the *fabella lateralis* always present, but the *fabella mesialis* was absent in 29 out of the 52 cases.

Turning to the *cyamella lateralis*, which Pfitzner describes as in the tendon of *M. popliteus* and working on the curved continuation of the articular surface of the *Condylus lateralis tibiae*, he states that it is constant in *Felidae* and *Leporidae*, and that he has himself *once* found it in a dog. He says that it has never been noticed in man. If we can trust the observations of Macalister, Krause and Knott, and if we have not misinterpreted those of Gruber this statement is inaccurate (pp. 583—584). Lastly as far as we are aware Pfitzner is the only person who has recorded a *cyamella mesialis*, and this only in two cats (p. 584). He was unable to note the muscular attachments. We have gone through a considerable number of dog skeletons in the Biometric Laboratory, in which as a rule the *fabellae* are preserved, but have met with no cases of *cyamellae* being preserved.

The literature of the sesamoids of man is fairly comprehensively treated by Pfitzner, and we have found his references very helpful in our search into the history of our knowledge of the sesamoids in man. We shall refer to his statistics of occurrence in man later. On the whole while Pfitzner's memoir is of great value—in particular he recognised the *cyamella*—it is not epoch-making in the sense of Gruber's.

(5) *Frequency of the Fabellae in Man.*

Putting aside the vague statements of the mediaeval anatomists we have five sources from which this frequency may be determined (i) Gruber, (ii) Ost, (iii) Pfitzner, (iv) the *Seventh Report of the Collective Investigation Committee*, (v) Skiagram Data.

We can dismiss Ost at once; his paper* is very brief and refers to only 30 cases in which he found a *fabella* five times or 16·7°/₀; sex is not given. Pfitzner observed hemisesamoids or orthosesamoids in 30 out of 290 cases or in 10·34°/₀. He never met with a *fabella mesialis*. While Pfitzner presents little analysis of his cases Gruber provides a considerable amount, but it is occasionally far from clear as to what exactly he is referring to.

Thus Gruber tells us (p. 70) that without regard to size or sex it appears in about $\frac{1}{6}$ of the cases, i.e. 16·7°/₀. On p. 26, however, he tells us that in 2340 limbs he found it in 400 instances or 17·1°/₀. Paying regard to sex Gruber states, again on p. 70, that he found it in about $\frac{1}{5}$ of the male cases and $\frac{1}{3}$ to $\frac{1}{4}$ of the female; according to this the lateral *fabella* occurs in 20°/₀ of cases in man and

* Ueber das Vorkommen eines Sesambeines in den Ursprungssehnen des *Gastrocnemius* beim Menschen. *Zeitschrift für Anatomie und Entwickelungsgeschichte*, Bd. II, S. 309, Leipzig, 1881.

25°/₀ to 33°/₀ in woman. Turning back to p. 26 we find that Gruber makes the following statements:

Ages 10 *to* 83* *without regard to sex:*

220 cadavers or 440 limbs, 66 lateral *fabellae*, i.e. 15 °/₀.

Sexed Cases:

400 male limbs, lateral *fabellae* in 77 cases,
100 female limbs, lateral *fabellae* in 22 cases,
i.e. Males 19·25 °/₀, Females 22·0 °/₀.

Extremities, Age unknown. Both sexes:

1400 with 235 lateral *fabellae*, i.e. 16·8 °/₀.

Without Age, but sexed:

Male limbs 294 with 61 lateral *fabellae*,
Female limbs 98 with 29 lateral *fabellae*,
i.e. Males 20·8 °/₀, Females 29·6 °/₀.

Returning to the 2340 limbs, Gruber tells us, however, that (p. 26)

940 male limbs gave 165 lateral *fabellae*,
840 female limbs gave 143 lateral *fabellae*,
i.e. Males 17·6°/₀, Females 17·0 °/₀.

Finally we read (p. 27) that

1155 right limbs gave 212 lateral *fabellae*,
1185 left limbs gave 188 lateral *fabellae*,
i.e. Right Limbs 18·4 °/₀, Left Limbs 15·8 °/₀.

We find it extremely difficult to deduce anything like definite percentages from this tangle of figures, and it would seem that this huge mass of material has been largely wasted for statistical purposes. We can only suppose that the difference of percentages arises from the proportion of young limbs which may be included in the different series.

We may, perhaps, conclude:

(*a*) That there is no significant sexual difference.

(*b*) That there may be preponderance of right limbed lateral *fabellae*.

(*c*) That for all ages and sexes the percentage (judged from 2340 limbs) is 17·1 °/₀.

Why this percentage should run up to 22·0°/₀ in the case of sexed femora of unknown age we are unable to say; or, why it should fall to 15·0 °/₀ in the case of the 220 cadavers, unless the younger ages have been omitted in the previous result, we cannot explain. Possibly the latter form an earlier part of the record, and that observation became more stringent and accurate as the investigation

* The maximum frequencies at the decades indicate that these records of age are very rough.

progressed. The *fabellae* enumerated include both hemisesamoids and orthosesamoids. The results are in rough agreement with Ost's short series, but shew a far higher frequency than Pfitzner's record.

We now turn to the interesting *Seventh Report of the Committee of Collective Investigation of the Anatomical Society of Great Britain and Ireland for the year* 1896—1897*. We have already commented on the high percentage of lateral *fabellae* reached by Gruber, but these percentages are at least doubled in the statistics of this *Report*. Over 40 °/₀ of occurrence of *fabellae* are here recorded, and if this rate of increase of percentage continues we shall soon have to return to the standpoint of Vesalius! One wonders (i) if the distinction between limb and cadaver has always been maintained, (ii) if the distinction between condyle and limb has also been maintained†, and lastly supposing the results do indicate the number of *fabellae* per 100 condyles, (iii) if the "Knotenpunkt" itself in which the lateral *fabella* is to be sought has not been occasionally mistaken for a sesamoid‡.

In 287 limbs of both sexes examined, 81 lateral *fabellae* and 39 mesial *fabellae* were found. These figures provide 28·2 °/₀ of lateral *fabellae* and 13·6 °/₀ of mesial *fabellae*, i.e. about twice as many lateral as mesial. Now Pfitzner, Gruber and ourselves have not discovered in man any mesial *fabellae*. Hence we must put against Pfitzner's 10·34 °/₀ and Gruber's 17·1 °/₀ the 41·8 °/₀ of the present investigation. But the matter takes a different form if we distinguish between osseous and non-osseous *fabellae*§. Of the 81 lateral *fabellae* only 28 were orthosesamoids; and of the 39 mesial *fabellae* only three were reported as osseous. In other words only 9·75 °/₀ of orthosesamoidal lateral *fabellae* occur, and 1·05 °/₀ of orthosesamoidal mesial *fabellae*. These results are very luminous for the controversies of the mediaeval anatomists. Supposing those three mesial orthosesamoids were actually and correctly diagnosed then if the mediaeval anatomists had, as they probably had an osseous body in view they might easily have examined a hundred or more limbs without finding a mesial *fabella*. It is even conceivable that Pfitzner might not find one in 290 limbs. It is less conceivable that they should have escaped Gruber in the case of 2340 limbs, and in the case of both Pfitzner and Gruber who were well acquainted with hemisesamoids, it is surprising that neither found a mesial hemisesamoid in 2630 limbs examined, when they ought to have found 331 on the scale of the *Collective Investigation Report*. It must be quite clear that these different investigators had a totally different conception of the nature of a non-osseous sesamoid, and that this is the origin of the surprising percentage differences we encounter in these investigations as to the presence of *fabellae*

* *Journal of Anatomy and Physiology*, Vol. xxxii, p. 182, London, 1898.

† A condyle gives only one chance, a limb two chances, and a cadaver four chances of a *fabella*.

‡ The *Report* says: "With regard to the constitution of the bodies we have evidence that in many of the investigations no microscopical examination was made. We shall therefore content ourselves with dividing the cases in which sesamoid bodies occur into two classes,—first those in which the bodies were osseous; secondly those in which they were of some other materials (fibrous, fibro-cartilaginous, or cartilaginous)," p. 184.

§ It is not, perhaps, adequate to treat these as orthosesamoids and hemisesamoids, as they may contain fibrous structures, which Gruber and probably we ourselves should not class as hemisesamoids.

in man*. We believe that until the nature of the hemisesamoid has been standardised (for example, it is defined as a hyaline *cartilago* after Gruber, or otherwise), it is unwise to lay stress on any other statistics than those of orthosesamoids.

Out of a total of 79 *fabellae* found in male subjects by the *Collective Investigation* 22 were orthosesamoids, and of 28 *fabellae* found in female subjects 10 were orthosesamoids. Accordingly less than a third of the *fabellae* were osseous, or more exactly the chance of a *fabella* in man being orthosesamoidal is almost ·30. Thus we should expect in the 79 male *fabellae* 24 orthosesamoids and 55 hemisesamoids (as against 22 and 57 observed), and in the 28 female *fabellae* 8 orthosesamoids and 20 hemisesamoids (as against 10 and 18 observed). With a standard deviation in the males of about 4·0 and in the females of about 2·4, the differences of observed and computed are non-significant, or we cannot assert a sex-differentiation in ossification. This *pro tanto* is against an intensive stress theory of origin, for if orthosesamoids are produced by stress from hemisesamoids, we might reasonably expect greater stresses in the male, and a larger proportion of orthosesamoids in their total.

Again in a total of 146 right limbs 61 *fabellae* were found or 41·8 per 100 limbs. In 141 left limbs 59 sesamoids were found or again 41·8 per 100 limbs. It is therefore difficult to accept the statement of the *Report* (p. 186, (2)) that *fabellae* are rather more common on right than left limbs. In 157 male limbs 76 *fabellae* were found or 48·2 per 100 limbs; in 84 female limbs 27 *fabellae* were found or 32·1 per 100 limbs. This shows a greater preponderance of *fabellae* of all sorts in the male, although both sexes have the same ratio of orthosesamoids to hemisesamoids. This result contradicts that of Gruber (see our p. 165, (*a*)) and appears opposed to the conclusion drawn by the framer of the *Report* (p. 186) himself, who writes: "sex has little influence on their [i.e. the *fabellae's*] frequency."

The chief value of the statistics† in this *Report* is the confirmation they

* Again on the scale of the *Collective Investigation Report*, Gruber and Pfitzner between them ought to have found between 26 and 27 mesial orthosesamoidal *fabellae*. They found *none*. Hence we are compelled to suppose either racial differences exist in this respect, or that one or other set of observers is in error.

† A further point is worth recording, namely the correlation between the appearance of *fabellae* in both heads. Clubbing together both sexes and both sides we have 241 limbs showing the following table:

Presence of *Fabellae* in the two Heads of *M. gastrocnemius*.

		Outer Head. Present	Outer Head. Absent	Totals
Inner Head.	Present	13 (10)	20 (23)	33
	Absent	57 (60)	151 (148)	208
	Totals	70	171	241

provide of the occasional occurrence of mesial orthosesamoidal *fabellae* as asserted by Hyrtl, Macalister and others. But even assuming the correctness of the observation we are forced to the conclusion, that mesial orthosesamoidal *fabellae* are very rare in man, that mesial hemisesamoids are also much rarer than lateral, and that generally man confirms the view that there is a marked differentiation in origin between the lateral and mesial seasamoids of the knee-joint.

(6) *Skiagraphic Data for the Occurrence of the Sesamoids of the Knee-Joint in Man.*

Our data in this case consists of (i) 102 skiagrams of the knee-joint taken at a military orthopaedic hospital and examined by the kindness of Dr Stanley Melville, and (ii) 352 skiagrams of the knee-joint, which were found among the thousands of skiagrams taken at the Fulham Military Hospital and now deposited at the Royal College of Surgeons. We owe to the kindness of Professor Keith our access to this material. Dr Melville found seven cases of the presence of lateral *fabellae*, we found 22 certain cases and a doubtful four further cases. The results 6·9 °/₀ and 6·2 °/₀ to 7·4 °/₀ are in very good agreement. We think it safe to conclude that skiagraphically visible *fabellae* occur in about 7 °/₀ of human cases. If we suppose (i) that hemisesamoids would not show on the skiagrams and that (ii) they are twice as numerous as orthosesamoids we should reach about 21 °/₀ as the occurrence of the lateral *fabella* in man, i.e. a percentage approximating to Gruber's and that of the *Collective Investigation Report*, but much in excess of Pfitzner's. Dr Stanley Melville was not able to report a single case of the appearance of two *fabellae*, although there is a shadow on one of his plates (see our Plate XV, Fig. 33) which by a great stretch of the imagination might possibly be interpreted as a second *fabella*, much more distally situated than is customary. In the skiagrams from the Fulham Military Hospital we have not succeeded in finding any trace of a mesial *fabella*. Yet if the data of the *Collective Investigation Report* are to be trusted we ought to have come across five to six mesial orthosesamoids in the total of 454 limbs; we do not refer to the hemisesamoids for we assume they would not appear in skiagrams. Now while we cannot disregard the statements of such qualified observers as Hyrtl and Macalister, and dogmatically assert with Gruber that a mesial *fabella* has no existence in man, we feel bound to say that we feel it exceedingly elusive, and that we must emphasise the importance of the next recorder preserving an anatomical

If we apply tetrachoric computation to this table—and the variety in size and constitution of the *fabellae* roughly [illegible] it—we find for the correlation

$$r = \cdot 1790 \pm \cdot 0885.$$

We see at once that there is no close and significant relation between the appearance or absence of *both fabellae* simultaneously, i.e. there is no close organic relation between them in man. In fact if distributed purely at random we should anticipate 10 cases in which mesial and lateral *fabellae* would appear simultaneously. We give the numbers which might be anticipated if the appearances were absolutely independent in brackets. This approach to independence in origin is certainly singular in the case of man, considering the universality of *fabellae* in so many lower types of life.

preparation, and further urging all able to do so to examine carefully skiagrams of the knee-joint, recording not only presence but absence of one or both *fabellae**.

The skiagrams give very accurately the position of the *fabella* with extended limb. That position is opposing the middle third of the posterior face of the lateral condyle. Generally we may say that while it is sometimes more proximal or more distal, it stands midway opposite to the vertical wall of the articular surface. As our skiagrams are not stereoscopic it is impossible to hazard any statement how far the *fabellae* are removed from the midplane of the condyle towards the lateral border of the articular surface. Our diagrams show what great variation there is in the size of these sesamoids.

In seven of our illustrations (Plates VII, VIII, IX, XV, XVI, Figs. 16, 17, 19, 20, 32, 33, 34) the *fabella* shows complete separation from the articular surface of the condyle; it is extremely unlikely that a facet could exist in these cases, and certainly not a grooved facet. In possibly three cases only (Plates VIII, IX, XVI, Figs. 18, 21, 35) is it reasonable to suppose that facets may have existed; in these three cases, however, it is hardly reasonable to suggest that the sesamoid is in the least depressed below the articular condylar surface. It would thus appear that the first of Gruber's positions for the *fabella*, i.e. embedded in the node of fibrous structures, is about, very roughly, two to three times as frequent as his second position, i.e. situated in a facet on the condylar articular surface.

Plate XV, Fig. 33 is the case we have already referred to (p. 168) in which there is some sign, still very elusive, of a second *fabella*. We are unable to lay any stress on it. It will be at once recognised that the drawings of Trew, Heister and Casserius hardly describe anything which appears in our skiagrams.

We may now draw attention to sesamoids of the knee-joint appearing in somewhat anomalous positions. Plate XIII, Fig. 29 shows apparently two small sesamoids opposing the lateral face of the external condyle, but somewhat removed from its surface. Plate XIV, Fig. 31 gives with less certainty a single sesamoid in a somewhat similar situation. The roundness of these individuals appears to preclude any pathological origin. They are too high up to represent in any way normal *cyamellae*. They might almost be described as intermediates in position between *cyamella* and *fabella*. Their almost vertical position over the head of the fibula is not without suggestiveness. But without evidence from the dissecting-room, or confirmatory skiagraphic records, we wish neither to assert that there is a third position possible for the *fabella*, or that occasionally an additional sesamoid of the knee-joint may occur in a position intermediate between that of *fabella* and *cyamella*. We know of no such sesamoids in lower types of life; for the time being we may call them x-sesamoids and if in the future no further evidence is forthcoming with regard to them, they may be dropped. We did not, however, like to pass them by without a word of record.

* We shall be extremely grateful for any material with regard to sesamoids of the knee-joint which may be sent to us (Biometric Laboratory, University College, London, W.C. 1).

Cyamella in Man. It is very much less easy to differentiate the *cyamella* by aid of skiagraphic analysis. As it lies deep in the popliteal sulcus, it can hardly be distinguished by a *norma lateralis* of the knee-joint. A *norma frontalis* provided the *cyamella* is of considerable size appears to give the best chance. Thus Plate XI, Fig. 25 provides an illustration of what is probably a *cyamella lateralis*. Plate X, Figs. 22 and 23 shows another *cyamella lateralis* from posterior and frontal aspects. Plate XI, Fig. 24 gives our nearest approach to a *cyamella mesialis*. There is something similar in Figs. 22 and 23 of Plate X. We decline to interpret them and should refuse entirely to be dogmatic about them. But taken in conjunction with Pfitzner's statement as to what he had found in two cats (see our p. 164), we think it important that in the dissecting-room, the skiagraphic studio and the laboratory of comparative anatomy the investigator should bear in mind the slight possibility of a *mesial cyamella**. We have found two probable and one doubtful lateral *cyamellae* in our 352 cases, or in 0·57 °/₀ of cases probably and in 0·85 °/₀ possibly. Thus the *cyamella lateralis* in man first recorded by Macalister appears to be of great rarity.

Lunulae. Our search for these sesamoids so common in lower types was not very successful. In the first place the skiagrams do not exhibit ossifications of the semi-lunar cartilages at all clearly, and in the next pathological ossifications of the semi-lunar cartilages appear to be far from infrequent. Plate XII, Figs. 26, 27 and Plate XIII, Fig. 28 indicate what may be true ossicles of the semi-lunar cartilages. In the case of Fig. 27 the *lunula* is accompanied by what may possibly be an ossicle on the antero-lateral border of the tibia immediately above the *capitulum fibulae*. On the whole we found nine *lunulae* ranging through various shades of doubtfulness. Thus the occurrence of *lunulae* in man does not exceed 2·5 °/₀ of knee-joints *and is probably far less.*

We have not included the *patella* in our consideration of the knee-joint sesamoids, but in examining these skiagrams we have found apart from obvious pathological cases five or six of double *patella*†. We are prepared indeed to be told that even these six are pathological ossifications of the tendon of the quadriceps. But we are not equally certain that those who will take this view are fully aware of the regular occurrence of the double *patella* in certain lower types. Tillmans and Bernays‡ have found in man in several cases a hemisesamoidal body above the true *patella* and Bernays notes the same conditions in some of the *Felidae* and the rodents (Cat, Mouse, Rabbit). Pfitzner found such a double *patella* in the *Leporidae* (*loc. cit.* S. 577). In this consideration it is worth noting that *Macropus giganteus* has a hemisesamoid instead of an orthosesamoid

* In the first and last cases an anatomical preparation should certainly be preserved of any discovery or supposed discovery. Our own case is not so definitely an osteome as Gruber's: see our Plates X and XI, Figs. 22—24.

† No percentage is possible as the anterior and posterior views of the knee-joint give no power of discriminating a double *patella*.

‡ "Die Entwicklungsgeschichte des Kniegelenkes," *Morphologisches Jahrbuch*, IV, Supplement, 1878, S. 442.

for its *patella*. Our Plate XIII, Fig. 28 and Plate XIV, Fig. 30 show typical cases of such double *patellae* in man. We suggest that in discussing the pathological nature of these cases, the possibility that they have vestigial associations should not be entirely overlooked*.

(7) *Summary of our Knowledge as to the Sesamoids of the Knee-Joint in Man.*

After 19 centuries of consideration, 18 of which were little better than fumbling, we can sum up our present knowledge of these sesamoids as follows:

(*a*) An orthosesamoidal lateral *fabella* occurs in man in about 7 °/₀ of knee-joints.

(*b*) In two to three times this number of cases there is a hemisesamoid, probably if indeed not almost definitely a hyaline *cartilago*.

(*c*) The situation of this *fabella* in about three-fifths of the cases is roughly opposed to the articular surface of the condyle near its most posterior point, but separated from it by 3 to 7 mms. It is the "Knotenpunkt" of a variety of muscles. In about two-fifths of its occurrences the *fabella* lies on a facet upon the articular surface of the condyle.

(*d*) There is some evidence for the far rarer existence of a mesial orthosesamoidal *fabella*, but both the evidence and this *fabella* itself are very elusive. The *Collective Investigation Report* records it in about 1 °/₀ of knee-joints. Gruber did not find a single case in 2340 knee-joints, and we have only found a single and very doubtful trace of it in 1 out of 454 skiagrams of the knee-joint. The same *Report* asserts that there are 13 times as many mesial hemisesamoidal *fabellae* as orthosesamoidal. Gruber again, and Pfitzner also, record none whatever. Macalister and Hyrtl and others have recorded their presence.

(*e*) The lateral *cyamella* has been reported by Macalister in man. We believe that we have found it in under 1 °/₀ of skiagrams of the knee-joint. If we interpret Gruber's statements correctly, he has erroneously termed the sesamoid he found in the popliteal sulcus a *fabella*, it should be a *cyamella*, and this is further evidence of its rare appearance as an anomaly.

(*f*) The existence of a mesial *cyamella* in man cannot be said to be established by the few doubtful skiagrams we have reproduced (Plates X and XI). Mesial *cyamellae* do not occur in any family of mammals universally. Pfitzner reports two cases in the cat, which always has a lateral *cyamella*. The sole interest of the anomalous appearance of a mesial *fabella* would be to strengthen the possibility that an organ which is persistent on one side of a limb may anomalously reduplicate itself on the other.

(*g*) There is some slender evidence of the rare occurrence of *lunulae* in man. They have not as far as we are aware been systematically sought for in the dissecting-room. We should expect them as rare anomalies in man as they exist universally in certain lower forms.

* In the Fulham series of skiagrams also occurred four well marked cases of Gruber's *Processus supracondyloideus ossis femoris*: see Pearson and Bell, *Monograph on the English Longbones*, Part I, The Femur, Text pp. 85—89, and Atlas, Plates XXVI—LXVIII.

(*h*) Three hypotheses have been provided for the existence of the sesamoids of the knee-joint, namely:

(i) The theory of their manufacture from cartilage by intensive stress or even by friction. Against this hypothesis it is adequate to cite the cartilaginous *patella* of the giant kangaroo, the persistent *fabellae* of the smaller primates, and the relatively rare occurrence in man, the persistent *lunulae* of the squirrel and the absence of *lunulae* except as anomalies in man; the fact that the male has no higher percentage of *fabellae* than the female, and the general absence of *fabellae* in the greater and more-massive anthropoids.

(ii) The theory that the lateral *fabella* serves a useful purpose as strengthening the "Knotenpunkt" of a congeries of muscular attachments. Such a hypothesis of use value might be valid, if man were a living form developing for the first time a *fabella*. On the contrary more primitive types of the primates have it universally and the higher types appear to have lost it or at least to be losing it. If the lateral *fabella* in man was there because it was of great service it would be difficult to explain why 7 °/₀ need it and 93 °/₀ get on very well without it!

(iii) The theory that the *fabellae* and the *cyamellae* are vestiges of some structure of earlier form, which has disappeared, and that hemisesamoids are not stages towards orthosesamoids, but the last steps in a degenerative process. Any such theory of the sesamoids of the knee-joint if valid will have to account for the marked differentiation in frequency, size and constitution of lateral and mesial sesamoids, and it will have to give some account of the structure of which they may be supposed to be the debris. It will also have to account for the muscular attachments of the sesamoids which we now find associated with the knee-joint in man.

This third hypothesis seems to us the only reasonable one hitherto proposed for the existence of the sesamoids of the knee-joint, and we shall therefore endeavour to trace the evolutionary history of the *fabella* and *cyamella*. If we observe that in the earlier primate forms the sesamoids of the knee-joint are osseous, we can dismiss the question of hemisesamoids; the only account we need give of them is that they are a measure of the extent of degeneracy, signposts to where an orthosesamoid once existed, and not the beginnings of sesamoids awaiting old age or intensive stress to make them osseous. We turn now to a study of the sesamoids of the knee-joint in living forms other than man, endeavouring to combine our own observations with those occasionally contradictory *inter se* of other observers.

(*To be continued*)

DESCRIPTION OF PLATES.

The first five plates represent historically the development of the anatomists' conception of the sesamoids of the knee-joint.

Plate I. Fig. 2 shows after Pfitzner the sesamoids of first metatarsal of the foot in man. Such sesamoids of the digits were certainly known to the Greeks. They are the most conspicuous and we cannot demonstrate that the Greeks knew those of the knee-joint. It is reasonable to suppose that the seed of the plants the Greeks called sesamé should somewhat resemble these sesamoids.

Fig 1. First drawing of the *fabellae*. It occurs in Casserius' *Tabulae anatomicae* of 1632 which illustrate Adrian Spiegel's *De humani corporis fabrica*. The latter following Vesalius asserts the existence of sesamoids in *both* heads of the external *gastrocnemius*, and they are thus somewhat naively exhibited by Casserius. There is no trace of the "Knotenpunkt" of actuality.

Fig. 3. See under Plate IV.

Plate II, Fig. 4. Diagram from the Altorf 1715 dissertation, *De chylosi Foetus in utero*, of C. J. Trew. The diagram shows none of the muscular relationships of the external *fabella*. F. V shows the sesamoid resting in what Trew thought its customary situation, "os sesamoideum insidens cavitate condyli exterioris in omnibus femoribus conspicuae." F. VI exhibits "os sesamoideum femoris magnitudine naturali extra situm." Trew makes no reference to an internal *fabella* in his supposed discovery. The "conspicuous cavity in all femora" appears to be the roundish impression for the insertion of the lateral head of *M. gastrocnemius* on the external epicondyle, a situation where the sesamoid is not in our experience ever found.

Fig. 5. This drawing is taken from Laurence Heister's *Compendium Anatomicum*, Edition 1748. There is much in it to indicate that Heister notwithstanding his paper of 1717 (see our p. 150 ftn.) copied Trew's illustration, but added the mesial *fabella*, which he rests on the horizontal surface of the condyle in immediate association with the supracondylar tubercle.

Plate III, Fig. 6. Peter Camper's drawing of the external *fabella* from his 1754 memoir: "Vom Bruch der Kniescheibe und des Olecranon" (S. 68 of the *Vermischte Schriften* of 1801). *AKIL* is the tendon of the outer head of *gastrocnemius* "worin sich ein Knöchelgen (*ossiculum sesamoideum*) *f. g.* findet, welches man in dem inneren Bauch derselben nie sieht." *f. g.* is not marked on the figure, but is presumably the shaded portion. If so, the relation of this sesamoid to the other muscles and ligaments is far from clearly indicated.

Plate IV, Fig. 7. A drawing after Sir George M. Humphry's figure in his *A Treatise on the Human Skeleton*, 1858 (Plate LI, Fig. 2). Posterior view of left knee-joint, described as follows: *A*, *ligamentum posticum*; the thickest portion of it ascends from *B* the *semimembranosus* to *C* the outer head of *gastrocnemius*. *D* a sesamoid bone at its junction with the latter. *E* inner head of *gastrocnemius*, also connected with *ligamentum posticum*. *F* popliteus muscle, raised a little to show *G* the posterior peroneo-tibial ligament. *H* external lateral ligament. Humphry thus indicated the relation of the sesamoid to the *ligamentum posticum*.

Fig. 3 of Plate I (placed there for economy of space) is from the historical order to be considered here. It is copied from Gillette's memoir of 1872 (see our p. 155). *a* is the external *fabella in situ*, *b* the *gastrocnemius* and *c* the situation where it rests on the posterior surface of the condyle. The situation is fairly in accordance with that suggested by Humphry, but Gillette does not indicate the relationship of the sesamoid to other ligaments and muscular attachments.

Figs. 8 and 9 of Plate IV and 10, 11, 12 of Plate V are all taken from W. Gruber's great memoir of 1876, which is the first to illustrate carefully the muscular attachments to the sesamoid.

In all these figures 8 to 12, except Figs. 9 and 10 where by an unfortunate slip of the engraver 5 has been placed for *5*, the system of lettering is the same, namely:

1 = femur, 2 = tibia, 3 = fibula, 4 = sesamoid. *a* = knee-capsule, *b* = *ligamentum popliteum*, *c* = long external lateral ligament or *ligamentum anticum*, *d* = short external lateral ligament, or *ligamentum posticum*, *e* = external *meniscus genualis*, *f* = great adductor muscle, *g* = *M. gastrocnemius internus*, *h* = *M. semimembranosus*, *i* = *M. biceps femoris*, *k* = *M. gastrocnemius externus*, *l* = *M. plantaris*, *m* = *M. popliteus*,

n=*M. popliteus biceps* (Fig. 9), o=internal supracondylar tubercle, p=*processus tuberositatis* of the internal condyle for the insertion of the tendon of the adductor muscle, γ=superficial and δ (engraved 5 in Figs. 9 and 10)=deeper layers of the under "strängformig" portion of the tendon of origin of *M. gastrocnemius externus*.

Fig. 8 shows the only mesial *fabella*, 5, observed by Gruber; it was a pathological ossification, α being the upper vertical, and β the transversal limb of this pseudosesamoid; it appears to be associated with a somewhat emphasised tuberosity of the internal condyle, p.

Fig. 9. A most interesting illustration of the anomaly of *M. popliteus biceps*, n, of the two branches n'' is the outer deeper head and n' the supernumerary inner and superficial head attached to the external *fabella*. This *fabella* (4) appears as a "Knotenpunkt" of tendon of *M. gastrocnemius* k (5=δ) of the short external lateral ligament (d), and its posterior face is shown by the removal of *M. gastrocnemius* (k) and *M. plantaris* (l).

Plate V, Fig. 10. Left knee-joint seen in posterior aspect. We see the ossicle exposed with its upper inner and outer surfaces, the head of origin of *M. gastrocnemius* being cut through and drawn back over the ossicle (4). We see the ossicle attached to the *ligamentum popliteum* (b) and the short external lateral ligament (d). The position of the sesamoid and its shape are unusual for a *fabella*. It appears to be low down on the external face of the lateral condyle, and approaching the popliteal sulcus. Its position seems at least transitional from that of *fabella* to that of *cyamella*.

Fig. 11. Right knee-joint in posterior aspect. The *fabella lateralis* (4) is seen wedged in so to speak between *gastrocnemius* (k) and *plantaris* (l). This is the position given for it by Cheselden in 1733, and more fully recognised by Sömmering in 1841: see our pp. 150 and 151.

Fig. 12. Left knee-joint in posterior aspect. The *fabella lateralis* (4) has been freed from *gastrocnemius* (k) and seen in relation to the *ligamentum popliteum* (b), the short external lateral ligament (d), and by the opened knee-capsule (a) in relation to its position on the external condylar surface.

It will be seen how great an advance Gruber made in our knowledge of the muscular and ligamental attachments of the *fabellae*.

Fig. 13. Sesamoid of the external condyle in the case of the Wombat (after Pfitzner). This sesamoid rests on the head of the fibula, and appears to be directly associated with the *parafibula* of other Australian types, and again the direct articulation of fibula with the external condyle in other species. These points are discussed in Part II of the present memoir.

Plate VI. Drawings of an actual case of the *fabella lateralis* in man from a cadaver in the Dissecting-Room at University College.

Fig. 14 is a sagittal section of the left femur showing the position of the *fabella* before *M. gastrocnemius* was turned back. Fig. 15 shows the *fabella* in the tendon of the muscle after it has been turned back and the facette on the articular surface of the condyle in which it rested. The closeness of contact differs considerably from the usual position as indicated in the skiagrams: see Plates VII—IX.

Plate VII, Fig. 16 and Fig. 17. Lateral *fabellae*.

Plate VIII, Fig. 18 and Fig. 19. Lateral *fabellae*.

Plate IX, Fig. 20. Small lateral *fabella*. Fig. 21, Large lateral *fabella*, probably differing from the previous illustrations in resting in a facette on the articular surface of condyle. Cf. Plate VII.

Plate X. Most probably a lateral *cyamella* in the neighbourhood of the popliteal sulcus in posterior (Fig. 22) and anterior (Fig. 23) aspects. Is there evidence of a mesial *cyamella*?

Plate XI. There is some evidence in favour of a mesial *cyamella* placed in the groove on the internal epicondyle in Fig. 24. Fig. 25 gives a fair example of a lateral *cyamella* in the popliteal sulcus.

Plate XII, Fig. 26. There is a small bone in the semi-lunar, which may be a posterior lateral *lunula*. Fig. 27 gives more definite evidence of an anterior lateral *lunula*. Both of these *lunulae* are fairly smoothly shaped and do not resemble pathological ossifications: at the same time the damaged condition of lateral condyle in Fig. 26 prevents any stress being laid on the former case. In Fig. 27 there appears to be a well shaped sesamoid at the head of the fibula in close proximity to the *lunula*.

Plate XIII, Fig. 28. This must be taken in conjunction with Plate XIV, Fig. 30. In both cases we have an ossification of the tendon of the quadriceps extensor muscle. In both cases it *may* be considered as pathological, but at the same time its general resemblance to the double or "tandem" *patella*, which

occurs regularly in the hare, should be noted. The cases of double *patella* recorded by Gruber (*Virchow's Archiv*, Bd XCIV, S. 358, 1883) and Wright (*Journal of Anatomy and Physiology*, Vol. XXXVIII. p. 65, 1904) are comparable with each other, but not atavistic in character like those of our skiagrams *. Both our Figs. 28 and 30, seem to suggest the existence of posterior *lunulae*.

Plate XIII, Fig. 29 shows two sesamoids on the face of the lateral epicondyle, possibly related to the lateral femoro-fibular ligament.

Plate XIV, Fig. 31 gives an indication of a single sesamoid in much the same position.

Plate XV, Fig. 32. Lateral *fabella*.

Fig. 33. Lateral *fabella* with a faint trace of a second sesamoid a little below it—possibly but improbably a hemisesamoid of the mesial condyle.

Plate XVI, Fig. 34. Lateral *fabella*. This knee-joint exhibits considerable pathological ossification of the semilunar cartilage.

Fig. 35. Lateral *fabella*.

Figs. 32—35 are from skiagrams taken by Dr Melville at the Orthopaedic Military Hospital, and are reproduced from the *Monograph on the Longbones of the English Skeleton*, by Pearson and Bell, Atlas II, CI A and CI B.

Plates XVII and XVIII. Sections of the sesamoids in Foetal Forms. These confirm Gruber's view that a hyaline *cartilago* is the necessary precursor of a sesamoid. Neither hemisesamoid nor orthosesamoid appears to arise from fibrous cartilage. Fig. 36. Section of lateral *fabella* of newly-born kitten, (i) Magnification 45·5, (ii) Magnification 300; showing hyaline cartilage area (*dad*) bordered by fibro-cartilage (*dcd*). Fig. 37. Section of the *cyamella* of a newly-born kitten, (i) Magnification 45·5, (ii) Magnification 300; *a—bb* hyaline cartilage; *bb—cc* intermediate area showing typical rounded nuclei of hyaline cartilage and elongated spindle nuclei of fibro-cartilage, but no threadlike nuclei of fibro-cytes; *dd—e* fibro-cartilage showing spindle nuclei and threadlike nuclei. Fig. 38. Longitudinal section of metatarsial sesamoid of full-term human foetus, (i) Magnification 45·5, (ii) Magnification 300, see our p. 159. The metatarsial sesamoid has clearly a hyaline *cartilago* as precursor. Fig. 39. Longitudinal section of *patella* of full-term human foetus, (i) Magnification 45·5, (ii) Magnification 300, see our p. 159. The precursor of the *patella* in man is clearly hyaline.

* Tillmans and Bernays had found hemisesamoidal supernumerary *patellae* in cadavers (*Morphologisches Jahrbuch*, IV. Supplement, 1878, S. 442). Presumably these double *patellae* were arranged tandem fashion for Bernays refers to those in cats, mice, and rabbits as similar.

A FIRST STUDY OF THE BURMESE SKULL.

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1. Introductory.

The collection of skulls which forms the subject of the present study was procured from the neighbourhood of Moulmein, in the southern part of Burma, by the late Colonel P. H. Caster, I.M.S., at the request of Professor Karl Pearson. The territory which stretches from the Namkin mountains in the north down the whole length of the Irawadi river and still further down the coast towards the Malay Peninsula, and is administered under the name Burma, embraces many racial units which might be classed together generally as Burmese, but of the Burman proper is only one. The request was for purely Burman skulls, but it was evidently difficult to ensure this, and the collection of 142 skulls includes some that are of a different racial type from the majority. The series is on the whole well-preserved, though some few are rather badly damaged, and cannot yield a full series of measurements.

* The author wishes to acknowledge the very valuable aid she has received from the Department of Industrial and Scientific Research. Without this aid it would not have been possible for her to devote the past two years to craniological training. We have also to acknowledge the assistance of a grant made by the Royal Society Government Grant Committee towards the cost of the plates of this memoir.

Although our material, when divided up into sexes and groups, is far from ample enough to allow us to form decisive conclusions as to the racial constants, and as to the exact relation in which the Burman race stands to other types, it is still by far the largest collection of Burmese skulls yet submitted to craniometrical methods, and until a fuller series shall be available to supplement these, a preliminary study of the type by computing means only will not be labour spent in vain.

2. MEASUREMENTS AND METHODS OF MEASUREMENT.

The following direct measurements* have been taken: C = capacity taken with mustard-seed tightly packed in skull, and then weighed, the worker having previously performed this operation on one of the "crânes étalons" described by Macdonnell† in order to bring her personal equation into line with that of her predecessors. The contents of the crâne étalon were packed into the measuring glass, as well as weighed, and the water capacity was also measured in the glass, in order to obtain the constants for the calculation of capacity. F = Flower's ophryo-occipital length. L' = glabellar projective horizontal length‡. L = maximum length from glabella to occiput§. B = maximum horizontal breadth on parietal bones. B' = least forehead breadth from one temporal crest to the other. H' = basio-bregmatic height. H = height measured on craniophor, from basion to the point vertically above it (generally a little behind the bregma). OH = auricular height, as measured on the craniophor, that is, the height of the skull vertically above that part of the auricular orifice which rests on the top edge of the ear-plugs‖. LB = length from basion to nasion. Q = transverse arc perpendicular to the horizontal plane, passing through the "apex," and terminating on both sides at the top of the ear-rods, this measurement being taken when the skull is on the craniophor. In this measurement the tape is carried on to the ear-rods after passing over the zygomatic ridges. Q' = a similar measurement taken off the craniophor, and terminating at the "auricular points¶". S = sagittal arc from nasion to opisthion. S_1 = arc from nasion to bregma. S_2 = arc from bregma to lambda. S_3 = arc from lambda to opisthion. S_3' = chord from lambda to opisthion. U = horizontal circumference measured directly above the superciliary ridges and round the most projecting mesial part of the occiput, i.e. the most projecting part between the inion and lambda. PH = premaxillary height, measured from alveolar point to the tip of the anterior nasal spine, if such exists, and, where it does not, to the sagittal point of the lower edge of the pyriform aperture. $G'H$ = upper face height from nasion to alveolar point. GB = face breadth from the lower end of one zygomatic-maxillary suture to that of the other, where the suture crosses the lower front rim of the cheek-bone. J = zygomatic breadth, from the most lateral point of one zygomatic arch to that of the other. NH (R) and NH (L) = nasal

* Major lengths have been given to the nearest ·5 mm., the smaller measurements to the nearest ·1 mm.

† *Biometrika*, Vol. III. p. 203. ‡ *Ibid.* Vol. I. p. 415.

§ I have thought it best, on account of a good deal of asymmetry in the series, to take L always in the median plane, and consequently it may occasionally be less than L'.

See p. 182. ¶ See p. 181.

height from nasion to lowest edge, right and left, of pyriform aperture. NB = nasal breadth, greatest breadth of pyriform aperture, wherever it may be. DS = dacryal subtense, being the shortest subtense in the sagittal plane from the bridge of the nose to the dacryal chord, measured by Mérejkowsky's simometer. DC = dacryal chord, i.e. length from dacryon to dacryon. DA = dacryal arc, i.e. the shortest arc over the bridge of the nose from dacryon to dacryon, measured with the tape. SS = simotic subtense, the shortest subtense in the sagittal plane from the nasal bridge to the simotic chord, as measured by the simometer. SC = simotic chord, i.e. the minimum chord between the two naso-maxillary sutures. O_1 = greatest breadth of orbit, R and L, from side to side, using Fawcett's curvature method* to determine the margin on the nasal side†. O_2 = greatest height of orbit, R and L, taken perpendicular to O_1. G_1 = length of palate, from the point of the spina nasalis posterior to an imaginary line tangential to the inner rim of the alveoli of the middle incisors. G_1' = similar measurement to G_1 but taken from base of spine. G_2 = breadth of palate between inner alveolar walls at second molars‡. GL = profile length from basion to alveolar point. fml = length of foramen magnum from basion to opisthion. fmb = greatest breadth of foramen magnum. $G'H$ = upper face height from alveolar point to nasion. $P\angle$ = profile angle, or angle made by the line from the nasion to alveolar point with the horizontal, found by means of Ranke's goniometer when the skull is in horizontal position on the craniophor, as described by Fawcett (*Biometrika*, Vol. I. p. 414). From these direct measurements there were also calculated the various indices: occipital (*Oc. I.*), cephalic (B/L', H/L, H/L', B/H), facial ($G'H/GB$), nasal (NB/NH, R and L), orbital (O_1R/O_2R; O_1L/O_2L), palatal (G_2/G_1), foraminal (fmb/fml), dacryal (DS/DC), simotic (SS/SC). The angles ($N\angle$, $A\angle$, $B\angle$) of the fundamental triangle whose apices are the nasion, alveolar point and basion, were calculated from the measured lengths LB, GL, $G'H$ by the aid of Pearson's Trigonometer in the manner described by Fawcett (*Biometrika*, Vol. I. p. 418). θ_1, the basio-nasal horizontal angle, was obtained by the subtraction of $N\angle$ from the supplement of $P\angle$; θ_2, the basio-alveolar horizontal angle, was arrived at by the subtraction of $A\angle$ from $P\angle$.

Of these various measurements, one or two are new; most are similar to those made by previous craniometricians.

C. The measurements, and methods of measurement, described by Fawcett (*Biometrika*, Vol. I. pp. 416–419) have been adopted exactly, with one exception, viz. that of C, the capacity. Macdonnell's labour-saving device of weighing the mustard-seed to determine its volume has been adopted wherever possible by all later workers in this Laboratory.

The only available one of the three "crânes étalons" used by Macdonnell (*Biometrika*, Vol. III. p. 204) was skull δ. After some preliminary practice in tight

* See *Biometrika*, Vol. I. p. 430. Also *Biometrika*, Vol. VIII. pp. 311, 312.

† Another measurement O_1' (=breadth of orbit measured from dacryon) had to be taken later for comparison with the data obtained for other racial series.

‡ This may occasionally be the same as the measurement between the inner rims of the alveoli of the second molars, but is more frequently rather less than these.

packing, the skull was filled five times in succession by me, and the seed packed again each time in the measuring-glass; after which it was weighed. This operation was also performed by Professor Pearson.

Prepared Skull.

		Cm.³ of seed	Weight of seed in grams
K. Pearson ...	...	1325	1065·45
M. L. Tildesley	...	1325	1071·00
		1325	1062·55
		1320	1063·50
		1325	1064·30
		1325	1060·30
Totals	...	7945	6387·10
Mean	...	1324·17	1064·52

The skull was then filled with water and the water measured in the measuring-glass: this, which should be the true capacity of the skull, came out at 1365 cm³.

We thus arrived at the equation:

$$\text{Volume of 1000 grs. of seed} = \frac{1365 \times 1000}{1064{\cdot}52}\ \text{cm.}^3 = \frac{1000}{779{\cdot}87}\ \text{cm.}^3$$

The result obtained by Macdonnell on this skull was that 1000 cm.³ contain 764·63 grs. of seed. The seed we used had been kept for a large number of years, and was probably slightly more shrunken than that used by him, and would therefore give a denser packing.

H and *H′*. The height of the skull from the basion was measured by Macdonnell to the bregma, not vertically as by Fawcett. In this he was followed by Benington* and Thomson†. His *H* is also the *H* used by Flower and Turner. I have therefore taken, for the purposes of comparison with their results, both these measurements, calling Macdonnell's *H*, *H′*.

Q and *Q′*. We may note that Macdonnell has also measured his *Q* over the bregma, instead of in the vertical plane, following, he says, Virchow and Turner, and Benington does the same, but also takes the vertical *Q* under the title Q_1. Thomson abandoned the bregmatic *Q*, but added *Q′*‡, taken off the craniophor, from the upper margin of one auricular passage to the upper margin of the other. I have done as Thomson has done. With regard to the determination of the position of the apex in defective skulls where the orbits or auricular passages were missing, so that the skull could not be adjusted accurately to the Frankfurt horizontal on the craniophor: Macdonnell said he put it at 2–3 cms. behind the bregma. This evidently varies for different races, since the mean distance for both males and females of our series was just under 15 mms. I have therefore estimated the apex at 15 mms. behind the bregma where it could not be properly determined, and if the orbits

* *Biometrika*, Vol. VIII. p. 296. † *Ibid.* Vol. XI. p. 84. ‡ *Ibid.* Vol. XI. p. 85.

alone were missing have settled the Frankfort plane by means of this apex. All measurements dependent on this approximation are of course queried.

O_1R and O_1L. Of the two methods—the "curvature" method and the "geodesic*"—of completing the orbital margin where it becomes indefinite on the nasal side, I have used the curvature method, which Fawcett herself used and which was followed by Thomson, although Macdonnell and Benington used the geodesic.

G_1'. This was a measurement first made by Macdonnell, and has continued to be used by his successors in this Laboratory.

NH. In all the papers of this school until Thomson's this measurement was defined as taken to the lowest point of the margin of the pyriform aperture, whether this may be to the right or to the left. Thomson says she has taken it to the left, unless this is damaged, and has also measured the right if this be sensibly different; but from the fact that she gives no separate measurement of the right side in her data we infer that her interpretation of the term "sensible difference" applied only to marked nasal asymmetry and that this feature in the Moriori Series was much more regular than among the Burmese, whose maximum difference between nasal height R and L is 2·7 mms. I have measured both sides in all cases.

S_1, S_2, S_3, S_3', *Oc. I.*, *fmb*, *fml*, *fmb*/*fml* are all measurements added to the series by Macdonnell†, and have continued to be made by his successors here.

Oc. I. was called by him the Cerebellar Index, but the more accurate title of Occipital Index was adopted later.

DA and *DC* were first measured by Benington‡, and *DS* was obtained from them by calculation.

SA, *SS*, *SC* were also measured by him§ later, when he had the use of the instrument invented by Mérejkowsky, by which I also have obtained *DS* as well as *SS*.

Sex. The sex of the skulls was determined by Professor Pearson, when classifying them into the three types to be described later (p. 217).

Mandibles. There were no skulls with mandibles in my series.

3. On the Difficulties attached to the Definition of certain Cranial "Points."

Basion. In Thomson's paper it was suggested that the basion and opisthion should be marked off as points before any measurements are made from them‖, it having been noticed that slightly different points may be selected on the different occasions on which use is made of them, e.g. in measuring the basio-bregmatic height, and the foraminal length, and in drawing the sagittal contour. I have adopted the suggestion and find it very advantageous. In selecting basion and opisthion on the foraminal border it is very desirable to bear in mind that the foraminal length will be determined probably by the dividers and the basio-

* *Biometrika*, Vol. I. p. 431. † *Ibid.* Vol. V. p. 91. ‡ *Ibid.* Vol. VIII. p. 297.
§ *Ibid.* Vol. VIII. p. 317. ‖ *Ibid.* Vol. XI. p. 129.

bregmatic height by a large pair of callipers. The points on the border should accordingly be so chosen that they provide a resting point for the vertical or horizontal measuring instrument, i.e. the points should be about central on the curved rim where the surface changes its direction from horizontal to vertical.

Auricular Point. The ambiguities attaching to this "point" have been fruitful in the provision of difficulties, and reduce the value for comparative purposes of a good deal of material. The difficulties have been much discussed already, but I cannot forbear to make my own contribution to the discussion, in the hope of assisting somewhat in the solution of the working problem. The main fact, of course, is that the "auricular point" like most other cranial points is not a point, but an area, only it happens to be nearly the worst of its kind and less susceptible of a definition by which different workers will identify approximately the same position for it. I will summarise the history of this in our own Laboratory.

From the beginning of craniometric work here the Frankfurt Concordat was taken as the basis of our method of measurement, though it has had to be modified in some details—as few as possible—and more exactly defined in others. The Concordat gives the auricular point as that point on the upper rim of the auricular orifice which lies immediately above the centre of the orifice when the skull is adjusted to the Frankfurt horizontal plane. When the skull is placed on a Ranke's craniophor its auricular height is taken from the ear-plugs on which it rests; naturally to achieve stability it is the highest point in the vertical section* through the point of contact with the roof of the orifice upon which the skull rests. But this point may vary with the depth to which the ear-plugs are inserted; and in any case it is not possible to guarantee that this point is immediately above the centre of the orifice, whatever the centre of an irregular trumpet-shaped cavity may be: the section of the orifice might be an oblique oval (such as Le Double refers to†, and I have myself found). Probably, however, the difference, if any, due to this cause would be slight. Fawcett found it needful to drop "the centre of the orifice" and modified the definition of the auricular points to "the highest points of the upper rims of the auricular passages‡." The application of this definition—and probably of any definition by which one could attempt to indicate exactly this exceedingly difficult point—is not simple in a considerable number of skulls, at any rate in my series, owing to the fact that their auricular orifice has no very definite rim. The auricular passage is roofed by the squamous portion of the temporal bone, and the curve upwards and outwards towards the zygomatic ridge is sometimes so smooth that it is hard to say at what point there is a greater change in direction than at any other: this leaves a good deal of room for personal equation in determining the point, and unavoidably so. Of course in many other skulls there *is* a definite break in direction, giving a distinct upper rim, and all stages are to be found between these two extremes. I do not claim to add anything to the definitions, which I have tried to apply in determining my "auricular points." In this connection I may, however, mention one

* By "vertical section" is here to be understood one parallel to the sagittal plane of the skull.

† *Variations des Os du Crâne*, p. 325.

‡ *Biometrika*, Vol. I. p. 413.

feature which I was tempted sometimes to think answered best to the descriptior "upper rim." This is a notch on the postero-superior side whose lower lip, projecting somewhat, suggests a border to the orifice, especially where this notch, as may happen, is carried forward above the opening. Although the edge of this lip, or the curve continued from it where it fails to extend right over the opening, might in many cases be a possible interpretation of the words (one also perhaps giving less room for personal equation between different workers and different schools of workers), there were others in which the upper rim was exceedingly well defined and certainly did not coincide with the curve continued from the notch. I therefore disregarded this feature in all the skulls, endeavouring to identify the break in direction of the upward curve of the bone apart from this.

So much for the determination of the auricular point as defined. In the use of this point for craniophor measurements, however, another difficulty presented itself in a considerable number of cases in my series. The skull is supported on the craniophor by means of ear-plugs, and it is obvious that the ear-plugs must enter a little distance into the orifices for the skull to be supported safely by them: their tips cannot stop short at the auricular points. Now, in some skulls, the roof of the auricular orifice does not rise immediately after leaving the auricular point, nor yet take a horizontal direction, but continues with a slight downward direction for a short distance. When this is so, our auricular points do not actually come in contact with the ear-plugs, which touch a lower part of the roof a little distance inside. From an examination of the transverse contours I find that in 129 cases out of the 259 auricular passages (right and left) which were traced, the auricular point rested on the ear-plugs; in the remaining 130 it was slightly raised. The mean distance of auricular point from top of ear-rod in the whole series was 0·5 mm., and the maximum distance—reached in two cases—was 2·8 mms. Thus the auricular height *OH*, defined as height of skull above the auricular points, when measured by the craniophor, actually takes the height of skull above the point of contact with the ear-rods, and thereby increases by 0·5 mm. in my series the auricular height as defined: an amount which would doubtless vary with every racial series measured.

Certainly this might be reduced or increased by the personal equation of the worker inserting the plugs. I have therefore examined my transverse contours to see to what extent this factor operated in my series. One might take 5 mms. as the minimum distance consistent with safety to which the tops of the plugs could be pushed in beyond the auricular points. In practice I estimate that I myself pushed the plugs in further than this, roughly to 15 mms. beyond the vertical from the zygomatic ridge, which would be between 11 and 12 mms. beyond the auricular point (the difference between the mean ordinates to auricular point and zygomatic ridge in the transverse contours being 3·4 mms. for males, 3·3 mms. for females). What difference might this make? My transverse contours show that the lowest point of the part of the vertical section of the roof of the orifice which lies within 11–12 mms. of the perpendicular through the auricular point—and would thus be the point of contact of ear-plug with the skull as I inserted it—

occurred at a distance from this perpendicular of 5 mms. or less in 239 out of my 259 cases; in the remaining 20 it did exceed that. Except in the case of these 20, therefore, no difference would have been made by my inserting the ear-plugs less far, and the difference made for these few individuals would have a very slight effect on the mean value for my series. In another series it might have more, in others less.

The other direct measurement involving the auricular points was the transverse arc Q which Fawcett* defines as the "cross circumference of the skull measured in a vertical plane from the upper rim of one auricular passage to that of the other," a definition which is repeated by Benington (his Q_1†) and by Thomson‡. We find, however, a note to Thomson's definition which says that "Q is taken with the tape over the skull, and measured from the top of one ear-rod to the top of the other."

Q', on the other hand, "the same measure taken when the skull is off the craniophor," is terminated on each side by the selected auricular points. As the note states, "Q'—owing to the recessing of the margins of the auricular passages—is not quite the same as Q," i.e. in the individual measurements, where the difference is seen to be as much as 4 or 5 mms. in some cases (the excess sometimes being in favour of Q, sometimes of Q'); though the mean values of Q and Q' obtained for the Moriori skulls are extremely close together, there being only 0·2 mm. difference between them for both males and females.

In my own series the individual differences were of the same order, though more numerous, and the mean value of Q (the craniophor measurement) was 1·8 mms. shorter for males and 2·2 mms. for females. These differences can be practically accounted for by the recessing of the margins of the auricular passages: the transverse contours show the zygomatic ridges at a mean height of 3·0 mms. (♂ and ♀) above the auricular points (selected) and projecting 3·4 mms. beyond them in the males, 3·3 mms. in the females, so that the mean distance between the two on the skull is 4·5 mms. (♂ and ♀). For Q', taken *off* the craniophor, the tape traverses this 4·5 mms. distance; I have interpreted Thomson's note to mean a vertical drop to the ear-rod from the zygomatic ridge, which gives a mean distance of 3·0 mms. This would lead us to expect mean difference of 1·5 mms. for my series between Q and Q', a difference quite reasonably close to what I have obtained. In the case of the Moriori crania the constants for this region of the skull would probably differ from those of the Burmese; indeed the peculiar conformation of the Moriori skull in the region above the auricular orifice was one of its distinguishing racial characters. On the other hand, it is possible that there may also have been some difference between the interpretation put by Thomson and myself on the definition of the auricular point, as we have already seen that in many skulls there is room for a considerable personal equation in identifying the "upper rim of the auricular passage."

* *Biometrika*, Vol. I. p. 416. † *Ibid.* Vol. VIII. p. 296. ‡ *Ibid.* Vol. XI. p. 85.

4. Determination of the Inion.

The determination of the inion has been discussed by more than one distinguished craniometrician and anatomist; it has been dealt with in our own biometric school in a paper by M. A. Lewenz and K. Pearson* in the course of an argument which demonstrated its unsuitability as terminal for an arc measurement on the living, and which therefore was concerned to emphasise the very real difficulties in identifying this point, rather than to make any constructive suggestions as to the method of doing so.

If I venture to take up this subject again, the circumstance is accounted for by the fact that the position of the inion still had to be marked on my sagittal contours, in spite of the difficulties.

To recapitulate briefly: the original definition of the inion by Broca and in the Frankfurt Verständigung, as the meeting-place of the lineae nuchae superiores, was abandoned generally by later scientists, Merkel† pointing out that the inion corresponded to the meeting-place of the lineae nuchae supremae.

I was able to identify a point which accorded with this later definition in 53 of my male skulls; in seven it was vague; and of one I have only the skull-cap sawn off above the inionic region. In female skulls the inion is of course usually less pronounced, and the point as defined could be determined in only 58 out of 74 cases. These numbers may be compared with those given by Professor Thane‡ where the inion could be determined in 41 out of 43 ♂ and 43 out of 60 ♀ English skulls. The seven infant skulls in the Burmese collection naturally have little to indicate its position.

I was thus left with 7 ♂, 16 ♀ and 7 infant skulls, for which the position corresponding to this same anatomical point had to be inserted in my contours, within as narrow limits of accuracy as could be achieved.

As a guide in doing this, I examined the skulls of my series in which the inion is to be found by Merkel's definition, and noted the following facts with regard to them:

a. The lineae supremae are frequently very vague, even untraceable, although their juncture may be clear and prominent.

b. The lineae superiores are usually, though by no means invariably, more clearly marked; their junction is generally recognisable, and in all cases in my collection where this is so, they are seen to curve downwards into a V before meeting.

c. The lineae supremae, when descending to form a U about the median plane, as is often the case, occasionally short-circuit as well, giving an upper ridge with a slight depression between that and the lower junction which is our inion. This may be misleading at first if the inion be not boldly marked.

* *Biometrika*, Vol. III. p. 375.

† "Die Linea nuchae Suprema," Leipzig, 1871. Quoted by Schwalbe, *Zeitschrift für Morphologie und Anthropologie*, Vol. I. p. 24, 1899.

‡ *Biometrika*, Vol. III. p. 376.

d. Of the 53 male skulls, where the inion as defined by Merkel could be determined, it lay in 21 on the common horizontal tangent* to the lineae superiores, in 17 rather below, in 15 above.

In the 58 female skulls, it lay on the common tangent in 37 cases, in 9 below, in 12 above. Its average position in my series is evidently therefore at the point where the median plane cuts the common horizontal tangent to the lineae nuchae superiores.

e. The lineae superiores also short-circuited sometimes, and formed a misleading ridge along their common tangent, as well as being continued V-wise to a point below in the normal manner.

f. Where I found an inion projecting from the skull in a downward direction to form a hook, into which both the lineae supremae and lineae superiores are continued, it was possible to detect the spot at which the lineae supremae stopped short of the others. When such a skull was regarded in profile, the curve downwards towards the tip of the hook showed a slight break at this point.

Having made these general observations upon my 53 ♂ and 58 ♀ skulls, I made use of any that might serve to elucidate the system of lines upon my doubtful skulls, determining the inion by *d* where there was nothing to indicate for it another position.

With reference to the very considerable variations in form of the inion, I quote Le Double†: "Il peut être remplacé par une dépression, faire défaut, ou constitué par une éminence, lisse ou rugueuse, mamelonnée ou recourbée en crochet à sommet inférieur," adding that only in one case in our whole series of 141 possessing the inionic region is the inion clearly replaced by a depression.

5. CONTOURS.

A study was made of the three chief contours, Transverse, Horizontal, Sagittal. The contours were drawn by means of the Klaatsch contour tracer in the manner described by Dr Benington in his paper on Cranial Type Contours‡. From certain average measurements on these were constructed type contours for each of the three racial classes we were dealing with. The method employed was as follows:

(*a*) *The Transverse* (*vertical or auricular coronal*) *Section.* The skull being placed on the craniophor and adjusted in the usual manner to the standard horizontal plane, the horizontal rod of the craniophor determines the "apex" of the skull, which is marked with a pencil stroke. Short lines are also drawn on the skull just above the auricular passages, and in the same vertical plane as the middle of the ear-rods, leading into the auricular passages, and meeting the little horizontal lines by which the position of the auricular points has been marked. The skull being now transferred to the Klaatsch apparatus, the vertical plane is made horizontal, and the contour traced from as far into the orifice as the pointer

* That is, the tangent perpendicular to the sagittal plane.

† *Variations des Os du Crâne*, p. 12.

‡ *Biometrika*, Vol. VIII. p. 123.

will go, through the auricular points and the apex line*. The line joining auricular point to auricular point is the auricular line of the contour, and the line perpendicular to it through its mid-point is the vertical axis *MA* of the transverse vertical contour. This vertical axis is divided into ten equal parts by aid of the proportional compasses, and lines parallel to the auricular line are drawn through the points of division. These lines are numbered from below upwards—1 (the auricular line), 2, 3, 4, 5, 6, 7, 8, 9, 10; numbers 2 to 10 will not necessarily be bisected by the vertical axis owing to the asymmetry of the skull. Another line ($A\frac{1}{4}$) is taken one quarter of the last section from the apex, and yet another ($M\frac{1}{4}$) one quarter of the first section from the auricular line. Ordinates are also drawn from the points *ZR* (*R*) and *ZR* (*L*) where the zygomatic ridges are crossed (that is, from the line of the ridge which is a continuation of the upper edge of the zygomatic arch). These last three lines were added to what had been done previously, in order to give an idea of the shape of the contour just above the auricular points. (When they are omitted, and the terminals of mean 1 and 2 splined up in the type contour, an impression is given that the contour is traced only from the zygomatic ridges.) A tangent is drawn to that part of the contour which traces the roof of the auricular orifice, at its lowest point within about $1\frac{1}{2}$ cms. from the mouth of the opening, i.e. at the points which will have rested on the horizontal ear-rods on the craniophor, the rods being inserted at not more than about $1\frac{1}{2}$ cms.† This tangent will not necessarily be parallel to the auricular line. Finally, a line parallel to the tangent is drawn touching the skull in the region of the apex. These two tangents aim at reproducing on the transverse contour the measurement taken on the craniophor, where a horizontal rod (i.e. parallel to the ear-rods) is brought down touching the skull from above, the height of it giving the *OH* of the direct measurements. The importance should be realised of adjusting the skull very carefully so that the pointer passes exactly along the lines leading into the auricular orifices, which give the plane in which the ear-rods lie. If, when this vertical plane is made horizontal, the pointer goes slightly above or below these lines, the outline of the roof of the orifice may be considerably altered (since it offers a rather irregular surface), and consequently also the position of the tangent.

The following measurements are now taken: length of vertical axis *MA*, length to *R* and *L* of each of the horizontals (the sum of 1*R* and 1*L* being the interauricular diameter), distance between the two tangents, and the co-ordinates of the two zygomatic points. They are entered in the columns of the record for each contour, in the order above given, as follows:

MA	1*R*=1*L*	$M\frac{1}{4}R$	$M\frac{1}{4}L$	2*R*	2*L*	3*R*	3*L*	4*R*	4*L*	5*R*	5*L*	6*R*	6*L*

* As a matter of fact I drew as much of the vertical section of the auricular passage both inferior and superior as my instrument permitted and these have been preserved for future study when comparative material is available.

† This is the distance to which I inserted the rods. It may be advisable (see p. 182) to shorten it

7R	7L	8R	8L	9R	9L	10R	10L	$A\frac{1}{4}R$	$A\frac{1}{4}L$	TH	ZR (R)		ZR (L)	
											y	x	y	x

The means of the columns give the corresponding lengths for the type skulls of each of our own groups, and from these we can plot the points in the type section. A line is now drawn passing through the plotted points, and we thus reach the transverse type contour. Since the skull is resting face upwards during this tracing, the contour as drawn gives the *norma facialis*, and this is the aspect that Benington has chosen. I myself, like Thomson, have reversed on the type contour, the right and left of the individual contours, thus giving the *norma occipitalis*. (See Figs. I–VI.)

(*b*) *The Glabellar Horizontal Section.* This is obtained by placing the skull on the craniophor and by aid of the scriber marking points on the same horizontal plane as the nasion: one, the gamma, in the occipital region where it is crossed by the sagittal plane, the other in the region above the left auricular passage*.

The skull is now adjusted on the Klaatsch support until these three points are found to be in the same horizontal plane. The horizontal contour is then drawn parallel to this plane, but through the glabella; it will fall below the lambda. The points in the median plane immediately above the nasion and below the lambda on this contour are specially marked with the Klaatsch tracer. They will be spoken of as F and O, the glabellar and occipital points of the horizontal contour. The points (T_R and T_L) at which the contour crosses the temporal lines, right and left, are separately marked with the tracer as Thomson has done: it is found more accurate to mark them in this way, as there is a certain liability to error in turning a sharp corner such as that at the temporal lines, unless special care is exercised. This completes the tracing of the contour. The fronto-occipital line FO is now drawn, and is divided into ten equal parts by the proportional compasses; lines perpendicular to FO are drawn through the points of division and numbered 2 to 10, travelling from F to O; one at $O\frac{1}{4}$ to obtain the occipital curvature; lines through $F\frac{1}{4}$ and $F\frac{1}{2}$ to give the frontal curvature, and to complete this, ordinates from T_R and T_L.

We may note that when these temporal lines are not marked, as in Dr Benington's contours, the resultant effect in the frontal region of the type contour is much more rounded, and suggests at first sight that the plane in which the contour was taken is higher than the glabella, where the temporal ridge is much less prominent.

* It is not really necessary to make such a point above both auricular passages, since three points suffice to determine a plane; though a fourth point might act as a check on the orientation of the skull upon the Klaatsch holder

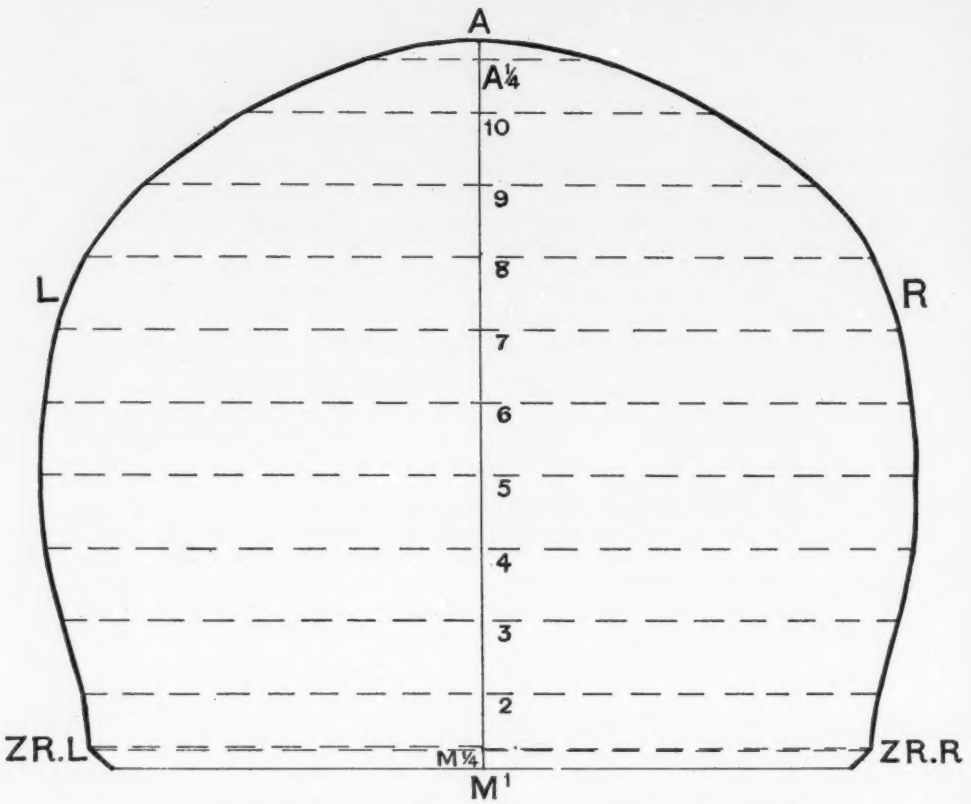


Fig. I. Burmese Crania. Type A°(Burmans). ♂. Vertical Contour. (44 crania.)

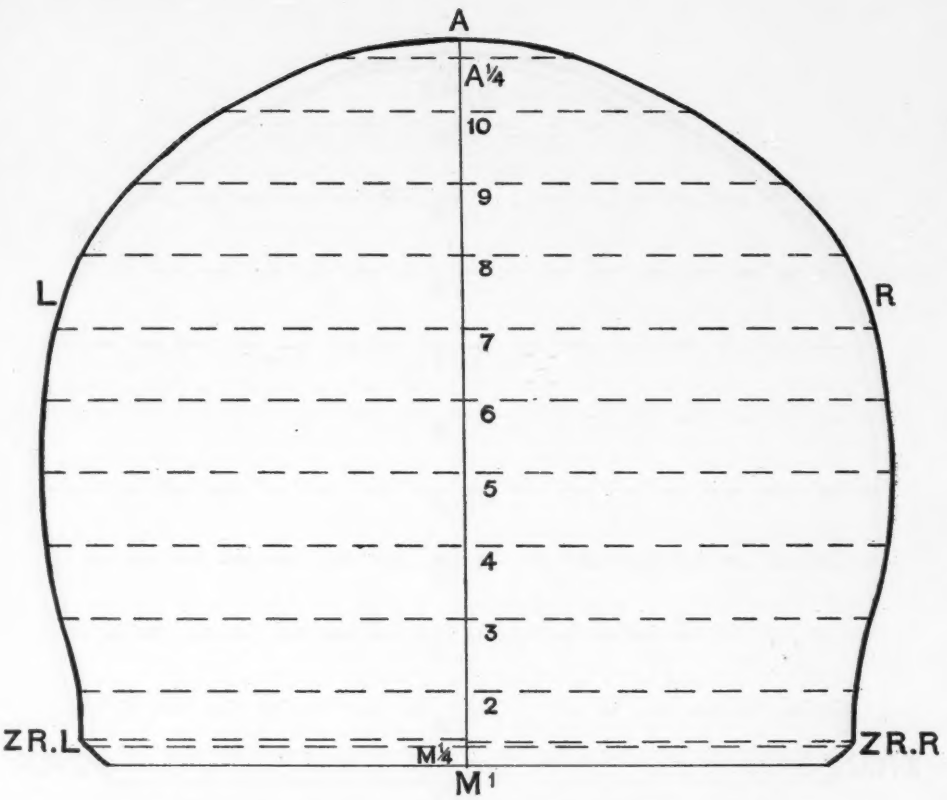


FIG. II. Burmese Crania. Type B (? Hybrids). ♂. Vertical Contour. (7 crania.)

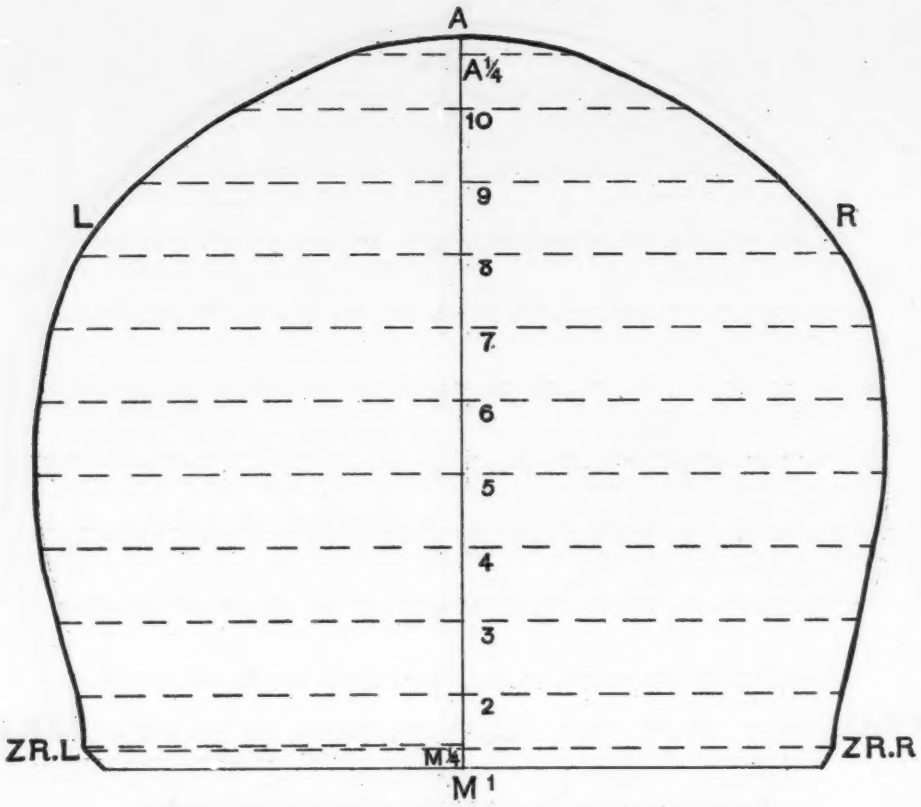


Fig. III. Burmese Crania. Type C (? Karens). ♂. Vertical Contour. (8 crania.)

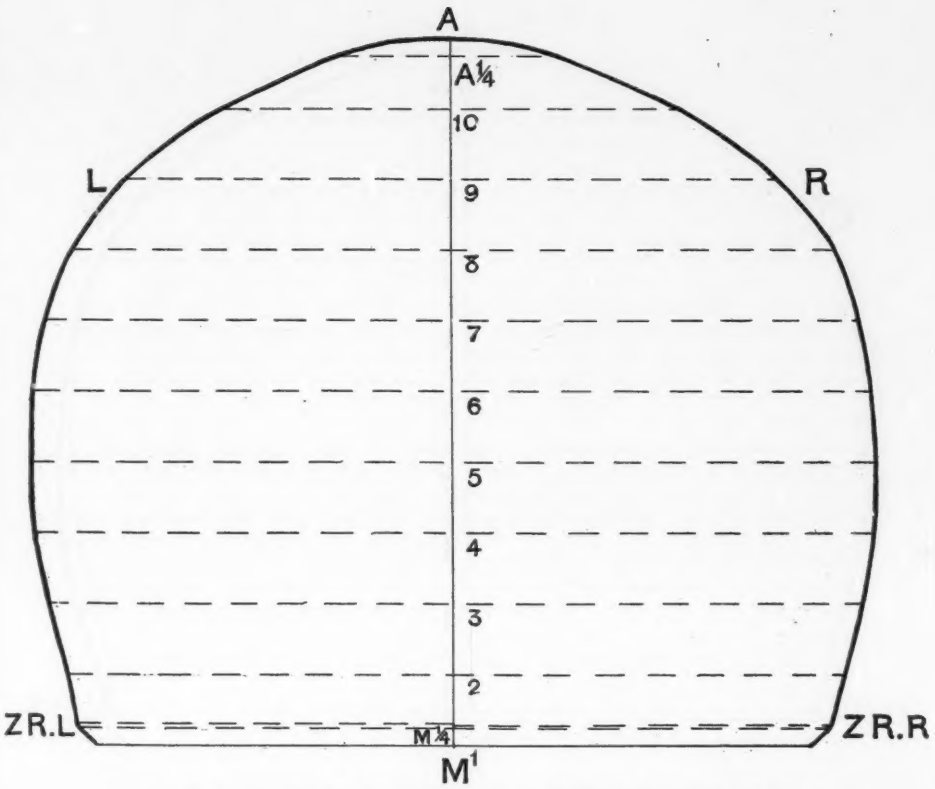


FIG. IV. Burmese Crania. Type A (Burmans). ♀. Vertical Contour. (38 crania.)

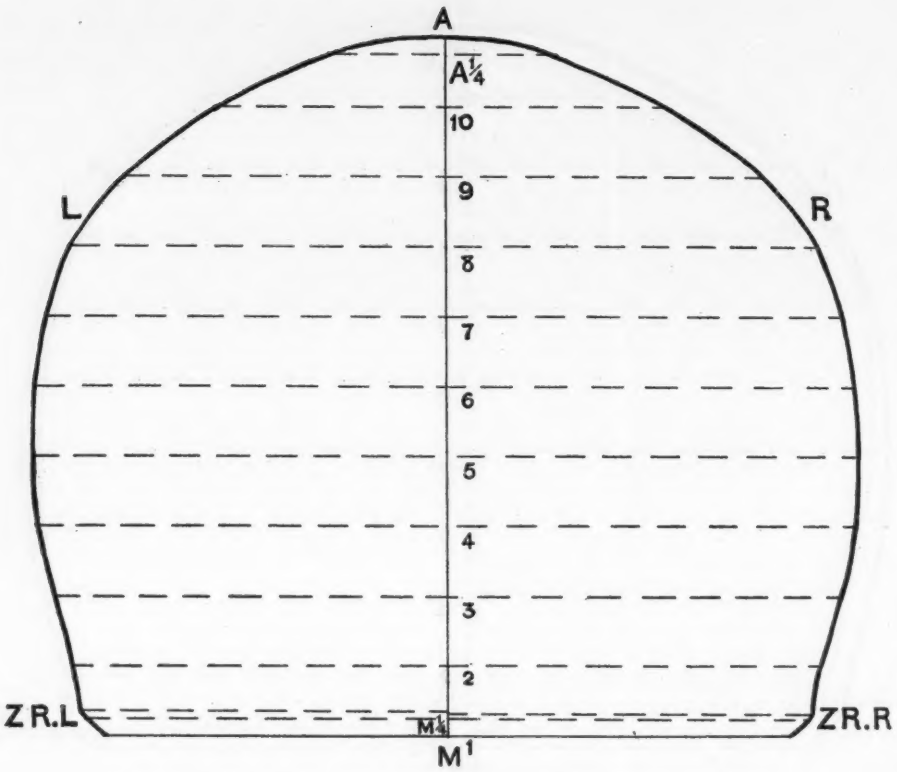


Fig. V. Burmese Crania. Type B (? Hybrids). ♀. Vertical Contour. (17 crania.)

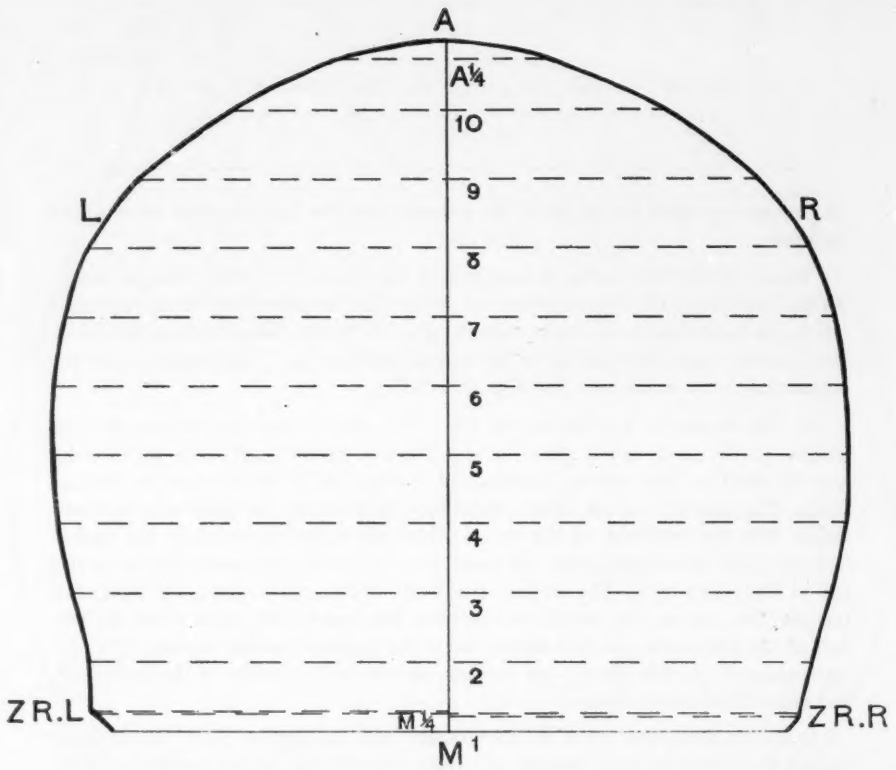


FIG. VI. Burmese Crania. Type C (? Karens). ♀. Vertical Contour. (18 crania.)

We have the following measurements recorded for each available skull:

FO	$F\frac{1}{2}R$	$F\frac{1}{2}L$	$F\frac{1}{2}R$	$F\frac{1}{2}L$	*2R*	*2L*	*3R*	*3L*	*4R*	*4L*	*5R*	*5L*	*6R*	*6L*

7R	*7L*	*8R*	*8L*	*9R*	*9L*	*10R*	*10L*	$O\frac{1}{2}R$	$O\frac{1}{2}L$	T_R		T_L	
										y	*x*	*y*	*x*

The means are again found for all the columns, and the type contours constructed as before.

When a horizontal contour is being traced the skull is lying with its apex downwards. In plotting the type contours we reverse the measurements to the right and left in the individual contours, in order to give the horizontal sections as looked at from above when the skull is in its normal position apex uppermost; thus we obtain the *norma verticalis*. (See Figs. VII–XII.)

(*c*) *The Sagittal or Median Section.* The point γ and the apex having been already marked on the skull, this is placed in the Klaatsch support right side downwards, and adjusted so that nasion, bregma and lambda all lie in the same horizontal plane. The contour is then drawn from the point where the basi-occipital bone unites with the sphenoid, to the basion; from the opisthion round to the nasion and along the nasal ridge; from the nasal spine to the alveolar point and on to the end of the palate spine. The alveolar point, the nasion, the bregma, the apex, the lambda, the gamma, the inion, the opisthion, the basion, the point where the left half of the transverse palatine suture meets the sagittal palatine suture (all lying approximately in this plane), are marked, as also the projection of the auricular* and sub-orbital points upon it.

It will be noted that when we have determined our sagittal plane for contouring, by the nasion, bregma, lambda, this plane is not bound to pass exactly through the inion, the opisthion, basion and alveolar point, nor exactly along the ridge of the nose, the mesial ridge of the premaxilla, and the middle of the palate to the tip of the nasal spine. Various slight asymmetries of the skull will throw these points out, and our plane will miss them. It is these things themselves, however, that we want represented in our sagittal contour—it will not, for instance, give a measure of nasal prominence, if, the nose being skewed as often happens, we pass the pointer along the nasal bones but off the nasal ridge. I therefore admit that, having fixed my plane, I have not kept rigidly to it, but have slightly depressed

* See p. 181.

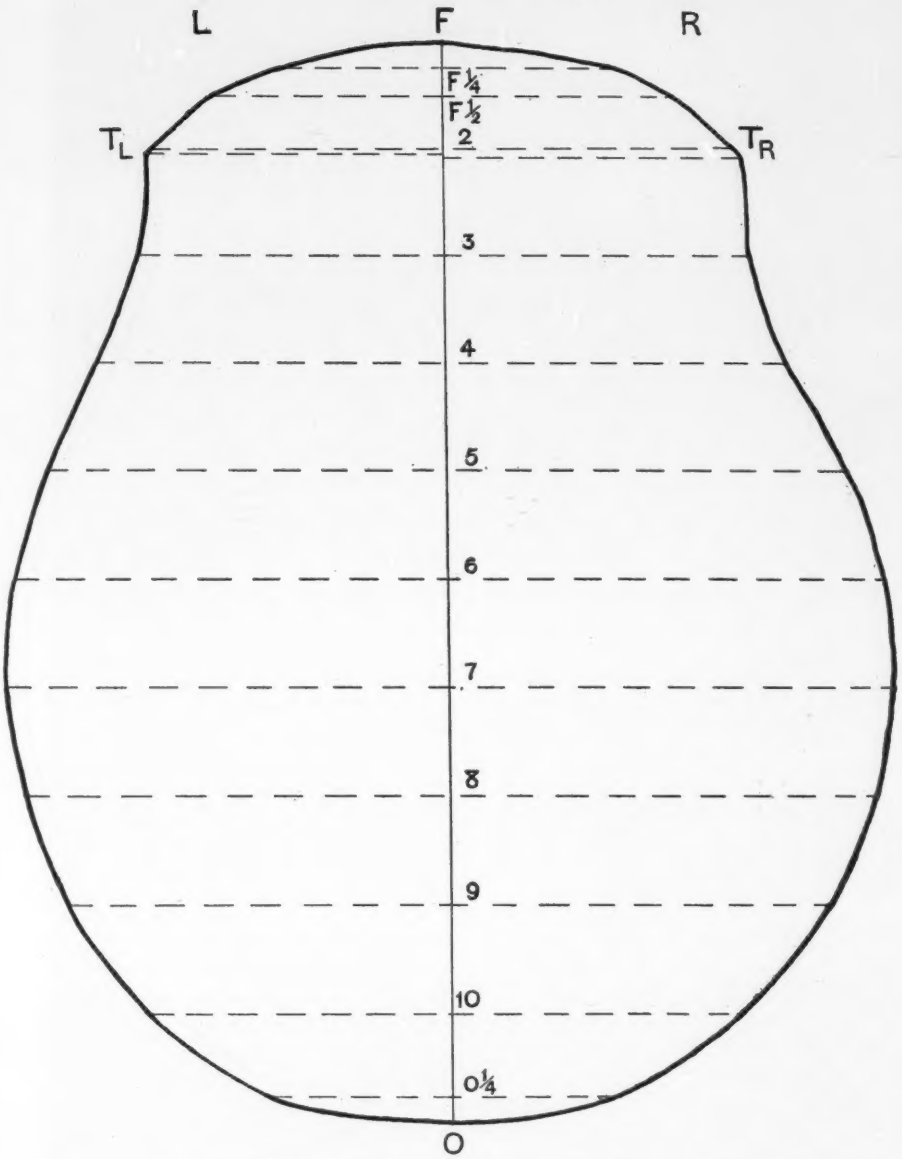


FIG. VII. Burmese Crania. Type A (Burmans). ♂. Horizontal Contour. (44 crania.)

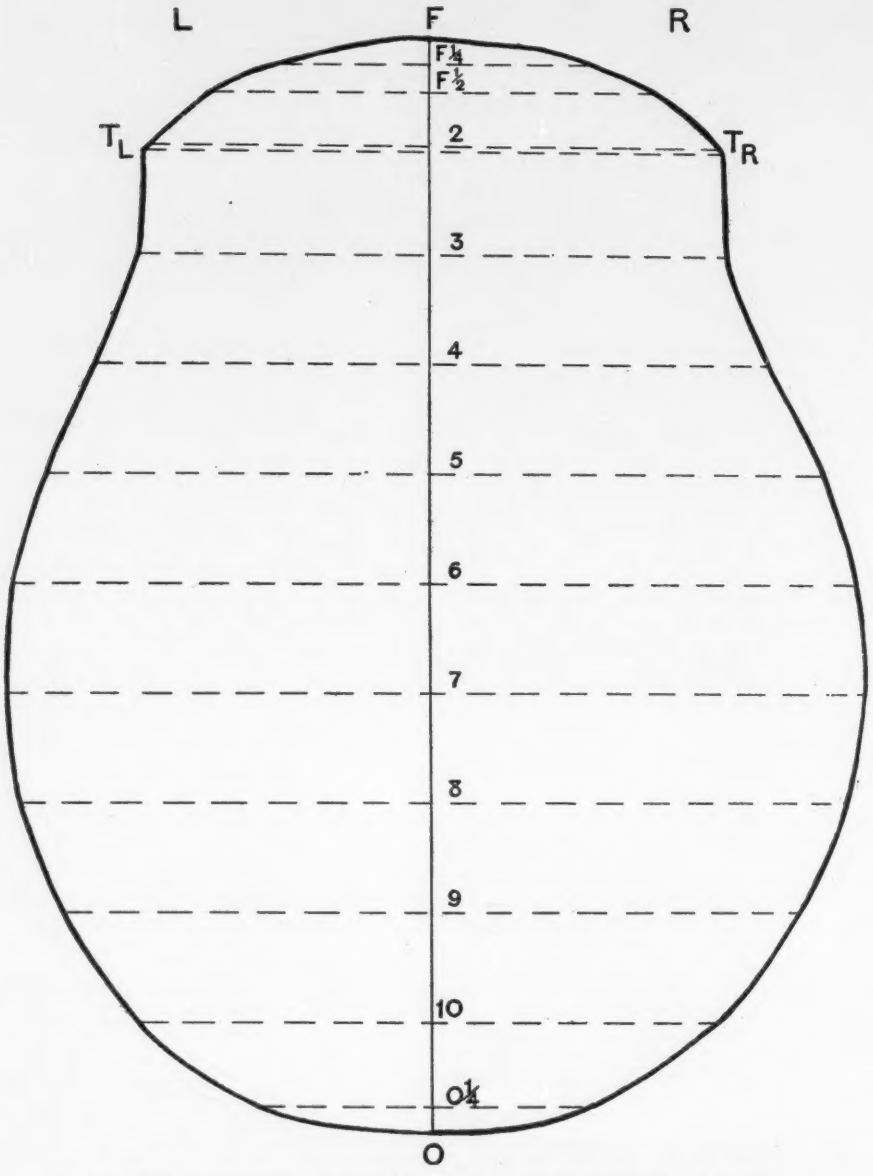


Fig. VIII. Burmese Crania. Type B (? Hybrids). ♂. Horizontal Contour. (7 crania.)

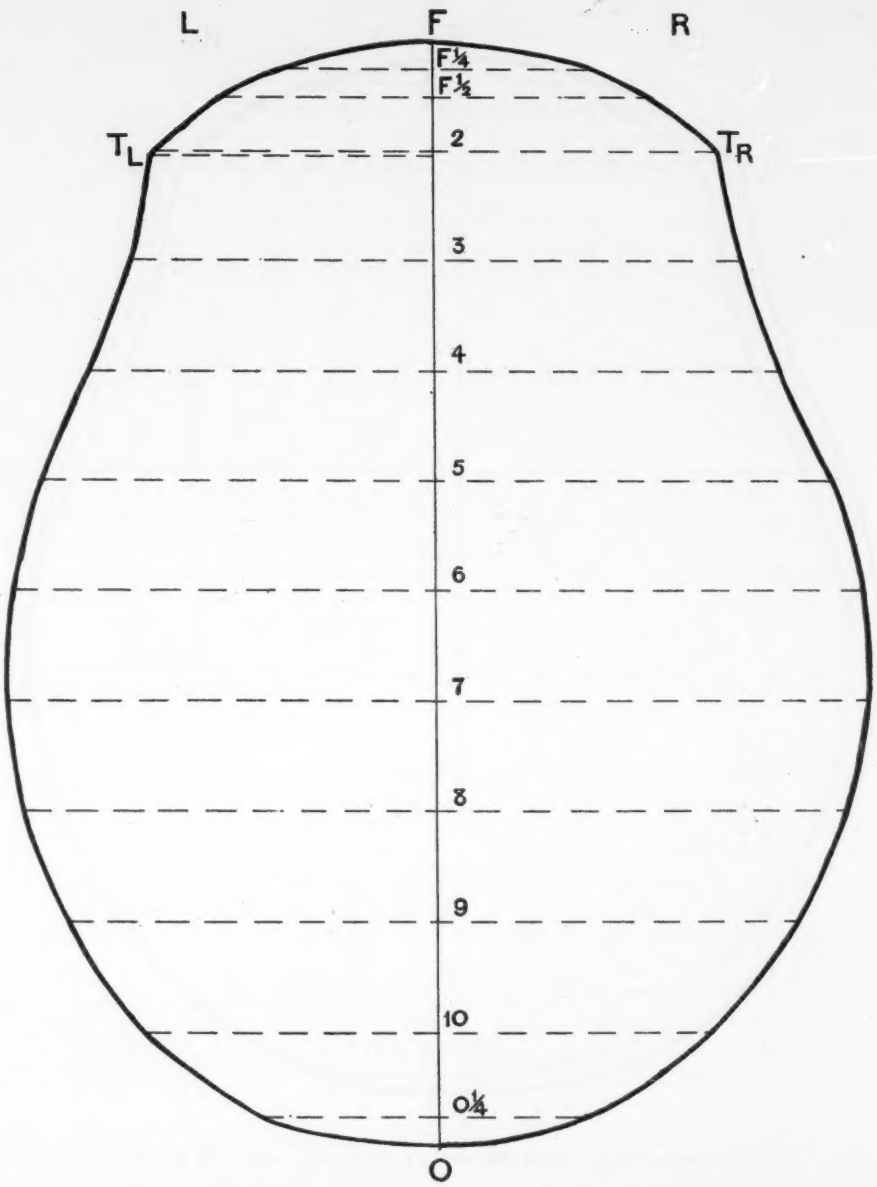


FIG. IX. Burmese Crania. Type C (? Karens). ♂. Horizontal Contour. (8 crania.)

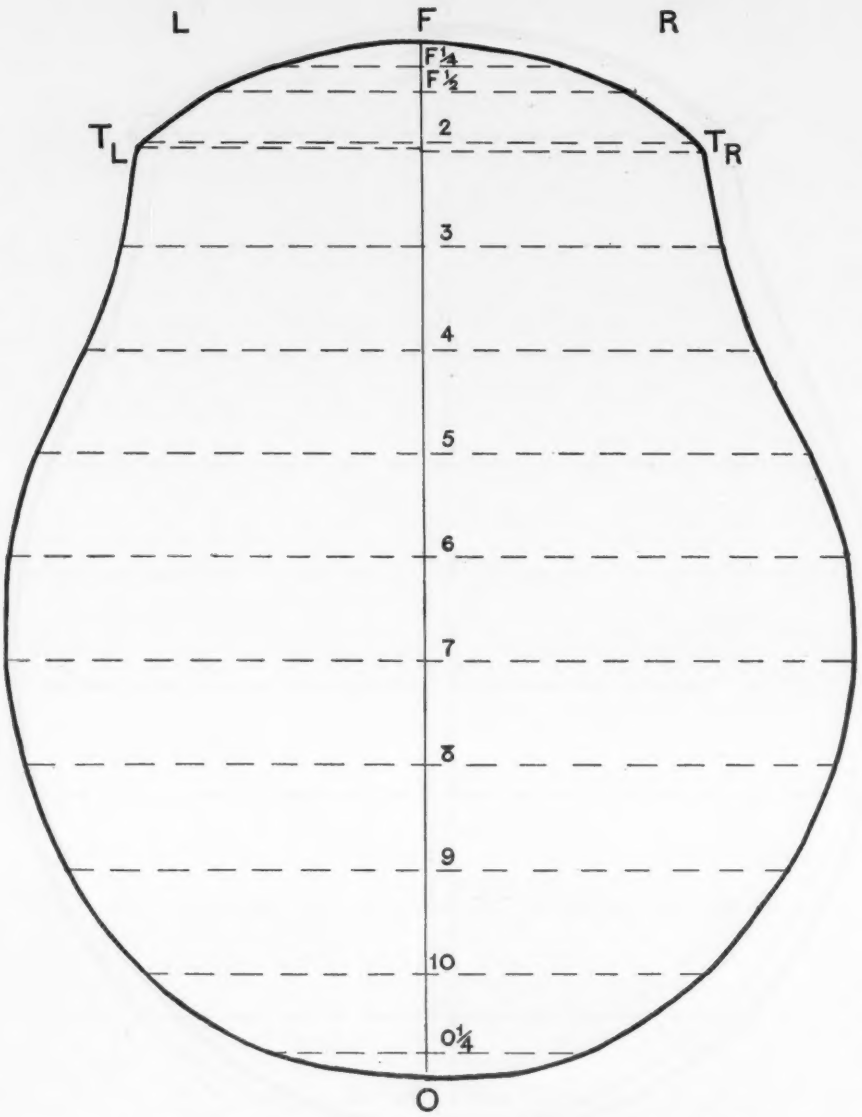


FIG. X. Burmese Crania. Type A (Burmans). ♀. Horizontal Contour. (39 crania.)

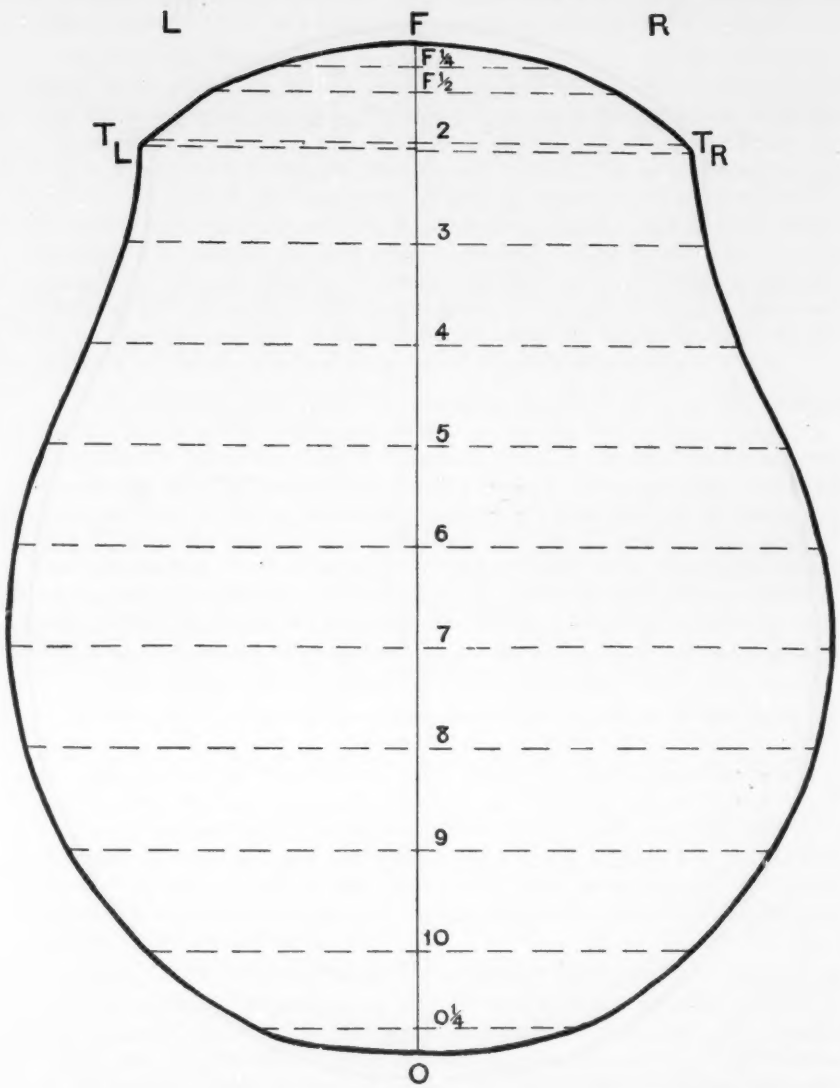


FIG. XI. Burmese Crania. Type B (? Hybrids). ♀. Horizontal Contour. (17 crania.)

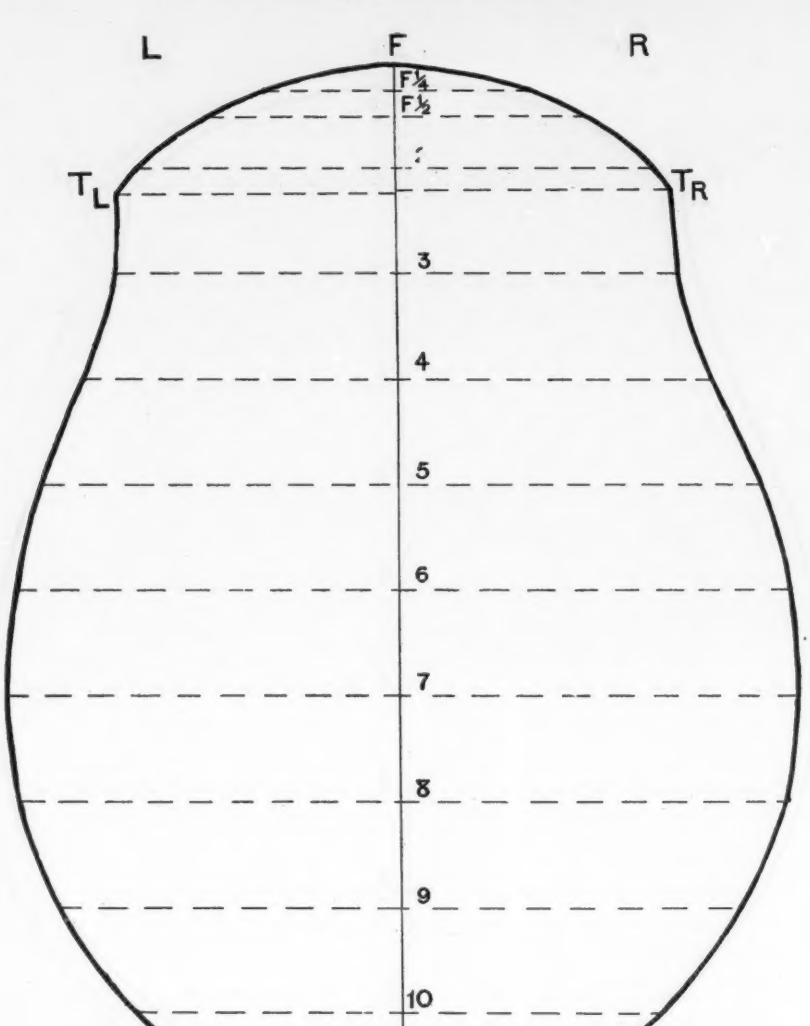


Fig. XII. Burmese Crania. Type C (? Karens). ♀. Horizontal Contour. (18 crania.)

or elevated the pointer if necessary when approaching one of these characters, in order to pass right over or along it, and represent it properly in the tracing*.

We now take the horizontal line through the nasion, i.e. the line linking the nasion to the gamma (γ), as our horizontal axis, and divide it into ten equal pieces with the proportional compasses. The nasion counting as 0 and the other divisions from nasion to gamma as 1, 2, ... 9, ordinates above are drawn through these ten points perpendicular to the axis, also through a point ($N\frac{1}{4}$) at a quarter of the distance of 1 from N, and two points ($\gamma\frac{1}{4}$ and $\gamma\frac{1}{8}$) a quarter and an eighth of the distance of 9 from gamma respectively. Ordinates 8, 9 and $\gamma\frac{1}{4}$ are produced below the axis, to the contour line, and another ordinate is added below it from a point $\gamma\frac{1}{2}$, halfway between 9 and γ. Ordinates are drawn from the bregma, lambda, opisthion, and sub-orbital point perpendicular to the axis. Vertical tangents to the most projecting points of the glabellar and occipital regions are drawn to $N\gamma$ produced, the points of contact being named G and Oc respectively.

The basion is joined to the nasion and gamma; so also is the inion. The alveolar point is joined to the nasion and basion, giving the fundamental triangle. To determine the vertex a horizontal tangent is drawn to the top of the skull, and an ordinate drawn to $N\gamma$ from the point of contact. To give some measure of nasal characteristics, the tip (L) of the nasal contour has been joined to the nasion, so that NL gives the nose length, and the angle between NL and $N\gamma$ is the angle of nasal prominence. Some nasal ridges exhibit a double curve, turning downward towards the tip so that for a little distance NL either cuts the outline or coincides with it. The point where NL first meets the outline of the ridge is marked L', and NL' is also measured; where there is no double curve NL of course is equal to NL'. The maximum subtense is drawn from NL to the curve above L'.

It was desired to take some measurement of the proportions of the palate. A horizontal was therefore drawn through the alveolar point and a vertical line drawn to it through the point where the left half of the transverse palatine suture meets the sagittal palatine suture. Another horizontal was drawn through the highest point† in the contour taken along the palatine process of the maxilla, and the junction of this with the vertical already drawn gave the point P of which I have measured the co-ordinates. It will be seen that I have taken measurements only of the palatine process, the ordinate of P giving its greatest vertical height, and the abscissa its horizontal length.

Finally, a line is drawn through the sub-orbital point parallel to the $N\gamma$ line, and a perpendicular dropped to this from the apex gives the auricular point which is to be used for the contours. Now if the plane passing through the nasion, bregma and lambda, which we have used as our sagittal plane, were exactly at right angles to the horizontal plane defined as passing through the left sub-orbital point and

* Our Klaatsch tracer is provided with a vertical knife-edge as well as a tracing pointer to meet these difficulties, but as the pointer must be used for horizontal and transverse sections the labour of attachment of the knife-edge for the sagittal is against its use. Duplicate instruments would be desirable.

† It very frequently occurs that the point where the sutures meet is itself the highest point.

the two auricular points (in practice the "craniophor auricular points*"), then when the sagittal plane is adjusted in a horizontal position for this contour, the interauricular axis will be in a plane vertical to it, and the projections of the two auricular points on our paper will be in the same straight line as the sub-orbital point, this line being parallel to the horizontal axis $N\gamma$. If, in addition, our vertical plane (which determines the apex and has been made perpendicular to the horizontal plane) be also exactly perpendicular to the sagittal plane, the projections of our two auricular points will coincide. They will also coincide with the perpendicular drawn in the sagittal contour from the apex to the horizontal line through the sub-orbital point. All this, however, supposes a high degree of symmetry in the skull, such as I have found comparatively few skulls to possess. We must also take into account the fact that the "craniophor auricular points"—i.e. the points which actually touch the ear-rods and from which the horizontal plane is determined—are frequently not accessible by the pointer of the Klaatsch when the skull is lying on its side, so that it is the "auricular points by definition" which are actually used for projection on the paper. We thus see that it would be possible, but for the limitations of our contour tracer, to obtain five "auricular points" on every sagittal contour, and while they might all coincide, the odds are against such coincidence. As it is, we have three, and of these I have used that obtained by the perpendicular from the apex on the sub-orbital horizontal, for my measurements; but shall discuss later the differences of position between this and the other two.

This completes the measurements which were used in the construction of the type sagittal contours. (See Figs. XIII–XVIII.)

I have to place on record my thanks to Miss A. G. Davin for the preparation of all the graphs of my type contours.

Our record-sheet entries are:

$N\gamma$	Vertex		β		Glabella	
	x from N	y	x from N	y	Gx	Gy

Ordinates above $N\gamma$													
0	$N\frac{1}{4}$	1	2	3	4	5	6	7	8	9	$\gamma\frac{1}{4}$	$\gamma\frac{1}{8}$	γ

* See above, p. 181

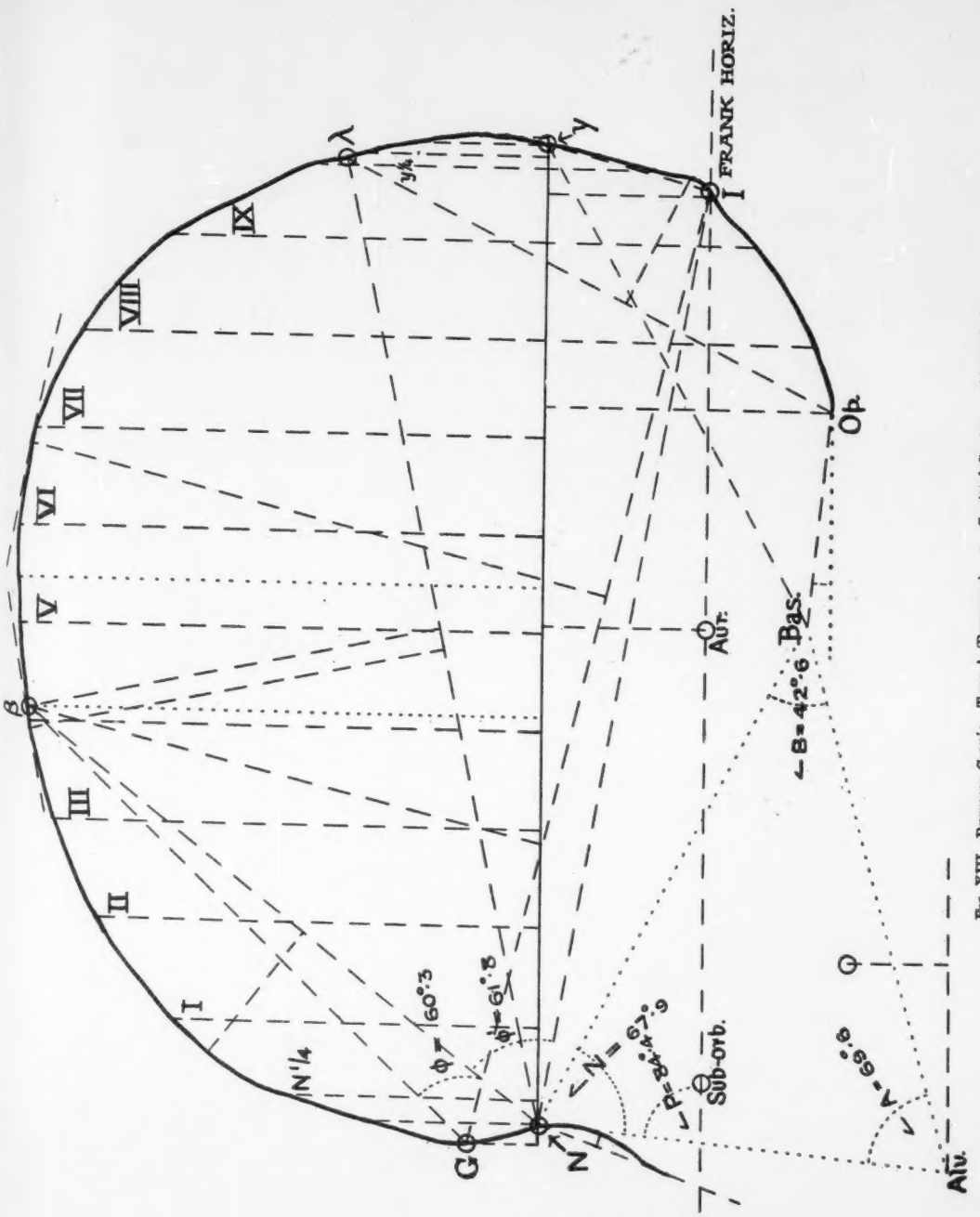


FIG. XIII. Burmese Crania. Type A (Burmans). ♂. Sagittal Contour. (44 crania.)

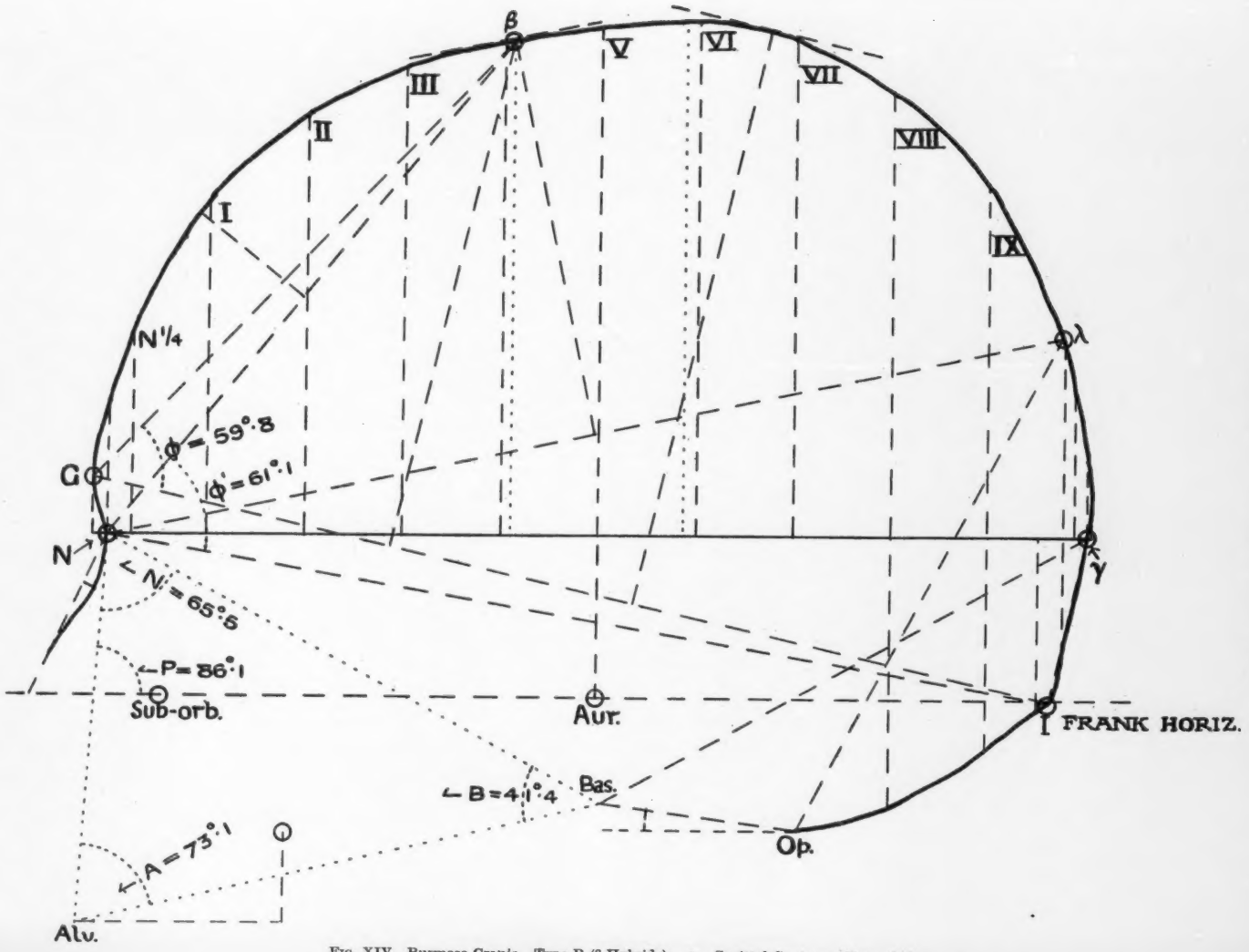
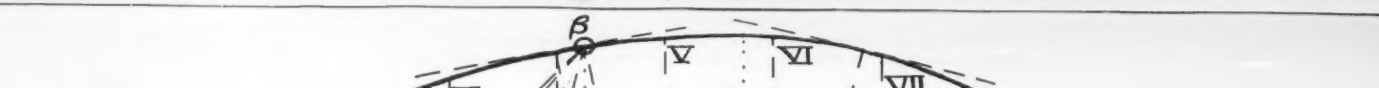
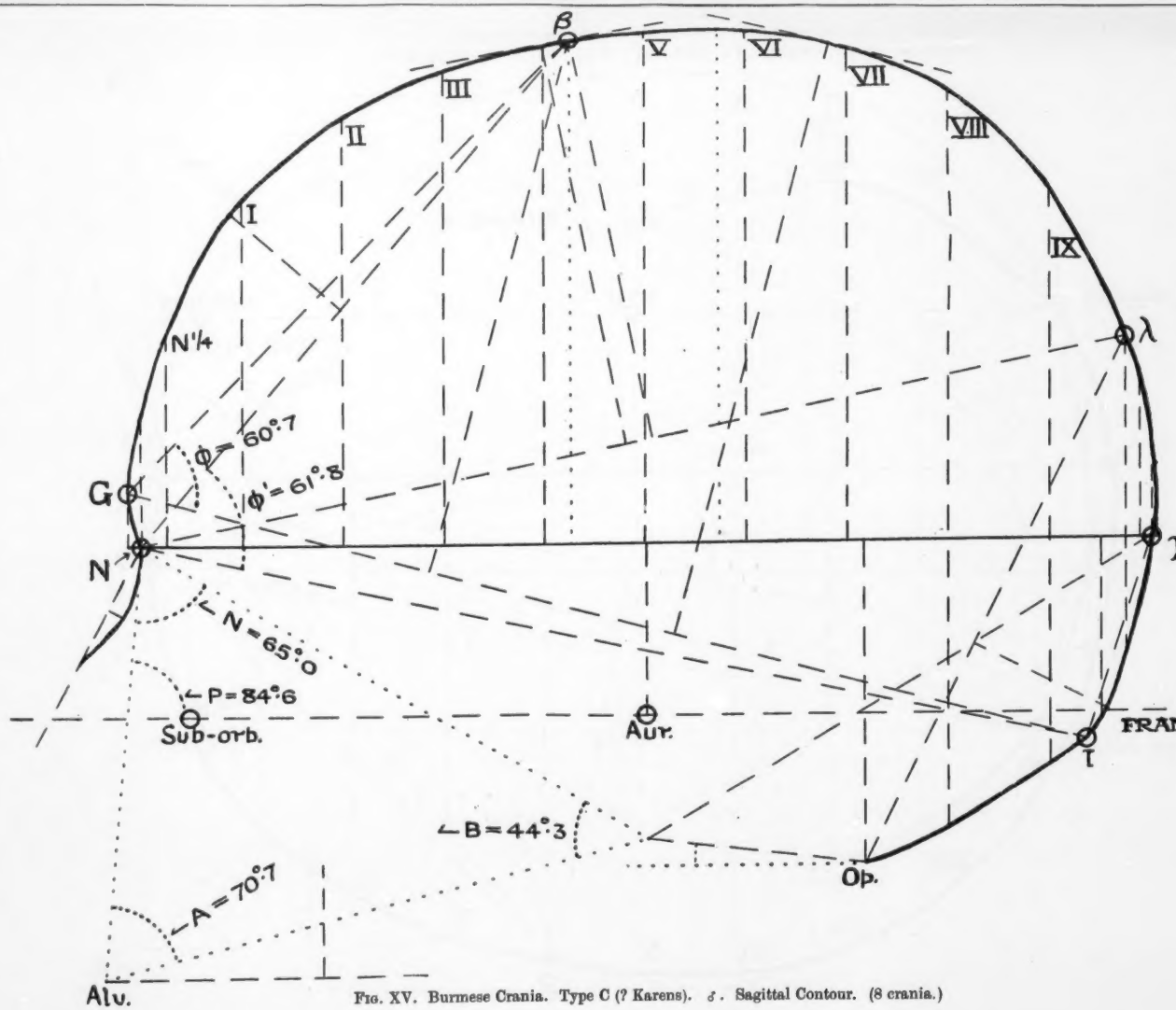


FIG. XIV. Burmese Crania. Type B (? Hybrids). δ. Sagittal Contour. (7 crania.)





Alv.

FIG. XIV. Burmese Crania. Type B (? Hybrids). ♂. Sagittal Contour. (7 crania.)

FIG. XV. Burmese Crania. Type C (? Karens). ♂. Sagittal Contour. (8 crania.)

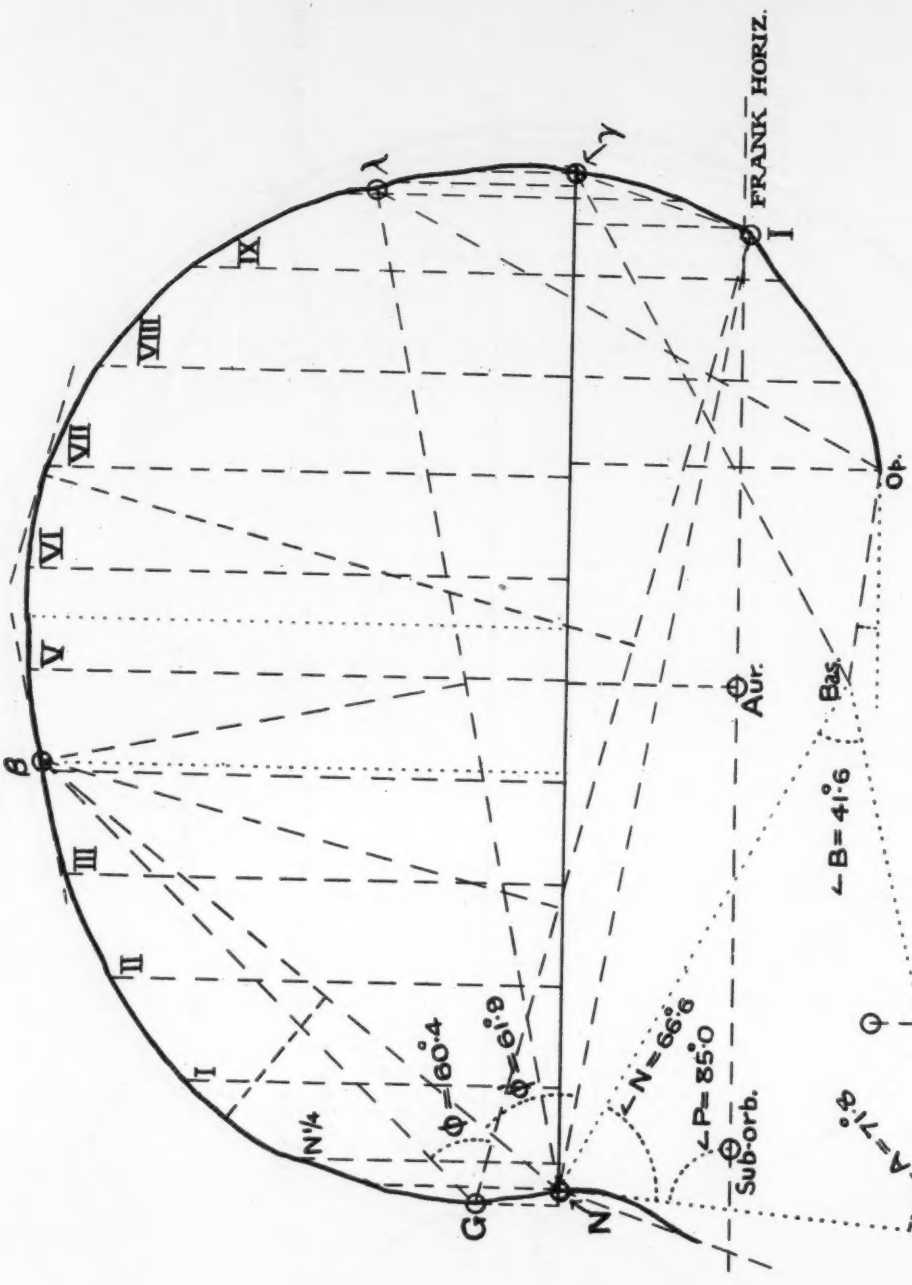


Fig. XVI. Burmese Crania. Type A (Burmans). ♀. Sagittal Contour. (38 crania.)

Fig. XVI. Burmese Crania. Type A (Burmans). ♀. Sagittal Contour. (38 crania.)

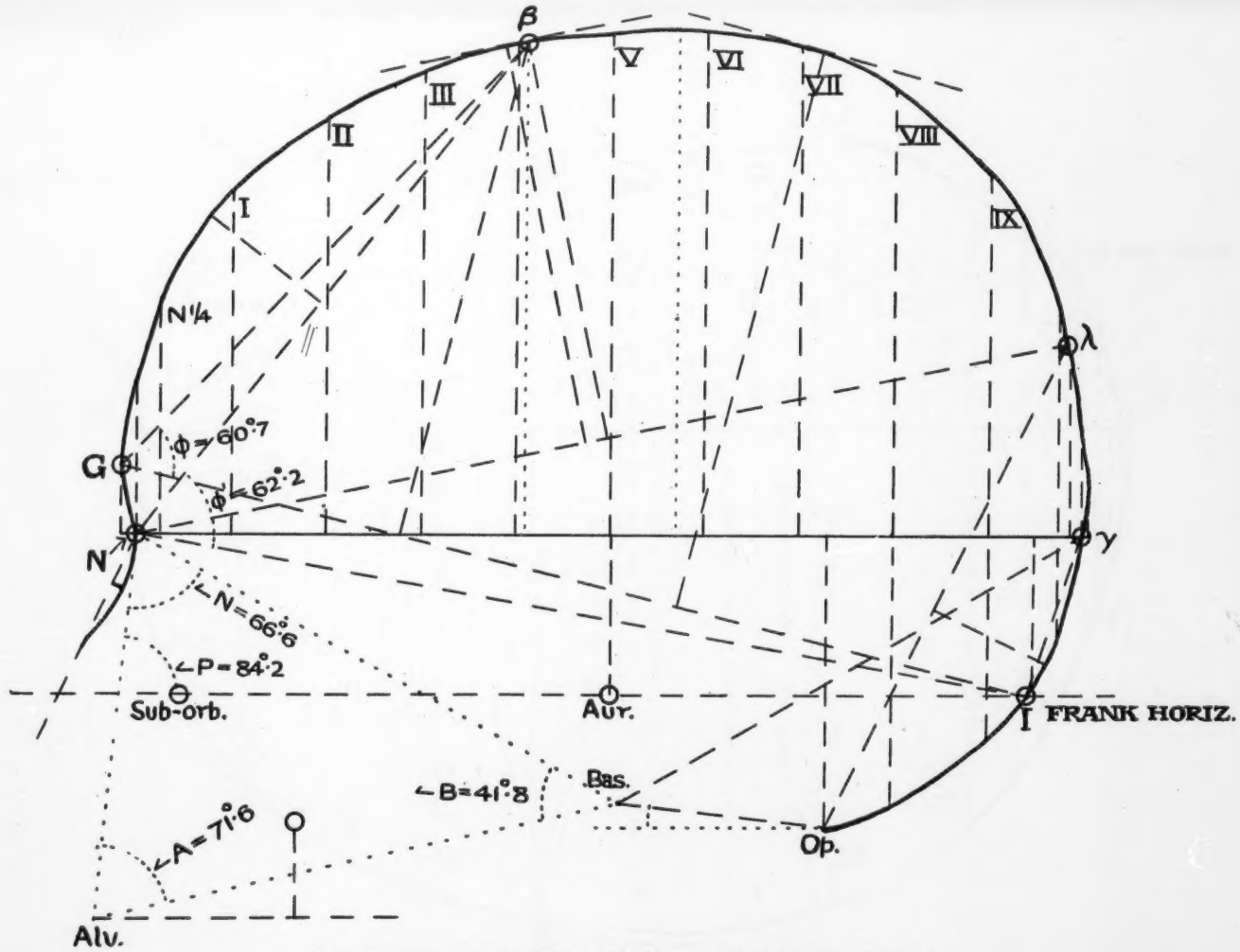


Fig. XVII. Burmese Crania. Type B (? Hybrids). ♀. Sagittal Contour. (17 crania.)

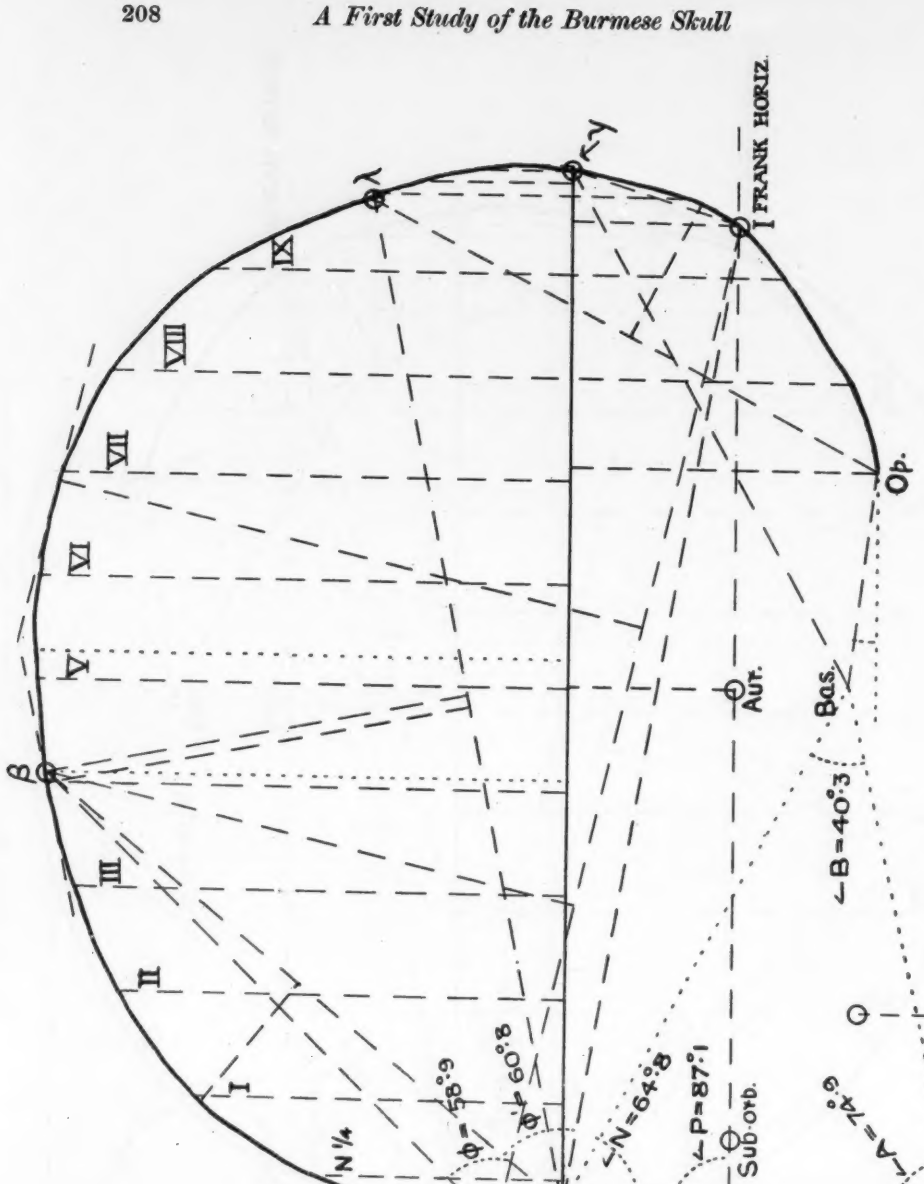


Fig. XVIII. Burmese Crania. Type C (? Karens). ♀. Sagittal Contour. (18 crania.)

Ordinates below $N\gamma$				Occipital point		λ		Sub-orbital point	
8	9	$\gamma\frac{1}{2}$	$\gamma\frac{1}{4}$	x from γ	y	x from γ	y	x from N	y

Auricular point		Opisthion		Inion		Basion	
x from γ	y	x from γ	y	γI	NI	γ Bas.	N Bas.

Alveolar point		Nose				Palate	
N to A.P.	Bas. to A.P.	NL	NL'	Max. nasal subtense		Max. subtense	
				x from N	y	x from A.P.	y

In addition to these measurements I took a number of others which were not used in construction of the type. This was done with a double object: partly to compare their means with the corresponding measurements that were read off from the type skull and thus test the accuracy with which extra values are given by the type (theoretically they are not bound to be exactly the same); partly for comparative purposes. They are as follows:

Lengths.

$N\beta$	$\lambda Op.$	$N\lambda$	β subtense to $N\lambda$		GI	β subtense to GI	
			x from N	y		x from G	y

Angles.

$\phi = \angle \beta GI$	$\phi' = \angle \beta NI$	$A\angle$	$N\angle$	$B\angle$	$P\angle$	$fml\angle$

i.e. nasion to bregma, lambda to opisthion, nasion to lambda, co-ordinates of bregmatic subtense to $N\lambda$, glabella to inion, co-ordinates of bregmatic subtense to GI; angles at glabella and nasion respectively, between chords to bregma and inion; angles of the fundamental triangle, at alveolar point, nasion and basion; profile angle or inclination to the horizontal of the line from nasion to alveolar point; and, lastly, the inclination of the basio-opisthionic line to the horizontal.

The mean position in the type contour of the terminals of all these measurements had already been determined in some other way, so these were not used in building up the type contour.

There were, however, four maximum subtenses also measured on the individual crania, and inserted in the type contour wherever the outline could reasonably be made to pass through them, which was generally the case; as with the last-named measurements, points, whose mean position is obtained in one way, are not theoretically bound to lie on a "mean" outline obtained in another, though they will be very close to it.

The maximum subtenses of which I have obtained the mean length and position are: maximum frontal subtense to nasio-bregmatic chord; maximum occipital subtense to opisthio-lambda chord; and maximum calvarial subtense, using nasio-lambda and glabella-inion lines respectively as base-lines.

Max. frontal subt. to $N\beta$		Max. occip. subt. to $\lambda Op.$		Max. subt. to $N\lambda$		Max. subt. to GI	
x from N	y	x from λ	y	x from N	y	x from G	y

6. On the Accuracy of Contour Values for the Determination of Cranial Constants.

It was promised in the last craniometrical paper published by this Laboratory* that the causes of discrepancies between contour and direct measurements should be investigated more closely with a view to the improvement of technique.

I will deal first with those measurements alone which are given by the means of characters in the individual contours, not with those which are read off from the type contour and not used in its construction.

I was hampered at the outset in my attempt to make the two methods of measurement give equal results by the fact that our contour tracer was no longer in quite good condition. Having been in frequent use for several years, the point of the scriber which is passed round the bone had been gradually worn down; and since one usually traces from left to right, it was not only slightly shortened, but rubbed away more on the right side (looking along the scriber towards the tip). The point

* *Biometrika*, Vol. xi. p. 131 footnote.

of the scriber is therefore not immediately above the point of the ink-container; but a little behind and to the left of it; with the result that a point on the skull will be projected a little in front of and to the right of its true position: 0·3 mm. in front and 0·4 mm. to the right, the hypotenuse of the triangle giving us 0·5 mm. displacement of the actual point.

When the scriber is moved round the skull in the usual direction, i.e. left to right, making an angle of about 60° with the direction in which it is travelling, the whole 0·5 mm. displacement operates to throw the tracing *within* its true position. Thus, in taking a measurement from one side to another of the tracing or wherever the positions of the scriber have been approximately parallel but in the opposite sense, one gets a result which is about 1 mm. less than the same measurement taken direct on the skull.

That the Klaatsch contour tracer had to be used with this defect, was a state of affairs due to the war. It was impossible to get the instrument over to the makers, and skilled instrument-makers here were too much occupied with other work to do this repair properly.

In comparing contour and direct measurements, however, I was aware of a certain discrepancy for which this instrumental defect did not wholly account. Further investigation revealed another source of error in the drawing-board on which the tracing was done. This was found to be slightly warped, in such a way that when the Klaatsch was tracing the top and base of the skull in the sagittal and transverse contours, and the sides in the horizontal contours, the instrument was tilted slightly away from the skull. As nearly as I can estimate it this tilt is only about 0·5° along each of the two long sides of the board, but at the height at which the scriber is placed during most of the contour tracing, this suffices to throw its tip about 0·5 mm. backwards, and consequently the line traced about 0·5 mm. *within* its true position*. Where this operates on two opposite sides of the tracing, we get maximum shortening of about 1 mm. on this score.

We will now proceed to the comparison of our actual results.

Burmese mean Glabellar-Occipital Length.

Types:	Males			Females		
	A	B	C	A	B	C
From direct measurements	173·5 (44)	173·8† (8)	176·7 (8)	166·5 (26)	163·8 (17)	170·2 (18)
From sagittal contours	172·0 (44)	174·4 (7)	175·5 (8)	165·7 (39)	162·8 (17)	169·1 (18)

* I fully admit that the possibility of such a defect should have occurred to me before the board was used. As, however, it was not realised till afterwards, when the long process of contour-drawing, ruling, measuring and averaging the measurements was complete and could not be repeated, it has had to remain uncorrected.

† For the seven skulls contoured, mean of direct measurements is 175·8.

In the position of the skull on the board when the scriber was passing over the glabella and occiput the warping of the board would have no effect. If we add to the contour measurements 1 mm. which the defective Klaatsch was responsible for docking, the agreement is seen to be very close indeed.

The length as obtained from horizontal contours does not tally with the above, of course, since their plane does not pass through the occipital point. It is on the average 2 mms. less.

Burmese Mean Auricular Height, OH.

Types:	Males			Females		
	A	B	C	A	B	C
Measured on craniophor ...	117·7 (44)	116·7 (7)	116·9 (8)	111·7 (38)	111·9 (17)	110·1 (17)
Measured on sagittal contours	116·3 (44)	114·9 (7)	115·4 (8)	110·6 (38)	110·0 (17)	109·5 (18)
Measured on transverse contours ...	117·1 (44)	116·1 (7)	116·1 (8)	111·5 (38)	111·4 (17)	109·4 (18)

In the case of the auricular height as given by the sagittal contours, we have to allow for some shortening due to both contour tracer and drawing-board defects. The tracer passes over the bregma inclined at about 60°: the bregma is therefore thrust about 0·5 mm. nearer to the auricular point; in marking the auricular point the tracer is not held aslant, and the shortening of the distance in question at this point will be about 0·3 mm.; i.e. about 0·8 mm. in all through the defect in the tracer. The warping of the board would affect both terminals and further shorten the line by about 1 mm. The addition of 1·8 mms. to the sagittal contour means brings them into closer agreement with the craniophor means except in the case of C♀; and the difference of 1·2 mms. there obtained is not serious. With the transverse contours, neither source of error displaces the mid-point of the auricular line which is one terminal, but they combine to displace the apex, our estimate (already used) being about ·8 mm.

Burmese Mean Upper Face Height, G'H.

Types:	Males			Females		
	A	B	C	A	B	C
Direct measurements	71·4 (39)	68·2 (8)	74·5 (7)	66·0 (34)	66·1 (17)	63·9 (16)
From sagittal contours	70·7 (38)	67·9 (7)	74·2 (7)	65·7 (34)	65·7 (15)	63·9 (16)

In marking the terminals of this measurement (nasion and alveolar-point), the positions of the Klaatsch were parallel and pointing in the same direction. The warp

of the board would not affect the nasion, but might have a slight effect on the position of the alveolar point, shortening the line slightly in some cases.

Burmese Mean Nasio-Basion Length, LB.

Types:	Males			Females		
	A	B	C	A	B	C
Direct measurements	98·5 (43)	98·8 (8)	100·5 (7)	93·9 (39)	94·7 (17)	95·7 (17)
From sagittal contours	97·4 (43)	98·0 (7)	100·2 (8)	93·0 (39)	93·3 (17)	94·7 (18)

In marking the basion, defects in both board and tracer would operate to thrust the point about 1 mm. in the direction of the obelion; for the nasion we need only allow for the defect in the tracer which would carry the point about ·5 mm. towards the occiput. If we consider the angle at which these two directions cut each other, the shortening shown of about 1 mm. in the contour measurements seems very reasonable.

Mean Length of Foramen Magnum, fml.

Types:	Males			Females		
	A	B	C	A	B	C
Direct measurements	36·7 (43)	35·4 (8)	37·3 (8)	34·7 (39)	35·1 (16)	35·1 (18)
From sagittal contours	36·8 (42)	35·1 (8)	37·1 (8)	34·7 (39)	35·3 (16)	35·1 (18)

The contour tracer occupied parallel positions in marking these points, and was pointing in the same sense. The defects in board and tracer displace their true positions to the same extent and in parallel directions, thus having no effect on the distance between them. The correspondence between the two sets of results is seen to be remarkably close.

Burmese Mean Basio-Bregmatic Height, H'.

Types:	Males			Females		
	A	B	C	A	B	C
Direct measurements	136·8 (43)	136·6 (7)	140·1 (8)	131·4 (39)	131·9 (17)	129·9 (18)
From sagittal contours	134·1 (43)	133·6 (7)	137·5 (8)	128·8 (39)	129·3 (17)	128·3 (18)

As in the case of the auricular height allowance must be made for shortening due to both defects, which we estimated at about 2 mms.

The addition of this again gives us reasonably close results.

Burmese Mean Length from Basion to Alveolar Point, GL.

Types:	Males			Females		
	A	B	C	A	B	C
Direct measurements	96·6 (39)	93·8 (8)	95·9 (7)	90·7 (34)	91·5 (17)*	89·2 (16)
From sagittal contours	96·5 (38)	93·6 (7)	95·6 (7)	90·3 (34)	90·4 (15)	88·8 (16)

* 91·0 for 15 skulls.

Although these two points are displaced by both the defects in our apparatus, the warping displaces each at right angles to the line joining them, and though the Klaatsch would tend to push the alveolar point rather towards the basion, this is partly compensated by the direction in which it moves the basion, so that we should expect a very slight error of less than 0·5 mm. from this source.

It will be seen that the results are good.

We now turn from lengths to angle measurements.

Comparison between Mean Angles obtained from Direct Measurements, and Mean Angles from Individual Contours.

Types:		Males			Females		
		A	B	C	A	B	C
$A\angle$	Direct measurements ...	70°·5 (38)	73°·4 (8)	71°·2 (7)	72°·1 (34)	72°·0 (17)	75°·2 (16)
	From sagittal contours ...	69°·6 (38)	72°·6 (7)	70°·4 (7)	71°·8 (34)	71°·6 (15)	75°·0 (16)
$N\angle$	Direct measurements ...	66°·8 (38)	65°·2 (8)	64°·4 (7)	66°·2 (34)	66°·5 (17)	64°·5 (16)
	From sagittal contours ...	68°·0 (38)	66°·1 (7)	64°·8 (7)	66°·8 (34)	66°·9 (15)	64°·7 (16)
$B\angle$	Direct measurements ...	42°·6 (38)	41°·4 (8)	44°·5 (7)	41°·8 (34)	41°·5 (17)	40°·3 (16)
	From sagittal contours ...	42°·5 (38)	41°·2 (7)	44°·8 (7)	41°·4 (34)	41°·6 (15)	40°·3 (16)
$P\angle$	Direct measurements ...	86°·0 (39)	85°·9 (8)	84°·0 (7)	85°·8 (33)	84°·0 (17)	86°·4 (16)
	From sagittal contours ...	84°·4 (38)	86°·1 (7)	83°·6 (7)	85°·0 (34)	83°·8 (15)	87°·2 (16)

Of these angles the first three are angles of the fundamental triangle, whose sides (LB, $G'H$, GL) we have already discussed, and have seen to be very little affected by the errors due to defective apparatus. Only one of the differences exceeds 1°: the correspondence is therefore good.

Contouring errors should not affect the direction of the lines ($N\gamma$ and N Alv.) by which the profile angle ($P\angle$) is determined. The direct measurements were

taken by means of Ranke's goniometer when the skull was on the craniophor. As this is a measure which we find it difficult to take accurately*, I am very satisfied to have obtained so good a degree of correspondence in the two sets of figures. All the measurements of which use has been made in this enquiry into the accuracy of contour tracing are means of individual contour measurements; they are not read off from type contours, since such readings are not bound theoretically to be identical with the corresponding means of the individual contours. Thus any discrepancy due to this cause has been obviated. The conclusion is that contours may be used to replace direct measurements and can do so with great accuracy. The results are, however, more subject to error through the conditions under which the work is done, and what seems to be a very slight defect in the instrumental conditions may be considerably magnified in the tracings.

Comparisons between Measurements on Type Contours and Means of Individual Contour Measurements which are not used in Construction of Type Contours.

Types:		Males			Females		
		A	B	C	A	B	C
$N\beta$	Mean contour value	112·6 (44)	111·3 (7)	113·1 (8)	106·7 (39)	106·0 (7)	108·2 (18)
	Type contour ...	112·6	111·5	113·1	106·5	105·9	107·5
$\lambda Op.$	Mean contour value	93·7 (43)	97·3 (7)	100·2 (8)	92·8 (39)	91·8 (17)	91·2 (18)
	Type contour ...	93·9	97·5	100·0	91·8	91·5	91·1
$N\lambda$	Mean contour value	167·9 (44)	169·2 (7)	171·4 (8)	162·0 (39)	160·3 (17)	163·4 (18)
	Type contour ...	168·0	169·4	171·3	161·4	160·2	163·1
β subt. to $N\lambda$	Mean contour value	72·0 (44)	70·2 (7)	70·0 (8)	68·5 (39)	68·0 (17)	68·1 (18)
	Type contour ...	72·4	70·4	69·7	68·5	68·2	68·4
GI	Mean contour value	168·7 (44)	171·1 (7)	169·3 (8)	160·7 (39)	157·7 (17)	164·1 (18)
	Type contour ...	168·9	171·4	169·3	160·7	157·7	164·3
β subt. to GI	Mean contour value	91·2 (44)	90·8 (7)	93·6 (8)	85·3 (39)	85·7 (17)	86·5 (18)
	Type contour ...	91·4	91·2	93·8	85·4	86·1	86·7
β bas. (H')	Mean contour value	134·1 (43)	133·6 (7)	137·5 (8)	128·8 (39)	129·3 (17)	128·3 (18)
	Type contour ...	134·7	134·4	137·5	129·3	129·7	128·7
fmb	Mean contour value	36·8 (42)	35·1 (8)	37·1 (8)	34·7 (39)	35·3 (16)	35·1 (18)
	Type contour ...	36·7	34·4	37·3	34·2	35·1	34·5
Glabella to occiput (L)	Mean contour value	172·0 (44)	174·4 (7)	175·5 (8)	165·7 (39)	162·8 (17)	169·1 (18)
	Type contour ...	172·3	174·3	175·4	165·3	163·0	169·2

We have here 54 pairs of measurements. In only nine of these is the discrepancy as much as 0·5 mm., and the largest is 1 mm. As any difference between the results of mean individual contour values and type contour values would tend to show itself more in small series than in larger ones a study of this table encourages us to rely with some confidence on mean values as read from the type contours,

* This difficulty was commented on by Fawcett and Macdonnell. (See *Biometrika*, Vol. III. p. 202.)

in comparing race with race. In this way, the method of contouring can be used to save the taking of a considerable number of individual length measurements.

We will now compare angle-measurements in similar fashion.

Comparison between Angle-Measurements on Type Contours, and Means of Individual Contour-Measurements.

Types:		Males			Females		
		A	B	C	A	B	C
φ	Mean contour value	60°·1 (44)	59°·7 (7)	60°·5 (8)	60°·6 (39)	60°·6 (17)	59°·1 (18)
	Type contour ...	60°·3	59°·8	60°·7	60°·4	60°·7	58°·9
φ'	Mean contour value	61°·7 (44)	61°·1 (7)	61°·6 (8)	62°·2 (39)	61°·9 (17)	60°·8 (18)
	Type contour ...	61°·8	61°·1	61°·8	61°·9	62°·2	60°·8
$A\angle$	Mean contour value	69°·6 (38)	72°·6 (7)	70°·4 (7)	71°·8 (34)	71°·6 (15)	75°·0 (16)
	Type contour ...	69°·6	73°·1	70°·7	71°·8	71°·6	74°·9
$N\angle$	Mean contour value	68°·0 (38)	66°·1 (7)	64°·8 (7)	66°·8 (34)	66°·9 (15)	64°·7 (16)
	Type contour ...	67°·9	65°·5	65°·0	66°·6	66°·6	64°·8
$B\angle$	Mean contour value	42°·5 (38)	41°·2 (7)	44°·8 (7)	41°·4 (34)	41°·6 (15)	40°·3 (16)
	Type contour ...	42°·6	41°·4	44°·3	41°·6	41°·8	40°·3
$P\angle$	Mean contour value	84°·4 (38)	86°·1 (7)	83°·6 (7)	85°·0 (34)	83°·8 (15)	87°·2 (16)
	Type contour ...	84°·4	86°·1	84°·6	85°·0	84°·2	87°·1
$fm\angle$	Mean contour value	7°·3 (42)	8°·8 (7)	7°·6 (8)	8°·8 (39)	7°·8 (17)	8°·2 (18)
	Type contour ...	6°·0	7°·8	7°·2	7°·3	6°·6	7°·2

In investigating the causes of the discrepancy between contour values and direct measurements, I found the explanation was the difference between auricular height as obtained by the two methods, which showed itself both in Thomson's and Crewdson Benington's work, and which Thomson was at a loss to account for*.

Ranke's craniophor which is used in the Biometric Laboratory was designed by the late Professor J. Ranke to provide means of obtaining easily the cranial measurements prescribed by the Frankfurt Concordat, among others the auricular height. This craniophor holds the skull in position supported on ear-plugs, and the auricular height should be measured from the top of the ear-plugs, the Frankfurt definition being "vertical" height of skull above the auricular points. A vertical rod to which a movable horizontal bar is attached is marked with a scale, which begins only at 38 mms., and from this the auricular height is read off. I have discovered that this scale, by an error of the instrument-maker, is actually graduated from the middle of the ear-plug, instead of from the top, as had always been assumed; and the diameter of the rod being 5 mms., the reading on the scale has always been 2·5 mms. in excess of the true auricular height. This error, which only became obvious when there were contour measurements to compare with direct

* *Biometrika*, Vol. XI. p. 127.

measurements, has no doubt been made by all users of Ranke's craniophor. It will be necessary therefore to deduct 2·5 mms. from all direct measurements of auricular height given by Fawcett*, Macdonnell*, Benington† and Thomson‡.

Auricular Height. (*First five corrected.*)

Race	Male	Female
Naqada	113·1	110·6
Whitechapel English	112·1	106·7
Liverpool St English	111·3	106·9
Congo Negroes ...	111·4	106·5
Moriori	114·6	112·6
Burmese A	117·7	111·7
„ B	116·7	111·9
„ C	116·9	110·1

The above table may be of use as giving the corrected values of the auricular height. I have added my Burmese for comparison. It will be seen that it makes these results still *more* incompatible with those of Ranke for the "Altbayerisch" (♂ 120·8, ♀ 114·2), or Parson's for Hythe (♂ 120, ♀ 116) and Rothwell (♂ 120, ♀ 116).

7. Description of Material and Comparison of Direct Measurements of the Three Series.

It was obvious in examining the whole series of 142 skulls that they were of rather mixed type, although one type predominated. It seemed desirable therefore to class together in separate groups those skulls which qualitative appreciation led us to suppose of the same type, reserving the right to combine these groups afterwards if the quantitative values obtained did not support this judgment. The procedure was as follows. Seven children's skulls, which could not be sexed, nor used to obtain any means, were withdrawn from the rest. The remaining skulls were then put into three groups: A and C, with an intermediate group B, which combined some of the features of both. Finally, the skulls in the various groups were sexed. It was necessary to leave the sexing until the types were separated, since some factors which affect the determination of sex, such as weight, size, and general boldness of shape, vary considerably from type to type, and some young males of a less robust racial type might easily be mistaken for females of a heavier and coarser one. For all this classification of material I have to thank Professor Pearson.

We thus had our series divided up into seven sub-series: three male and three female sub-series and one composed of children of various ages. The grouping was determined by the general appearance, more particularly of the face. As contributing factors to the general resemblance of those classed as Group A, we might single out as most noteworthy: the nose, broad at the bridge and flattish, with the pyriform aperture also wide, and its lower edge very rounded or even forming a double ridge with a negroid groove in between; marked sub-nasal prognathism; projecting cheek-bones; head of a massive and rugged type, etc.

* *Biometrika*, Vol. I. pp. 426–7, Vol. III. pp. 208–9 and Vol. V pp. 92–3. I have tested eight each of the Naqada and Whitechapel series and have satisfied myself that the extra 2·5 mms. were added to their measurements by the craniophor defect and must therefore be deducted.

† *Ibid.* Vol. VIII. pp. 208–9.

‡ *Ibid.* Vol. XI. p. 90.

In Group C less projecting cheek-bones, narrower noses with higher bridges, a sharp edge to the lower lip of the pyriform aperture; and again considerable prognathism; the whole head of a less bold and heavy character.

Sexing divided up Group A fairly evenly into 45 males and 39 females*; in Groups B and C the females preponderated, there being only eight males in each to 18 females in C and 17 in B. A number of the skulls had to be classed ♂ or it might be ♀ with a query, as is bound to happen in any series, but these dubious ones would not be numerous enough, if put into the wrong class, to modify the means to any considerable extent.

It was unfortunate that two out of six adult groups should contain as few as eight skulls, since these numbers are far too slight to base any reliable conclusions upon, or even to give any very helpful suggestions.

The whole series was accordingly examined once more some months afterwards, to see whether it would not be possible to divide the adults into two main types instead of three, putting the bulk of the intermediate group B along with C, so as to strengthen the numbers of the latter, and make the main categories less unequal. It was found that only two of the eight intermediate males could possibly have been put with the eight in group C, making ten in all; and though six of the seventeen B females might have been classed with the eighteen in C, this grouping was not very convincing since the C's were of a distinctly smaller and finer build.

In short, the small gain in numbers by the broader grouping would have been more than counterbalanced by the heterogeneity of the groups, and it was decided to leave the original divisions as they were.

It will not, however, be wise to attach significance to the results based on the two small male categories B and C, except in so far as they confirm the figures for the corresponding females.

After these conclusions had been arrived at upon the data themselves, we put our doubt as to the collection being purely Burman before Colonel Caster, and he reported that after further consideration there might be a certain number of Hindus or of Karens in the material, but that it was unlikely, owing to the source from which it was drawn, to include any Chinese, or any Mohammedans from India. In any case, the bulk of the skulls were Burman.

We therefore had little hesitation in applying the term Burman proper to our large groups A ♂ and A ♀. Since comparison with Hindu material showed little affinity between our C groups and the Hindus, as will be seen later, and taking into account the large proportion of Karen population around Moulmein, we suspect our C groups to be of Karen origin, though we have not enough definite evidence to establish the supposition. The few isolated Karen crania at the Royal College of Surgeons do not suffice to fix a type. Our B groups we suppose hybrids.

The following are the mean values obtained by direct measurement in the six groups:

* Not all the crania could be used for each contour, and this accounts for the different totals given in Table I and Figs. I–XVIII.

TABLE I. *Mean Characters of Burmese Crania.*

Types:	Males			Females		
	A	B	C	A	B	C
Capacity ...	1406·9 (27)	1415·0 (4)	1442·2 (5)	1267·9 (27)	1232·4 (11)	1231·4 (7)
W. in grs. ...	656·8 (38)	595·6 (7)	578·0 (6)	530·6 (31)	501·5 (13)	425·2 (11)
F	172·4 (44)	172·4 (8)	175·5 (8)	166·5 (39)	163·2 (17)	169·4 (18)
L′	174·2 (44)	176·7 (7)	177·2 (8)	167·4 (38)	164·7 (17)	170·7 (18)
L	173·5 (44)	173·8 (8)	176·7 (8)	166·5 (38)	163·8 (17)	170·2 (18)
B	143·7 (45)	141·1 (7)	140·4 (8)	138·0 (39)	135·6 (17)	130·6 (18)
B′	94·3 (44)	89·7 (8)	90·4 (8)	90·7 (39)	88·5 (16)	88·7 (18)
H	136·8 (43)	136·6 (7)	140·1 (8)	131·4 (39)	131·9 (17)	129·9 (18)
H′	136·0 (43)	134·7 (7)	139·1 (8)	130·4 (39)	130·8 (17)	129·4 (18)
OH	117·7 (44)	116·7 (7)	116·9 (8)	111·7 (38)	111·9 (17)	110·1 (17)
LB	98·5 (43)	98·8 (8)	100·5 (7)	93·9 (39)	94·7 (17)	95·7 (17)
Q	323·7 (44)	319·2 (7)	317·8 (8)	309·1 (36)	307·4 (17)	300·7 (16)
Q′	325·8 (43)	319·5 (7)	319·4 (8)	312·0 (37)	308·8 (17)	302·1 (16)
S	363·7 (44)	367·0 (7)	367·4 (8)	350·2 (39)	340·0 (17)	350·9 (18)
S_1	128·6 (44)	124·9 (7)	127·9 (8)	121·9 (39)	119·5 (17)	122·6 (18)
S_2	124·0 (45)	124·4 (7)	122·5 (8)	120·3 (39)	119·6 (17)	119·6 (18)
S_3	111·2 (44)	117·7 (7)	117·0 (8)	107·9 (39)	104·9 (17)	108·7 (18)
S_3'	94·7 (44)	98·1 (7)	99·7 (8)	92·8 (39)	92·0 (17)	91·4 (18)
U	505·7 (44)	497·7 (7)	503·1 (8)	485·1 (39)	476·9 (16)	479·6 (18)
PH	19·9 (38)	17·2 (7)	20·1 (7)	17·3 (33)	17·6 (16)	16·8 (16)
G′H	71·4 (39)	68·2 (8)	74·5 (7)	66·0 (34)	66·1 (17)	63·9 (16)
GB	101·9 (40)	100·9 (8)	94·3 (7)	96·7 (34)	93·6 (15)	91·8 (15)
J	134·0 (40)	131·7 (8)	126·7 (7)	126·7 (35)	121·9 (15)	118·7 (13)
NHR ...	53·4 (41)	52·3 (8)	55·1 (7)	50·3 (34)	49·4 (17)	47·6 (14)
NHL ...	53·5 (41)	52·4 (8)	55·2 (7)	50·6 (34)	49·4 (17)	47·3 (16)
NB	28·1 (41)	26·6 (8)	25·5 (7)	26·8 (34)	25·0 (17)	24·0 (14)
DS	10·3 (42)	10·6 (8)	11·6 (7)	9·4 (36)	9·6 (17)	11·2 (13)
DC	22·8 (43)	21·3 (8)	18·7 (7)	20·9 (37)	19·7 (17)	18·8 (15)
DA	32·6 (41)	30·9 (8)	30·7 (7)	29·8 (35)	29·3 (17)	30·7 (13)
SS	3·0 (41)	3·5 (8)	3·6 (7)	2·5 (37)	2·9 (17)	3·6 (13)
SC	9·0 (42)	7·8 (8)	7·2 (7)	8·4 (38)	7·7 (17)	7·7 (14)
O_1R	44·2 (41)	44·3 (8)	44·4 (7)	42·0 (34)	42·2 (17)	42·2 (15)
O_1L	43·6 (41)	43·2 (8)	44·1 (7)	41·8 (34)	41·5 (14)	42·2 (16)
O_2R	35·0 (41)	33·6 (8)	35·6 (7)	34·5 (34)	34·0 (16)	33·7 (14)
O_2L	35·0 (41)	34·3 (8)	35·9 (7)	34·6 (34)	34·6 (15)	33·2 (16)
G_1	49·9 (37)	49·2 (8)	50·0 (7)	46·7 (26)	47·0 (14)	46·4 (13)
G_1'	45·4 (37)	44·8 (8)	46·2 (7)	42·7 (29)	43·1 (16)	42·8 (13)
G_2	39·6 (40)	40·1 (6)	40·6 (7)	38·0 (32)	36·4 (15)	38·1 (14)
GL	96·6 (39)	93·8 (8)	95·9 (7)	90·7 (34)	91·5 (17)	89·2 (16)
fml	36·7 (43)	35·4 (8)	37·3 (8)	34·7 (39)	35·1 (16)	35·1 (18)
fmb	30·7 (42)	30·6 (8)	30·8 (8)	29·4 (39)	30·0 (16)	29·8 (17)
Oc. I. ...	62·8 (44)	61·5 (7)	62·8 (8)	64·1 (39)	66·1 (17)	61·4 (18)
100 *B/L′* ...	82·4 (44)	80·0 (7)	79·4 (8)	83·1 (38)	83·0 (17)	76·6 (18)
100 *H/L′* ...	78·2 (43)	76·3 (7)	78·7 (8)	78·0 (38)	79·5 (17)	75·9 (18)
100 *B/L* ...	82·9 (44)	80·4 (7)	79·5 (8)	83·1 (38)	82·9 (17)	76·9 (18)
100 *H/L* ...	78·5 (43)	76·7 (7)	78·9 (8)	78·5 (38)	79·9 (17)	76·1 (18)
100 *B/H* ...	105·8 (43)	104·9 (7)	101·0 (8)	105·8 (39)	103·7 (17)	101·0 (18)
100 *(B − H)/L*	+4·0 (43)	+2·6 (7)	+0·1 (8)	+3·9 (38)	+2·4 (17)	+0·9 (18)
100 *G′H/GB* ...	69·8 (38)	67·8 (8)	79·0 (7)	68·6 (34)	69·8 (15)	70·2 (15)
100 *NB/NHR*	52·8 (41)	51·2 (8)	46·4 (7)	52·9 (34)	50·8 (17)	50·4 (14)
100 *NB/NHL*	52·7 (41)	51·0 (8)	46·3 (7)	53·0 (34)	50·7 (17)	50·4 (14)
100 O_2/O_1L ...	80·0 (41)	79·4 (8)	81·5 (7)	82·8 (34)	83·0 (14)	78·8 (16)
100 O_2/O_1R ...	79·1 (41)	75·9 (8)	80·3 (7)	82·4 (34)	80·4 (16)	79·9 (14)
100 G_2/G_1 ...	79·8 (37)	82·2 (6)	81·3 (7)	80·5 (25)	76·8 (12)	83·0 (13)
100 *fmb/fml* ...	83·8 (42)	86·6 (8)	82·9 (8)	84·8 (39)	85·9 (16)	85·0 (17)
100 *DS/DC* ...	45·4 (42)	49·8 (8)	61·9 (7)	45·1 (36)	48·7 (17)	60·3 (13)
100 *SS/SC* ...	32·7 (41)	45·2 (8)	54·0 (7)	29·8 (37)	38·7 (17)	48·5 (13)
N ∠	66°·8 (38)	65°·2 (8)	64°·4 (7)	66°·2 (34)	66°·5 (17)	64°·5 (16)
A ∠	70°·5 (38)	73°·4 (8)	71°·2 (7)	72°·1 (34)	72°·0 (17)	75°·2 (16)
B ∠	42°·6 (38)	41°·4 (8)	44°·5 (7)	41°·8 (34)	41°·5 (17)	40°·3 (16)
θ_2	15°·4 (38)	12°·9 (8)	12°·8 (7)	13°·5 (33)	12°·0 (17)	11°·2 (16)
θ_1	27°·2 (38)	28°·8 (8)	31°·7 (7)	28°·3 (33)	29°·5 (17)	29°·1 (16)
P ∠	86°·0 (39)	85°·9 (8)	84°·0 (7)	85°·8 (33)	84°·0 (17)	86°·4 (16)

Let us now compare the characteristics of the extreme groups A and C.

In order to do this we need to compare any difference in the means with the probable error of that difference, to see whether it is significant or not. And for the probable error we need to know the standard deviation of each character within the race to which it belongs. Our numbers are altogether too slight to furnish us with this standard deviation; I have therefore adopted as next best thing the standard deviations as found in some other race, on the assumption, confirmed by previous experience*, that though the S.D.'s vary from race to race, they are at least of about the same order. Our probable error of means based on these borrowed S.D.'s will necessarily be only approximate, and we shall not be able to test the likely significance of a difference in means by asking definitely whether it is more or less than about 2·5 times the probable error. More latitude must be allowed than usual, especially when one sex seems to show a significant difference and the other not; a truer standard deviation might alter the probable error so as to make the two sexes indicate the same thing.

The standard deviations of the Egyptians (Naqada Race†) have been borrowed for as many characters as are given there; this is our longest measured series, and its standard deviations are therefore subject to the smallest probable errors. For characters not measured by Fawcett, I have taken from the Moorfields English Crania studied by Macdonnell‡ the S.D.'s of such characters as he added to Fawcett's list, and from Benington's Congo series§ the S.D.'s of the nose measurements which were his new contribution. The standard deviation of weight ‖ (W. in grs.) was taken from an Egyptian series of which 50 of each sex were weighed by A. G. Davin for comparison with Moriori crania. Finally, that of 100 $(B - H)/L$ was calculated from the S.D.'s of the component indices, and that of the premaxillary height (PH), which is approximately equal to the upper-face height ($G'H$) minus height of nose (NH), by means of the S.D.'s of these two characters.

On the following page is a table showing the excess or defect of Group C means as against those of Group A, with the approximate probable errors of the differences.

The characteristics which influenced our merely appreciative grouping are well brought out by this table. The greater width of the cheek-bones in Group A is indicated by the longer bi-zygomatic breadth (J) and the greater upper-face breadth (GB), the differences exceeding five times the probable error in both cases for both sexes. Again, as regards the nose, the width of the pyriform aperture in Group A exceeds that in Group C by more than five times the probable error in males, and by seven times in females; about the nasal height there is no such clear verdict, with a barely significant difference in the males, the C's exceeding, while the A's have a very definite excess for the females. In any case the relative proportions of the nose are not left in doubt, since the A's have a significantly higher nasal index in both.

* See Tables of Interracial Variability in Benington's Paper, *Biometrika*, Vol. VIII. pp. 323 and 324.

† Fawcett's paper in *Biometrika*, Vol. I. p. 438.

‡ *Biometrika*, Vol. V. pp. 92, 93.

§ *Biometrika*, Vol. VIII. pp. 298, 299.

‖ Not yet published.

The flatness of the nasal bridge in Group A relative to Group C is brought out by the absolute values of the simotic and dacryal subtenses (*SS* and *DS*), and by their ratio to the corresponding chords (*SS/SC*, *DS/DC*), the lowest of these index differences in either sex being nine times the probable error. The width of the bridge from dacryon to dacryon (*DC*) is much greater in Group A, being six to

TABLE II. *Difference between Mean Characters of Burmese Group A (Burmans) and Group C (Karens?)**.

	Males A–C	Females A–C		Males A–C	Females A–C
Capacity	−35·3±35·1	+ 36·5±25·4	O_1L	− 0·5± 0·6	− 0·4± 0·3
W. in grs.	+78·8±33·1	+105·4±18·0	O_2R	− 0·6± 0·6	+ 0·8± 0·4
F	− 3·1± 1·6	− 2·9± 1·2	O_2L	− 0·9± 0·6	+ 1·4± 0·4
L'	− 3·0± 1·6	− 3·3± 1·2	G_1	− 0·1± 1·0	+ 0·3± 0·6
L	− 3·2± 1·5	− 3·7± 1·1	G_1'	− 0·8± 1·0	− 0·1± 0·9
B	+ 3·3± 1·2	+ 7·4± 0·9	G_2	− 1·0± 1·0	− 0·1± 0·7
B'	+ 3·9± 1·2	+ 2·0± 0·8	*GL*	+ 0·7± 1·3	+ 1·5± 0·9
H	− 3·3± 1·4	+ 1·5± 0·9	*fml*	− 0·6± 0·7	− 0·4± 0·5
H'	− 3·1± 1·4	+ 1·0± 0·9	*fmb*	− 0·1± 0·5	− 0·4± 0·5
OH	+ 0·8± 1·2	+ 1·6± 0·8	*Oc. I.*	0·0± 0·6†	+ 2·7± 0·6†
LB	− 2·0± 1·3	− 1·8± 0·9	100 *B/L'*	+ 3·0± 0·7	+ 6·5± 0·4
Q	+ 5·9± 2·6	+ 8·4± 2·3	100 *H/L'*	− 0·5± 0·7	+ 2·1± 0·5
Q'	+ 6·4± 2·6	+ 9·9± 1·6	100 *B/L*	+ 3·4± 0·7	+ 6·2± 0·6
S	− 3·7± 3·1	− 0·7± 2·5	100 *H/L*	− 0·4± 0·7	+ 2·4± 0·6
S_1	+ 0·7± 1·4	− 0·7± 1·2	100 *B/H*	+ 4·8± 1·2	+ 4·8± 0·9
S_2	+ 1·5± 2·0	+ 0·7± 1·4	100 *(B − H)/L*	+ 3·9± 0·7	+ 3·0± 0·5
S_3	− 5·8± 2·2	− 0·8± 1·7	100 *G'H/GB*	− 9·2± 1·3	− 1·6± 0·9
S_3'	− 5·0± 1·5	+ 1·4± 1·2	100 *NB/NHR*	+ 6·4± 1·2	+ 2·5± 0·8
U	+ 2·6± 3·4	+ 5·5± 2·2	100 *NB/NHL*	+ 6·4± 1·2	+ 2·2± 1·0
PH	− 0·2± 0·8	+ 0·5± 0·7	100 O_2/O_1R	− 1·2± 1·4	+ 2·5± 1·2
G'H	− 3·1± 1·1	+ 2·1± 0·9	100 O_2/O_1L	− 1·5± 1·4	+ 4·0± 1·2
GB	+ 7·6± 1·4	+ 4·9± 0·9	100 G_1'/G_1	− 1·5± 2·0	− 2·5± 1·1
J	+ 7·3± 1·4	+ 8·0± 1·2	100 *fmb/fml*	+ 0·9± 1·8	− 0·2± 1·2
NHR	− 1·7± 0·8	+ 3·2± 0·6	100 *DS/DC*	−16·5± 1·7	−15·2± 1·2
NHL	− 1·7± 0·8	+ 3·3± 0·7	100 *SS/SC*	−21·3± 2·2	−18·7± 2·1
NB	+ 2·6± 0·5	+ 2·8± 0·4	*N* ∠	+ 2°·4±1°·0	+ 1°·7±0°·6
DS	− 1·3± 0·4	− 1·8± 0·3	*A* ∠	− 0°·7±1°·2	− 3°·1±0°·5
DC	+ 4·1± 0·7	+ 2·1± 0·3	*B* ∠	− 1°·9±0°·8	+ 1°·5±0°·5
DA	+ 1·9± 1·3	− 0·9± 0·7	θ_2	+ 2°·6±1°·1	+ 2°·3±0°·5
SS	− 0·6± 0·2	− 1·1± 0·2	θ_1	−4°·5±0°·9	− 0°·8±0°·4
SC	+ 1·8± 0·6	+ 0·7± 0·5	*P* ∠	+ 2°·0±0°·8	− 0°·6±0°·5
O_1R	− 0·2± 0·6	− 0·2± 0·3			

* Probable errors only approximate.

† The standard deviation for *Oc. I.* varies very markedly in different races (see *Biometrika*, Vol. VIII. p. 324) being nearly three times as great in the Negro as in the English. I have used the small standard deviation of the English in writing down the probable error. If I had used that of the Gaboon series, I should have got ±1·8 for ♂, ±1·6 for ♀.

seven times the probable error; the width of the nasal bones themselves (*SC*) is not so emphatically greater, but as was pointed out in K. V. Riley's and J. Bell's study of the Nasal Bridge*, a physiognomic factor such as the whole bridge of the nose is probably of much more importance from the evolutionary point of view than a simple anatomical unit (such as the nasal bones), which forms only one part of it.

Finally, as regards the general appearance of greater massiveness and ruggedness in Group A: this is confirmed to some extent by the weight (W. in grs.). Weight, it is true, does not depend only on size, but also on thickness of the bone*; but here, again, since exterior unevennesses and protuberances are by no means bound to correspond with interior excavations, it is probable that the smoother skulls are also the thinner on the whole, and that a more rugged appearance is associated to some extent with thickness.

Let us turn now to other characters than those which were considered in grouping, and first the main proportions of the skull. The Burman Group is definitely broader-headed than Group C for ♂ and ♀, the difference in maximum breadth (B) being greater in the case of the females (seven times the probable error) than with the males (three times). It is also shorter-headed, as is seen by the three length measurements L, L' and F. The result of both facts is that Group A has a higher cephalic index ($100B/L$ or $100B/L'$) in both sexes, being brought well within the brachycephalic category, while Group C is merely mesocephalic. The difference between the indices in both sexes ranges from about four to sixteen times its probable error.

Associated with the greater maximum breadth in Group A, we find in both ♂ and ♀ greater forehead breadth (B'), and the greater breadth of face, nose and nasal bridge to which we have already referred. It is not, however, accompanied by any significant difference in breadth of orbit (O_1, R and L), of palate (G_2), or of foramen magnum (fmb). Nor, on the other hand, is the shorter head-length in Group A associated with a significantly shorter foramen magnum (fml) or palate (G_1' and G_1).

The indices for these two (fmb/fml and G_2/G_1) likewise give no definite sign of differentiation.

When we consider the measurement for height (H and H') we do not find any such marked differences between the two groups as in length. Such difference as there is, is in opposite sense for the two sexes, but in neither case is this 2·5 times the probable error, and cannot be counted as significant. Nor do we find any significance in the differences between the means of the two groups for height above the auricular passages (OH); in height of orbit (O_2R, O_2L), upper-face height ($G'H$), and height of nose (NH, R and L), Group C shows a definite advantage, but for females only.

The orbital index (O_2/O_1, R and L) indicates that the Burman women are rounder-eyed than those of Group C. The men of the two groups are much alike.

As to the cranial indices in which height is a factor, Group A has a higher value for breadth over height (B/H), as we should expect; but the females alone of Group A have a significantly higher value for height over length (H/L and H/L'), the difference in males being negligible. The combined index $(B-H)/L$ shows a difference of about six times the probable error in both ♂ and ♀. This index places

* And again on the manner in which the skull has been preserved. In the present case the same average conditions of preservation apply to the different series.

the Burmans between the modern Europeans and the primitive races dealt with in the table compiled by Thomson*, whereas Group C comes in the middle of the primitive list; this will be discussed later†. Of the remaining features that exhibit significant differences in the means of the two groups, I need only refer to the transverse arc from auricular point to auricular point over the apex (Q' and Q), which is greater for Group A, doubtless owing largely to the greater width of skull; and to the angle formed by the inclination to the horizontal of the line joining alveolar point to basion (θ_2): this is significantly greater in Group A, while the distance between these two points is about the same in the two groups.

8. Comparison of Type Contours.

We can now turn to the type contours and see what further points of difference or affinity are shown there to exist among our three types.

The next three tables give the mean contour values from which type contours were constructed.

TABLE III. *Burmese Skulls. Means of Transverse Contours.*

Types:	Males			Females		
	A	B	C	A	B	C
MA	116·5 (44)	115·7 (7)	115·6 (8)	111·0 (38)	111·0 (17)	108·9 (18)
1*R*	59·1 (44)	57·3 (7)	57·3 (8)	56·4 (38)	54·6 (17)	53·3 (18)
1*L*	59·1 (44)	57·3 (7)	57·3 (8)	56·4 (38)	54·6 (17)	53·3 (18)
1½*R*	62·1 (44)	61·2 (7)	59·1 (7)	59·4 (36)	57·7 (17)	55·7 (17)
1½*L*	62·1 (44)	61·2 (7)	60·2 (7)	59·1 (38)	57·8 (17)	55·7 (17)
2*R*	63·5 (44)	62·0 (7)	60·3 (7)	61·5 (36)	59·5 (17)	57·7 (17)
2*L*	63·7 (44)	62·2 (7)	61·7 (8)	60·9 (38)	59·8 (17)	57·2 (17)
3*R*	66·6 (44)	64·9 (7)	63·3 (7)	64·4 (36)	62·8 (17)	60·7 (17)
3*L*	66·8 (43)	64·7 (7)	64·8 (8)	63·9 (38)	62·6 (17)	59·8 (17)
4*R*	69·3 (44)	66·8 (7)	65·8 (7)	66·6 (36)	65·3 (17)	63·1 (17)
4*L*	69·4 (43)	67·4 (7)	67·1 (8)	66·1 (38)	65·2 (17)	61·9 (17)
5*R*	69·7 (44)	67·4 (7)	67·4 (8)	66·9 (38)	65·8 (17)	63·6 (18)
5*L*	70·0 (43)	68·3 (7)	67·8 (8)	66·5 (38)	65·8 (17)	62·7 (18)
6*R*	69·1 (44)	66·9 (7)	67·3 (8)	66·2 (38)	65·1 (17)	63·1 (18)
6*L*	69·3 (43)	67·9 (7)	67·2 (8)	66·1 (38)	65·3 (17)	62·4 (18)
7*R*	67·2 (44)	65·2 (7)	65·6 (8)	64·3 (38)	63·2 (17)	61·4 (18)
7*L*	67·3 (43)	66·1 (7)	65·2 (8)	64·2 (38)	63·4 (17)	60·7 (18)
8*R*	62·9 (44)	60·8 (7)	61·0 (8)	60·4 (38)	59·3 (17)	57·7 (18)
8*L*	62·8 (43)	61·7 (7)	60·4 (8)	60·0 (38)	59·4 (17)	56·9 (18)
9*R*	54·3 (44)	52·2 (7)	51·7 (8)	51·7 (37)	50·8 (17)	49·2 (18)
9*L*	53·4 (43)	52·4 (7)	51·2 (8)	51·3 (38)	50·9 (17)	48·6 (18)
10*R*	38·1 (44)	37·7 (7)	36·5 (8)	36·3 (37)	36·0 (17)	35·0 (18)
10*L*	37·1 (43)	37·1 (7)	35·6 (8)	36·1 (38)	36·0 (17)	34·1 (18)
*A*½*R*	19·5 (44)	19·9 (7)	19·0 (8)	17·1 (37)	17·8 (17)	17·0 (18)
*A*½*L*	18·4 (44)	19·4 (7)	17·1 (8)	17·8 (38)	18·7 (17)	16·9 (18)
TH	117·1 (44)	116·1 (7)	116·1 (8)	111·5 (38)	111·4 (17)	109·4 (18)
ZR(*R*) *y*	62·3 (44)	61·6 (7)	59·2 (7)	59·8 (36)	58·2 (17)	55·7 (17)
ZR(*R*) *x*	3·0 (44)	3·7 (7)	2·6 (7)	3·1 (36)	3·5 (17)	2·2 (17)
ZR(*L*) *y*	62·5 (44)	61·9 (7)	60·7 (8)	59·6 (38)	58·4 (17)	56·4 (17)
ZR(*L*) *x*	3·4 (44)	4·1 (7)	3·4 (8)	3·3 (38)	3·8 (17)	3·0 (17)

* *Biometrika*, Vol. XI. p. 95.

† See pp. 245–6.

TABLE IV. *Burmese Horizontal Contours. Mean Values*

Types:	Males			Females		
	A	B	C	A	B	C
FO	170·9 (44)	173·7 (7)	174·4 (8)	164·1 (39)	161·6 (17)	168·2 (18)
F½R	27·2 (44)	26·8 (7)	25·6 (8)	23·4 (39)	24·0 (16)	21·9 (17)
F½L	25·7 (44)	24·0 (7)	23·5 (8)	22·9 (39)	22·4 (17)	20·8 (18)
F¼R	36·7 (44)	36·7 (7)	35·0 (8)	33·8 (39)	33·3 (16)	30·9 (17)
F¼L	37·3 (44)	35·4 (7)	34·3 (8)	33·7 (39)	33·6 (17)	30·0 (18)
2R	46·2 (44)	45·9 (7)	44·7 (8)	44·0 (39)	43·1 (16)	41·5 (18)
2L	46·6 (44)	44·4 (7)	43·7 (8)	44·1 (39)	43·2 (17)	41·7 (18)
3R	48·6 (44)	47·5 (7)	48·9 (8)	47·8 (39)	46·9 (17)	44·9 (18)
3L	48·9 (44)	46·6 (7)	48·2 (8)	47·8 (39)	47·0 (17)	45·2 (17)
4R	54·3 (44)	54·1 (7)	55·1 (8)	53·5 (39)	52·3 (17)	50·3 (18)
4L	55·7 (44)	53·7 (7)	54·8 (8)	53·8 (39)	53·1 (17)	50·3 (18)
5R	63·6 (44)	63·1 (7)	63·3 (8)	62·2 (39)	60·6 (17)	58·3 (18)
5L	63·7 (43)	61·4 (7)	62·4 (8)	61·3 (39)	59·9 (17)	57·2 (17)
6R	69·3 (44)	68·1 (7)	67·9 (8)	67·4 (39)	65·6 (17)	62·7 (18)
6L	68·8 (43)	66·5 (7)	66·9 (8)	65·9 (39)	64·7 (17)	61·7 (17)
7R	70·7 (44)	69·6 (7)	69·0 (8)	68·5 (39)	67·3 (17)	64·1 (18)
7L	70·2 (43)	67·6 (7)	68·0 (8)	66·6 (39)	65·7 (17)	62·9 (18)
8R	67·9 (44)	66·5 (7)	65·1 (8)	65·3 (39)	64·6 (17)	61·4 (18)
8L	67·0 (44)	65·5 (7)	65·3 (8)	63·6 (39)	62·8 (17)	60·9 (18)
9R	60·3 (44)	59·0 (7)	56·9 (8)	57·9 (39)	57·3 (17)	54·4 (18)
9L	60·5 (44)	58·7 (7)	58·4 (8)	56·8 (39)	56·1 (17)	54·6 (18)
10R	46·5 (44)	45·7 (7)	43·1 (8)	44·5 (39)	44·1 (17)	41·4 (18)
10L	47·9 (44)	46·5 (7)	46·0 (8)	44·5 (39)	43·2 (17)	42·3 (18)
O½R	26·2 (44)	25·9 (7)	23·0 (8)	25·4 (39)	26·5 (17)	22·9 (18)
O½L	29·2 (44)	28·0 (7)	27·1 (8)	25·4 (39)	25·7 (17)	25·9 (18)
T (R)	47·6 (44)	46·8 (7)	45·3 (8)	45·2 (39)	44·6 (16)	44·0 (18)
T (L)	47·7 (44)	45·6 (7)	45·1 (8)	45·2 (39)	44·5 (17)	44·8 (18)
xR	18·3 (44)	17·9 (7)	17·3 (8)	17·6 (39)	17·3 (16)	20·3 (18)
xL	17·8 (44)	17·9 (7)	18·0 (8)	16·9 (39)	17·0 (17)	20·7 (18)

TABLE V. *Means of Contour Measurements used in plotting Sagittal Type Contours.*

Types:		Males			Females		
		A	B	C	A	B	C
$N\gamma$		167·4 (44)	170·9 (7)	172·3 (8)	161·8 (39)	159·3 (17)	165·1 (18)
Vertex	x from N	91·8 (44)	100·5 (7)	98·4 (8)	89·1 (39)	90·9 (17)	87·4 (18)
	y	90·7 (44)	90·5 (7)	88·0 (8)	86·3 (39)	85·8 (17)	84·8 (18)
β	x from N	69·5 (44)	70·1 (7)	73·3 (8)	66·1 (39)	65·1 (17)	67·5 (18)
	y	88·2 (44)	86·5 (7)	86·0 (8)	83·5 (39)	82·9 (17)	83·0 (18)
Glabella	Gx	3·4 (44)	2·7 (7)	2·3 (8)	2·6 (39)	2·4 (17)	3·2 (18)
	Gy	12·2 (44)	10·0 (7)	9·2 (8)	13·2 (39)	11·6 (17)	10·8 (18)
Ordinates above $N\gamma$	O	29·7 (44)	22·0 (7)	21·7 (8)	29·4 (39)	25·5 (17)	28·1 (18)
	$N\frac{1}{4}$	44·1 (44)	36·0 (7)	36·0 (8)	42·9 (39)	40·2 (17)	40·4 (18)
	1	62·4 (44)	58·5 (7)	58·0 (8)	59·7 (39)	58·0 (17)	58·8 (18)
	2	75·5 (44)	73·1 (7)	72·2 (8)	71·6 (39)	70·4 (17)	70·9 (18)
	3	83·5 (44)	81·7 (7)	80·6 (8)	78·8 (39)	78·0 (17)	78·2 (18)
	4	88·1 (44)	86·2 (7)	85·3 (8)	83·3 (39)	82·8 (17)	82·8 (18)
	5	90·2 (44)	89·1 (7)	87·4 (8)	85·5 (39)	84·8 (17)	84·2 (18)
	6	90·3 (44)	90·1 (7)	87·5 (8)	85·8 (39)	85·4 (17)	84·0 (18)
	7	87·4 (44)	86·9 (7)	84·3 (8)	82·8 (39)	82·7 (17)	80·8 (18)
	8	79·5 (44)	77·9 (7)	76·5 (8)	75·2 (39)	75·7 (17)	72·9 (18)
	9	64·0 (44)	61·2 (7)	59·9 (8)	60·2 (39)	61·1 (17)	56·6 (18)
	$\gamma\frac{1}{4}$	39·4 (44)	34·8 (7)	34·3 (8)	36·3 (39)	38·0 (17)	31·3 (18)
	$\gamma\frac{1}{8}$	31·8 (44)	27·1 (7)	28·7 (8)	28·8 (39)	30·9 (17)	24·3 (18)
	γ	21·8 (44)	14·1 (7)	15·7 (8)	16·7 (39)	16·7 (17)	14·2 (18)
Ordinates below $N\gamma$	8	46·4 (44)	47·2 (7)	49·3 (8)	45·2 (39)	46·1 (17)	44·4 (18)
	9	35·8 (44)	37·7 (7)	38·0 (8)	33·6 (39)	34·5 (17)	33·9 (18)
	$\gamma\frac{1}{2}$	27·7 (44)	30·2 (7)	30·7 (8)	25·3 (39)	25·4 (17)	26·1 (18)
	$\gamma\frac{1}{4}$	19·0 (44)	21·7 (7)	19·6 (8)	17·5 (39)	17·8 (17)	18·7 (18)
Occipital	x from γ	1·2 (44)	0·8 (7)	0·8 (8)	1·0 (39)	1·0 (17)	0·9 (18)
	y	11·7 (44)	8·0 (7)	12·4 (8)	8·8 (39)	9·6 (17)	7·7 (18)
λ	x from γ	3·2 (44)	5·4 (7)	4·6 (8)	3·4 (39)	2·7 (17)	5·1 (18)
	y	34·6 (44)	34·6 (7)	33·7 (8)	31·8 (39)	32·2 (17)	31·7 (18)
Sub-orbital point	x from N	7·8 (42)	9·3 (7)	8·2 (7)	7·0 (34)	7·6 (17)	11·0 (16)
	y	28·1 (42)	28·4 (7)	29·3 (7)	27·3 (34)	27·1 (17)	26·6 (16)
Auricular point	x from γ	82·6 (44)	85·3 (7)	86·6 (8)	81·5 (39)	79·4 (17)	82·1 (18)
	y	28·1 (44)	28·4 (7)	29·4 (8)	27·1 (39)	27·1 (17)	26·5 (18)
Opisthion	x from γ	45·0 (43)	50·1 (7)	49·6 (8)	46·2 (39)	43·0 (17)	47·1 (18)
	y	49·6 (43)	51·6 (7)	55·5 (8)	49·5 (39)	49·8 (17)	49·0 (18)
Inion	γI	28·9 (44)	29·8 (7)	36·2 (8)	29·4 (39)	28·8 (17)	28·0 (18)
	NI	163·0 (44)	166·6 (7)	164·9 (8)	155·4 (39)	152·9 (17)	159·1 (18)
Basion	γ bas.	93·1 (43)	96·5 (7)	100·2 (8)	91·7 (39)	90·2 (17)	92·7 (18)
	N bas.	97·4 (43)	98·0 (7)	100·2 (8)	93·0 (39)	93·3 (17)	94·7 (18)
Alveolar point	N to A.P.	70·7 (38)	67·9 (7)	74·2 (7)	65·7 (34)	65·7 (15)	63·9 (16)
	Bas to A.P.	96·5 (38)	93·6 (7)	95·6 (7)	90·3 (34)	90·4 (15)	88·8 (16)
Nose	$\angle$ of nasal prominence	110°·0 (31)	115°·4 (7)	118°·5 (6)	108°·9 (22)	115°·1 (15)	116°·8 (12)
	NL	22·9 (29)	22·7 (6)	26·4 (5)	21·8 (16)	21·3 (13)	21·2 (9)
	NL'	20·8 (29)	21·0 (6)	22·4 (5)	20·3 (16)	19·2 (13)	17·9 (9)
	Max. nasal subt. x from N	10·4 (29)	9·6 (6)	11·9 (5)	9·9 (16)	9·2 (13)	9·1 (9)
	Max. nasal subt. y	2·6 (29)	2·6 (6)	3·2 (5)	2·6 (16)	2·5 (13)	2·0 (9)
Palate	Max. x from A.P. ...	35·5 (38)	35·9 (7)	37·0 (7)	33·5 (33)	33·9 (15)	34·5 (16)
	Subt. y ...	17·7 (38)	15·8 (7)	19·8 (7)	15·9 (33)	16·4 (15)	17·2 (16)
Frontal	Max. subt. to $N\beta$ x from N	51·8 (44)	53·5 (7)	53·1 (8)	47·9 (39)	48·4 (17)	48·9 (18)
	Max. subt. to $N\beta$ y	26·1 (44)	24·0 (7)	25·0 (8)	25·2 (39)	23·9 (17)	24·8 (18)
Occipital	Max. subt. to $\lambda Op.$ x from λ	53·9 (43)	55·5 (7)	58·6 (8)	49·2 (39)	50·8 (17)	49·5 (18)
	Max. subt. to $\lambda Op.$ y	24·6 (43)	28·4 (7)	25·8 (8)	23·4 (39)	21·2 (17)	24·7 (18)
$N\lambda$ base line	Max. x from N ...	82·6 (44)	88·1 (7)	84·5 (8)	81·1 (39)	81·2 (17)	79·1 (18)
	Subt. y ...	72·9 (44)	71·1 (7)	70·7 (8)	69·0 (39)	68·7 (17)	68·8 (18)
GI base line	Max. x from G ...	96·7 (44)	96·8 (7)	96·4 (8)	90·9 (39)	91·9 (17)	91·8 (18)
	Subt. y ...	102·1 (44)	103·4 (7)	104·5 (8)	97·0 (39)	97·3 (17)	96·1 (18)

In addition I append a table of mean measurements taken on the sagittal contours, but not used in construction of the sagittal type contour, as the position of the terminals had already been fixed by the means of other measurements. I have discussed already the degree of exactness with which these means from individual contours correspond to the values read off from the type contour.

TABLE VI. *Burmese Mean Sagittal Contour Values not used in Construction of Type.*

Types:		Males			Females		
		A	B	C	A	B	C
Lengths	$N\beta$	112·6 (44)	111·3 (7)	113·1 (8)	106·7 (39)	106·0 (17)	108·2 (18)
	$\lambda Op.$	93·7 (43)	97·3 (7)	100·2 (8)	92·8 (39)	91·8 (17)	91·2 (18)
	$N\lambda$	167·9 (44)	169·2 (7)	171·4 (8)	162·0 (39)	160·3 (17)	163·4 (18)
	β subtense to $N\lambda$ (x from N)	86·2 (44)	86·2 (7)	89·0 (8)	81·5 (39)	80·4 (17)	82·5 (18)
	β subtense to $N\lambda$ (y)	72·0 (44)	70·2 (7)	70·0 (8)	68·5 (39)	68·0 (17)	68·1 (18)
	GI	168·7 (44)	171·1 (7)	169·3 (8)	160·7 (39)	157·7 (17)	164·1 (18)
	β subtense to GI (x from G)	53·0 (44)	53·4 (7)	53·5 (8)	49·8 (39)	48·3 (17)	54·2 (18)
	β subtense to GI (y)	91·2 (44)	90·8 (7)	93·6 (8)	85·3 (39)	85·7 (17)	86·5 (18)
Angles	ϕ ($= \angle \beta GI$) ...	60°·1 (44)	59°·7 (7)	60°·5 (8)	60°·6 (39)	60°·6 (17)	59°·1 (18)
	ϕ' ($= \angle \beta NI$) ...	61°·7 (44)	61°·1 (7)	61°·6 (8)	62°·2 (39)	61°·9 (17)	60°·8 (18)
	$A\angle$	69°·6 (38)	72°·6 (7)	70°·4 (7)	71°·8 (34)	71°·6 (15)	75°·0 (16)
	$N\angle$	68°·0 (38)	66°·1 (7)	64°·8 (7)	66°·8 (34)	66°·9 (15)	64°·7 (16)
	$B\angle$	42°·5 (38)	41°·2 (7)	44°·8 (7)	41°·4 (34)	41°·6 (15)	40°·3 (16)
	$P\angle$	84°·4 (38)	86°·1 (7)	83°·6 (7)	85°·0 (34)	83°·8 (15)	87°·2 (16)
	$fml\angle$	7°·3 (42)	8°·8 (7)	7°·6 (8)	8°·8 (39)	7°·8 (17)	8°·2 (18)
Indices	Frontal index i_f ...	23·1 (44)	21·6 (7)	22·1 (8)	23°·6 (39)	22·5 (17)	22·9 (18)
	Occipital index i_{oc} ...	26·2 (43)	29·1 (7)	25·8 (8)	25·2 (39)	23·0 (17)	27·1 (18)

Before comparing our type contours we need to get some measure of the range of variation which would be covered by means based on other samples of the same size as our own, drawn from the same population. On pp. 143, 145, 147 of Crewdson Benington's paper on Type Contours* we have graphical representations of the limits between which the bulk of these variations would lie for the three contours representing a series of 100 English skulls. Assuming the standard deviations of English and Burmese characters to be approximately the same (and they will certainly be of the same order), and taking into consideration the fact that the probable error of a mean varies directly in proportion to the inverse square root of the number of skulls on which it is based, we see that we must allow for our largest series of forty-four (Group A ♂) a range half as big again as that shown for the English contour; while for our smallest series of seven (Group B ♂) the range must be made about 3·8 times as big. This demonstrates once more the unreliability of definite conclusions based upon short series.

Below are given the proportions in which the range of 100 English type must be increased for our six series; and for the convenience of those who have not the Benington paper for reference we give a few measurements to indicate the extent of the range.

* *Biometrika*, Vol. VIII.

Type contours based on 100 English skulls:

Transverse Contour. Range of variation from 1·4 mms. at apex tapering down to 0·8 mm. by the auricular points.

Horizontal Contour. Range about 1·2 mms. all round.

Sagittal Contour. About 1·6 mms. from glabella to lambda, tapering to 0·7 mm. at gamma and 0·9 mm. at nasion (below gamma not given).

Extent to which this range should be increased for Burmese groups:

Group	No.	Multiplying coefficient	Group	No.	Multiplying coefficient
♂ A	44	1·5	♀ A	39	1·6
♂ B	8	3·5	♀ B	18	2·4
♂ C	7	3·8	♀ C	17	2·4

Type Contours of Groups A and C (Burmese) compared.

One rather noticeable characteristic of the Group which we take to be Burman —Group A—is an impression of flatness at the back of the skull. Our photograph of skull No. 110 (Plate I, Fig. 1) in profile shows an extreme example of this flattening; it will also, however, be remarked in No. 4 (Plate III, Fig. 8) which has been photographed as a type (with all the characteristics rather marked) of our Group A males, and here we can see it also as looked at from above. The effect is helped by the fact that the parietal eminences are further back in proportion than are those of the Chinese for example (Plate VI, Fig. 19), or the Hindu (Plate V, Fig. 16), or to go outside the various series we have specially used for comparison in this paper, further back than in the Moriori skull (Plate XI, *Biometrika*, Vol. XI. after p. 135). This is a feature which is not brought out in our direct measurements, nor indeed very clearly in our contours, since the vertical contours are taken in a plane well in front of these eminences while in the horizontal contours the pointer of the contour tracer passes well below them. We shall, however, expect to find some confirmatory indication in the horizontal contour even though this is not drawn at the height where it would most clearly illustrate our point.

In so far as the effect is due to the actual flattening of the upper part of the occipital bone, that is, to its steeper fall from the lambda to the inion, the sagittal contours should serve quite well to demonstrate it.

Let us first take up the transparent-paper tracings of the ♂ and ♀ Horizontal Type contours for Group A (Burmans) and place them upon the corresponding contours for Group C, *F* to *F*, and *FO* falling along *FO*. We notice at once the slimmer proportions of Group C, and the more flattened occipital region in Group A. It will, however, also be noticed that the frontal curve of the C's is more concave, and it may be this that has thrust the occipital point *O* of Group C type contour too far out when we made our mid-frontal points *F* to coincide. We will therefore make the points where the contours indicate our temporal ridges lie in the same straight

line, *FO* lying along *FO* as before. The occipital part of the Group C contour no longer projects so far beyond that of the Group A, but still enough to show quite clearly the greater flatness of the latter; a flatness which clearly characterises both occipital and frontal regions. The swelling curves to left and right in Type A bring out in a striking manner the greater width which our direct measurements revealed.

We now turn to the transverse type contours, and lay first the tracing for Burman males over that of Group C males. If we make the mid-points of our auricular axes coincide and their perpendicular bisectors lie along one another, Group C contour is seen to fall entirely within Group A. Though the chief difference is in width, not in height.

But now, in order to compare the shape of the lateral curves, let us slide the auricular axis of the Group A tracing along that of the other until the *R* auricular points come together. The contours now coincide to a great extent on the right side; but they diverge in two places. In one of these (that about ordinate nine) we can perhaps identify the beginning of that swelling which, carried further back, culminates in the marked parietal eminence to which we have referred. We say perhaps, because the differences noted here are well within the limits of probable error. In the other region, that lying just above the auricular axis, we see indicated the greater massiveness of the Burman about the zygomatic ridge and in the area above it; this again is within our range of variation for the small Group C.

The ♀ type contours for Groups A and C demonstrate the same kind of difference, though here there is a greater difference in height and less massiveness in proportion in the part about the zygomatic ridge.

Lastly, the sagittal type contours.

We are spared the necessity of considering the relative values of the nasio-lambda line and the standard horizontal through the nasion ($N\gamma$), as the best base-line for comparison, since the angle between these two at the nasion is practically the same in the two male types: the two female types also agree together very nearly.

The question of the superiority of either of these to Schwalbe's glabella-inion line or Klaatsch's glabella-lambda line was examined in considerable detail in Thomson's Moriori paper* with a verdict in favour of the $N\gamma$ line, which is parallel to the standard horizontal plane.

Three statements were made about the standard horizontal plane through the sub-orbital point in the case of type crania.

1. That it passes very nearly through, but a little below the inion.

In five of our six type contours it does pass practically through the inion; in the sixth, that for the males of Group C, it is 6 mms. above. This, however, is our smallest series—seven only, and might be expected to vary.

The above important statement is therefore fully supported by the evidence of our Burmese.

* *Biometrika*, Vol. XI. pp. 105–111.

2. That it is approximately parallel to the plane of the foramen magnum.

On the Moriori ♀ type contours it is actually parallel; for the males there is only a difference of 2°·5 between the two lines representing the planes, that of the foramen magnum being so tilted as to bring the opisthion slightly nearer to the *N* line than is the basion.

The inclination of these two lines to one another has been measured on our Burmese individual contours. In our type contours the positions of basion and opisthion were determined by other measurements and the angle as shown on the type is not bound to correspond exactly with the mean of individual contours.

The following are our double values for this angle:

Types:	Burmese ♂			Burmese ♀		
	A	B	C	A	B	C
Mean contour value	7°·3	8°·8	7°·6	8°·8	7°·8	8°·2
From type contours	6°·0	7°·8	7°·2	7°·3	6°·6	7°·2

and in all the above, the tilt of the basio-opisthion line was such as to bring the *basion* nearer to $N\gamma$.

We see therefore that this second argument in support of $N\gamma$ as base-line for comparison is much less potent in the case of the Burmese than in that of the Moriori. The Burmese basio-opisthionic line is in fact about equally inclined to the $N\gamma$ and glabella-inion lines in our type contours, and would contribute nothing to a decision between these two.

3. That the standard horizontal is approximately parallel to the maximum length, i.e. the glabella-occipital line. This is quite true for our series, in all six types.

We will try each of these lines in turn as base-lines in comparing our own Group A and Group C type contours. Putting nasion on nasion and $N\gamma$ over $N\gamma$, we get very markedly in the males and to a less extent in the females, an impression of flattened frontal in Group C: the ♀ type contour for the C's has an outline which is decidedly more re-entrant at the nasion than that of the A's, so that the glabellar region of Group A ♀ seems more prominent—a feature not shown in the male contours, where in fact Group A shows slightly greater prominence.

Now if we move our Group A ♀ contour further forward so that the glabellar regions are together—while still keeping $N\gamma$ along $N\gamma$—we get a very similar contrast between the two groups, in the two sexes; the forehead less sloping in Group A, and the back of the head descending more abruptly from the crown. In the males the occipital bone of Group C ends at a point some 5 mms. lower than that of Group A, but this is not so with the females, and we must remember that the range of variation in Group A males is nearly four times that of Benington's type contour for 100 skulls, so that even this divergence might be accounted for by the

smallness of the Group C sample. It is, in fact, impossible to be dogmatic about any difference in the type which our Group C represents, as suggested by contour comparison, especially in the males; our conclusions can be but tentative. This applies, it need not be repeated, also to what follows:

We will now proceed to examine in how far the shape of the frontal bone itself has a share in giving the impression of greater frontal flatness in Group C, we superpose the nasio-bregmatic lines, nasion covering nasion as at first. The two chords are of practically the same length, and though there is slightly greater concavity in Group A above the ophryon, in the ♂ and ♀, this is so slight as to be negligible. The impression of flatness in Group C therefore is due very little to the actual flattening of the frontal but chiefly to its position.

If we use the glabella-lambda line as base-line, the characteristics we have already remarked are brought out quite well: in this position we get rather less the effect of frontal flatness, and more of the difference accounted for by the back of the head.

Finally, the glabella-inion line: this has the advantage in both sexes of making the opisthions coincide; and in the female contours brings out quite well the characteristics of the two groups. In the males, however, the much greater length of the occipital bone for the short series of Group C, with the corresponding lowering of the inion, makes Group A type fall entirely within Group C, from the middle of the frontal right away to the inion, so that here it is Group A that would seem to have a more flattened frontal, and about equal downward slope for the back of the head; an effect that is suggested neither by the actual appearance of the skulls nor by the other methods of contour comparison.

Whichever of these various base-lines is used, however, there is one feature so pronounced that it is brought out by all of them. This is the much less prominence of the nose in Group A, for both male and female.

The result of our comparison of these two groups by means of direct measurements and contours may be summed up as follows: Group A, the Burman Group, is broad and high in proportion to its length (in the brachycephalic and hypsicephalic categories), with marked frontal eminences, and is more vertical than Group C from the crown of the head down to the inionic region; it is of a rugged build, has prominent cheek-bones, nose wide and rather flat at the bridge, wide and rather depressed from the bridge downwards (nasal index classifies it as platyrhine), orbits flat rather than round, but not markedly so (mesoconchic, to use another of these cumbersome class designations), given to sub-nasal prognathism, as the "Remarks" in Appendix II testify, but face fairly vertical above this point; palate not specially broad in proportion to its length (on the borders of the leptostaphyline and mesostaphyline categories).

Group C is both longer and narrower in the main skull measurements but again high (mesocephalic and hypsicephalic); it is of smoother and lighter build with less outstanding cheek-bones, higher nasal bridge, and more prominent nose, the nose being narrower both at the bridge and lower down (mesorhine); orbits slightly

flatter than those of Group A (on the border between the chamaeconchic and mesoconchic categories); frequently characterised by sub-nasal prognathism, but fairly vertical above; palate also similar to that of Group A.

One of the measurements which has been taken, I believe, for the first time on the sagittal contours of my series, is that of height of palate (see p. 201). The term "high palate" figures frequently in literature dealing with mental defect, and it is generally assumed that a connection exists between these two phenomena. It seemed, accordingly, to be of some interest to discover whether any association existed between height of palate and capacity of skull in the present series: of mental capacity I naturally had no record. Miss Margaret Moul of this Laboratory was kind enough to make out for this purpose a correlation table, using the palate height index* as one variate, and the capacity of the skull as measured by mustard-seed (*C* of the direct measurements) as the other. The two sexes were kept separate; and as there were only 31 male skulls and 38 female on which it had been possible to take both measurements, the numbers were too few for grouping, and the correlation coefficient was worked out on the individual measurements.

The following results were arrived at:

Coefficient of Correlation between palate index and capacity of skull:

$$\text{♂: } r = -\cdot3402 \pm \cdot1071,$$
$$\text{♀: } r = -\cdot3283 \pm \cdot0976.$$

In spite of the large "probable error," due to the small series of skulls, the value of r is seen to exceed three times that of the probable error for both sexes, and must be accepted as significant: that is to say, the figures show a tendency for the higher palate to be associated with smaller *skull* capacity, in both male and female. It is not easy to conceive how a relatively high palate could connote physically a reduced capacity for the skull although it might mark a reduced nasal cavity. The point is of much interest, since the opinion of those dealing with mental defectives as to their relatively high palates has not been confirmed by the American investigators, who took casts of the mouth of 300 normal and 300 mentally defective children†. The present relation will be shortly tested on a wider series, a special instrument having recently been designed for the measurement of the palate. But if this correlation should hold, it does not follow that the relative palate height would be correlated with intelligence. A recent series of 88 mentally defective children of ages eight to fourteen collected for age showed no sensible correlation between their mental ages judged by the Binet-Simon tests and their cranial circumference. Although no great weight can be placed on this series, it tends *pro tanto* to confirm the results reached by Pearson in 1906‡ that the correlation between size of head and intelligence is very small.

* By palate-height index I understand 100 × height of palate ÷ length of palate. The latter was the G_1 of my direct measurements, while the height of the palate was obtained as described on p. 201 from the sagittal contour.

† *American Journal of Insanity*, Vol. XVI. pp. 687–697. "Comparative Measurements of the Hard Palate in Normal and Feeble-minded Individuals," by Drs Walter Channing and Clark Wissler.

‡ *Biometrika*, Vol. v. pp. 119–125.

9. On the "Remarks" as to the Cranial Anomalies.

Lastly, having examined direct measurements and type contours we turn to the Cranial "Remarks" which accompany the table of individual measurements in Appendix II.

The re-examination of the series of skulls for anomalies or other points which seemed worthy of comment, was carried out on the following system—lacking which, any contribution which these customary comments may make has extremely little value.

A list was made of characters and anomalies for which each skull should be examined, a list partly suggested by the comments made by previous workers in the school. Of the features included in this list, it may be rightly assumed that when they are not mentioned in the "Remarks," they do not exist in the skull under consideration.

The points are as follows:

Age and signs of age. Where no comment is made the skull is adult, with sutures joined but not obliterated; basilar synchondrosis is noted where it exists; also any teeth that are in process of coming through, or, on the other hand, any considerable loss of teeth during life and the consequent absorption of the alveolus: this of course may occur in the younger skulls, but is more associated with old age; any falling-in or thinning of the calvarium, due to old age, has been looked for and noted.

Teeth. Any case of undeveloped or imperfectly developed third molars (as when they have but a single fang and resemble incisors in shape and size) has been commented on. The only cases in which it would be impossible to detect this would be those in which the alveolus was absorbed in the molar region. I have also noted cases of extreme attrition of the teeth, where they remain. Many teeth show signs of very rough usage, presumably due to betel chewing, and are so worn, especially on the inner side, that they present a surface with a steep slope towards the palate. Very often they are considerably blackened also by this habit.

Palate. Each was examined for the existence of bony ridges across the palatine grooves leading from the pterygo-palatine canals, and referred to by Le Double*. Where such a one is thrown over the inner groove on either side, it is described as an inner palate bridge, left or right; over the outer groove as an outer palate bridge.

On thirteen of our skulls the palate was missing; on the remaining 129 we found a bridge in seven cases over the inner palatine groove (five times on *R*, twice on *L*) and in one case over the outer groove (on *L*).

Thus of 129 Burman and Burmese-hybrid skulls, 5·4 per cent. have an inner palate-bridge, 0·8 per cent. an outer. We may insert these figures into the table quoted by Le Double from Buntaro Adachi†.

* *Variations des Os de la Face*, p. 266.

† B. Adachi, *Zeitschrift für Morphologie und Anthropologie*, Band II, S. 202. Stuttgart, 1900.

Race	No. of skulls	Inner palate bridge	Outer palate bridge
Burmese	129	5·4 %	0·8 %
Europeans	1193	2·3 %	0·5 %
Asiatics	688	12·8 %	2·6 %
Australians and South Sea Islanders	64	7·8 %	4·7 %
American Indians (N. and S.)	122	1·6 %	0·0 %
Africans	123	9·7 %	7·3 %
Ancient Egyptians	384	4·7 %	0·0 %

The conclusion which Adachi himself draws is as follows: "Jedenfalls geht aus meinen Ausgaben vor, dass die Canalbildungen der Gefässfurchen am harten Gaumen bei Europäern und Amerikanern überhaupt selten, dagegen bei Asiaten, Australiern, und Südsee-Insulanern und Afrikanern (Altägypter ausgenommen) viel häufiger vorkommen†."

It will be seen that the Burmese figures are not of the same order as those given by Adachi for "Asiatics," and do not support the inference which he makes.

The palate was also examined for palatine torus, a thickening of the bone on both sides of the median suture; only two cases were found, and neither extreme.

Base of the pyriform aperture. The moulding of the base of the pyriform aperture has been mentioned as being one of the characteristics which influenced our classification of the skull. There is therefore included in the "Remarks" a brief description of this character for each skull in which the bones of the face were not broken away. For brevity this feature has been indicated by the letters P.B., and the description has reference first to that part of the floor of the nasal cavity which lies immediately behind the lower edge of the aperture, and secondly to the edge itself. The floor of the cavity is variously described as "flat"—in which case it may be assumed to be approximately horizontal as well—or "sloping upwards and outwards," or "downwards and outwards"; the edge itself is described as sharp, blunt or rounded, and if it is double, with a groove between its two parts, this also is mentioned. In some cases the floor of the cavity is so rounded as to form part of an almost unbroken curve with the rounded lip: such a case is described shortly as P.B. rounded, or V. rounded.

Taking into consideration the form of the edge alone, one finds the following distribution among our groups:

Type of edge	A ♂	A ♀	B ♂	B ♀	C ♂	C ♀
Sharp	5½*	8	4½*	8	3	9
Blunt	6	16	2	6	1	3
Rounded ...	18½*	5	1½*	1	3	4
V. rounded ...	9	3	1	1	—	—

* The "½" is due to skulls in which the lower border of the pyriform is "rounded" on the *R*, "sharp" on the *L*.

† B. Adachi, *loc. cit.* p. 203.

a table which suggests that this feature tends to be finer in the female than in the male; also that it is on the whole coarser among the Burmans proper than in our B and C groups.

As regards the "double edge" above referred to, we have noted seven cases of this among the ♂ Burmans, only one in the corresponding ♀, and one in each of the B groups.

Asymmetry. In going through the series I noted all cases of asymmetry, both occipital and frontal; also the existence of a Sylvian depression, and whether greater on one side or the other; also the relative sizes of the jugular foramina. As, however, these observations have been used to estimate the degree of association between different forms of asymmetry they have been omitted from the printed "Remarks."

*Interparietal bones**. The tripartite interparietal in its complete form of os pentagonale and *R* and *L* os triangulare* occurred in one case only (♀) in my series, and was associated with marked bathrocephaly. There were three cases (1 ♂, 2 ♀) in which one os triangulare was found, each time on the *R* side. In one of the females it was accompanied by considerable asymmetry of the occipital region, the greater prominence being on the same side as the interparietal bone. In the other two cases, there was a fairly prominent occipital region, but no marked asymmetry.

Ossicles in the sutures. Where these exist they have been noted, and included in the "Remarks."

Conformation at the pterion. All cases of epipteric bones have been mentioned; also instances of the thrusting of a process of the squama temporalis between the parietal and sphenoid bones to join the frontal.

Fourteen of our 61 ♂ skulls, and 21 of our 75 ♀ gave evidence of such peculiarities, 5 ♂ and 8 ♀ having them both *R* and *L*. There was an epipteric on the *R* in 11 ♂, 11 ♀ skulls; on the *L* in 6 ♂, 9 ♀; frontal process to temporal bone on the *R* in 6 ♀, on the *L* in 2 ♂, 3 ♀.

Malar bones. There are in our series five cases of a horizontal suture right across the malar bone (out of 127 skulls where face-bones exist) separating off what is frequently termed the os japonicum, two of these existing in the same skull.

In several other cases part of the suture is indicated, but it is incomplete.

Any specially heavy malar marginal processes have been remarked upon.

Pre-condyles. The existence of one or a pair of these has been recorded in the case of nine skulls. They represent all stages from a single small bony growth on the *R* or *L* between the condyles, in front of the foramen magnum, to a pair of well-developed separate characters and from that to a pair of pre-condyles partly fused and forming a small bridge, and, lastly, to a pair entirely fused and forming a conspicuous central eminence.

Metopism. There are two cases of metopism in our 142 skulls.

* See *Biometrika*, Vol. III. p. 220.

Other features. In addition to the above, all cases of flattening at the obelion, metopic ridge, and tympanic perforation have been recorded; and notes have been added on any other singularities that seemed worthy of comment.

10. COMPARATIVE MATERIAL.

When we turn to the consideration of the Burmese skull in its relation to other races, we are confronted with a problem that is among the most difficult and interesting of those many that await the study of the craniologist.

As with the lower types of life, so with man, we are, in Indo-China or Further India, on the border-line of two great divisions, the Oriental and Euro-Asian. Much work remains to be done still on the Crania of the Burmese races, and much on those of Thibet and Siam, before a full light can be thrown on the exact relations of these borderland peoples. Nor have we as yet any comprehensive study of the skulls of the great races that lie to the north, south, and west of our border-line, although a considerable amount of cranial material has been collected and is scattered throughout the museums of Europe. The measurements that have been taken vary very considerably, both as to number of characters chosen and as to the convention by which they are measured; and practically all were left in the records as mere measurements, not yet reduced to the constants of racial types.

In so far as the material served, I have used it to compile tables on which the means of between twenty and thirty characters can be based for four neighbouring types: Malayan, Chinese, Hindu and Dravidian.

In preparing these tables it was necessary to reject a considerable number even of the rather scanty measurements which were all that most craniologists supplied: either because the origin of the skull was stated too vaguely, or because the character measured was not one usually measured by other craniometricians; or because, if the same measurement were attempted it was taken by a non-comparable method. The necessity for ascertaining exactly what had been measured in the case of each worker made the compilation of my own series of means from theirs rather a laborious process: I have, however, taken all the care I possibly could to avoid errors arising from this source.

The following is the comparative material obtained:

Malayans. I was fortunate enough to find in Dr Emil Schmidt's Catalogue of of his own collection* a series of over 80 Malayan skulls. Of these, one was a child, and one a badly deformed hydrocephalic male: of the remainder, six only were ♀s, the rest ♂s. The ♀ skulls are of course too few alone to be of any use; the ♂ skulls, however, not only form a long series, but have had a large number of measurements taken on them, and these mostly in accordance with the Frankfurt Concordat, so that we have means of no less than 32 characters based on 69 to 78 skulls, with which to make comparisons.

♀ Malayans had to be collected from various sources, and were brought up to a total of 56; of these 38 were recorded in *Die Anthropologischen Sammlungen*

* *Die Anthropologischen Privat-Sammlungen Deutschlands*, Abtheilung I.

Deutschlands (Emil Schmidt's Collection 6, Breslau 1*, Munich 5*, Darmstadt 8*, Frankfurt 3*, Berlin 6*, Göttingen 6*, Bonn 3*), and 18 from Barnard Davis' Catalogue.

The skulls of the Malayan series have been assembled from the various islands of the Malay Archipelago, which we are aware contain a considerable Indonesian element. We may, however, surely dismiss the possibility that so experienced a craniologist and collector as Dr Emil Schmidt (to whom we owe all the ♂ data), when out to collect Malayans, would accept Indonesians and classify them under the title "Malay." There can be equally little doubt, however, that by "Malay" is not meant the pure Malay race, but any of the kindred races scattered upon those islands which are now more usually grouped under the broader term Malayan. That we are not dealing here with a pure race is revealed by the standard deviation of the cephalic index B/L. For the Malayan series it is $5{\cdot}13 \pm {\cdot}28$; and this may be compared with the corresponding standard deviations for other races quoted by Fawcett†. Here we see that the fairly homogeneous collections of Naqadas, Negroes, Panjabi Low Castes, Ainos, and Row Grave Germans show a standard deviation of the cephalic index below 3·0 for male skulls; for the ♂ Bavarians, Whitechapel English and Egyptian Mummies it lies between 3·0 and 4·0 and only in the collection of ♂ Modern Egyptians where it rises as high as 5·42 is our Malayan standard deviation exceeded.

There is therefore doubtless some heterogeneity in the Malayan material, but pending further study of the crania of the individual islands, we are compelled to use the means obtained from it, as a working approximation to a Malayan cranial type.

Chinese. No single series of Chinese skulls in the various German museums was long enough alone to supply anything like the number desirable, so it was again necessary to collect data scattered throughout the publication, and we have before us from German sources measurements taken in 69 male Chinese skulls (Schmidt's 19, Strassburg 2, Breslau 2, Heidelberg 2, Munich 13, Göttingen 11, Bonn 1, Frankfurt 7, Darmstadt 12), but about half of these, having been measured before 1882, and therefore not in accordance with the Concordat, yielded comparatively few measurements of which use could be made.

In addition there are sixteen skulls from Flower's catalogue of the collection at the Royal College of Surgeons, and seventeen from Barnard Davis' collection. Total for Chinese ♂ skulls 102, the numbers on which the various means are based ranging from 102 downwards.

Drawing on all these sources I only managed however to collect measurements of 11 ♀ Chinese skulls all told (3 from Flower, 1 from Schmidt, Munich 1, Bonn 1, Barnard Davis 5). The ♀ Chinese skull is evidently a rarity, and any further specimens that can be secured will form a very valuable addition to the existing collections of crania‡.

* Parts XII, X, IX, VI, V (i), II, I, respectively. † *Biometrika*, Vol. I. p. 440.

‡ The Biometric Laboratory would be very grateful indeed for the gift or loan of Chinese crania with locus of origin. Recent attempts to procure such directly have failed owing to the strong Chinese religious prejudices backed by legal ordinances.

The material from which our female means have had to be obtained is much too slender to base any definite conclusions upon, and is only interesting in so far as it may confirm the results obtained for males in the comparisons which follow.

Another weakness which has to be admitted is that this material is probably heterogeneous, being drawn from widely separated districts in China. The catalogues frequently omit to state from which part of China the skulls were obtained: 47 out of our 102 ♂ skulls are simply given as Chinese. Of the rest, 25 are from S.E. China, and 16 from the West Indies—presumably settlers from the S.E. rather than the north. 12 are from Shanghai and Ningpo district, and only two are from as far north as Pekin.

The females are distributed in much the same proportions: 5 from unknown districts, 4 from South China, 1 from Batavia, 1 from Shanghai. Thus we can say that the large majority of those whose origin is known are from the southern part of China. If the unknowns which form less than half our collection are divided in somewhat the same proportions between north and south, our means will be approximately those of the South China type, which is the one we need for comparison with our Burmese. And the standard deviation of the cephalic index B/L for males which is $4{\cdot}52 \pm {\cdot}29$ encourages us to believe that the sample is not more heterogeneous than our Malayan sample.

Hindus. We turn now to find a Caucasian neighbour from the west. Here again the difficulty arose that in many cases no details beyond the term Hindu, which, as is well known, is not racial but religious, and includes an agglomeration of types of non-Caucasian origin, as well as highly diversified Caucasian types. The term "Hindu" was therefore still less satisfactory than the bare term "Chinese," and the material to hand had to be rigorously sorted out.

The final table has been confined to skulls from the North-Eastern part of India, mainly Bengal, but including a few from round Benares, Allahabad being our western limit; further, all from within this area which had not belonged to persons of the Hindu religion were struck out as being more likely to contribute heterogeneous elements; I also omitted individuals belonging to such castes as I could discover by means of Risley's *Tribes and Castes of Bengal* to be of Dravidian origin or admixture. Again testing the homogeneity of my material by means of the standard deviation of the cephalic index, I obtained the value $4{\cdot}53 \pm {\cdot}38$ for ♂ Hindus, so that judged by this single constant the heterogeneity is of the same order as that of the Chinese sample.

In this Laboratory there is a short series of skulls of Hindus (9 ♂ and 6 ♀) from Bengal, from the Lower Ganges. These I have measured and pooled with the rest.

The total stands at 69 ♂ and 39 ♀ Hindus and is made up as follows: *Males*, Royal College of Surgeons 36, Barnard Davis' collection 23, Munich 1, Biometric Laboratory 9. *Females*, Royal College of Surgeons 8, Barnard Davis 23, Munich 1, Göttingen 1, Biometric Laboratory 6.

Dravidians. Finally, I took as the material for a Dravidian type the series of 40 skulls brought over by Dr Shortt from Madras in 1878, and belonging chiefly to the Maravar tribe, a Dravidian stock from the Madura district of the Madras Presidency. It was unfortunate that Dr Shortt saw fit to divide his material into two halves, one of which went to the museum of the Anthropological Society of Paris, while the other came to our own Royal College of Surgeons. This second series of 20 was sexed and measured by Flower. Three are given as ♀; three are indeterminate, and if we may class these with the males we have seventeen of the latter. The 20 in France were measured by E. Callamand* by Broca's method, but were unfortunately not sexed. Comparing, however, the means obtained by him with those obtained by Flower I have come to the conclusion that the 20 in France must be either all, or practically all, males, and have put them with Flower's 17, making 37 in all. This has been done with the more confidence as I have seen, on examining the series in the Royal College of Surgeons by the kindness of Professor Keith, that the skulls are very closely alike and form a distinct type with no marked exceptions, so that the means may be obtained with as great accuracy from a score of them as from threescore of a more mixed race.

With regard to the whole of our comparative material, I may say that I am probably more conscious than anyone else of how much work remains to be done on all the races dealt with. The tables of means on pp. 239–240 are such as the data available could supply.

11. Comparison of Burmese with adjacent Racial Series.

We will begin by comparing our largest group, the Burmans proper, with the Malayans, basing of course our judgment of any difference between their respective means for any character, on its relation to the probable error of that difference in samples of the size used†.

We notice how extremely close the two means are in the majority of characters; in fact, a significant difference is only revealed by both sexes in the following points: the Malayan, though brachycephalic, has a smaller cephalic index (B/L) than the

* "Le crâne des Noirs de l'Inde," *Revue d'Anthropologie*, Vol. I. 4th series, pp. 607–625, 1878.

† The probable errors can be roughly estimated from those given in Table II, where the Burman (Type A) is compared with Type C. The means of this latter type are based on about eight individual measurements for males, and about seventeen for females. As these numbers vary in the different series which we shall compare with our Burmans, so the probable errors will be modified, and in the following proportions:

Males.

For means based on	14	skulls, the probable error of the difference will be						·75 times that given in Table II.
„	„	21	„	„	„	„	„	·65 times.
„	„	54	„	„	„	„	„	·5 „
„	„	266	„	„	„	„	„	·4 „

Females.

For means based on	6	skulls the probable error will be				1·5	times that given in Table II.
„	„	50	„	„	„	·75	times.
„	„	278	„	„	„	·6	„

TABLE VII*. *Comparative Table.* (*Males.*)

Types:	Burmese ♂ A	Burmese ♂ B	Burmese ♂ C	Malayan ♂	Chinese ♂	Maravar ♂ (?)	Hindu ♂
Capacity ...	1406·9 (27)	1415·0 (4)	1442·2 (5)	[1424·4 (76)]	[1467·6 (46)]	[1289·7 (17)]	[1319·9 (34)]
F	172·4 (44)	172·4 (8)	175·5 (8)	—	179·6 (16)	175·6 (17)	175·9 (45)
L′	174·2 (44)	176·7 (7)	177·2 (8)	—	179·2 (4)	—	173·2 (9)
L	173·5 (44)	173·8 (8)	176·7 (8)	174·7 (78)	177·1 (84)	175·6 (21)	175·4 (33)
B	143·7 (45)	141·1 (7)	140·4 (8)	142·2 (77)	139·5 (102)	131·4 (38)	132·3 (69)
B′	94·3 (44)	89·7 (8)	90·4 (8)	93·4 (77)	93·9 (49)	93·2 (21)	92·4 (10)
H	136·8 (43)	136·6 (7)	140·1 (8)	136·2 (76)	136·6 (37)	132·5 (38)	132·1 (43)
H′	136·0 (43)	134·7 (7)	139·1 (8)	137·4 (76)	136·9 (69)	—	131·5 (10)
H″† ...	(136·8 (44))	(138·7 (8))	(139·3 (8))	—	136·6 (17)	—	137·8 (23)
OH	117·7 (44)	116·7 (7)	116·9 (8)	118·1 (77)	119·2 (38)	—	111·4 (10)
LB	98·5 (43)	98·8 (8)	100·5 (7)	99·5 (76)	99·1 (66)	98·8 (38)	99·2 (44)
Q′	325·8 (43)	319·5 (7)	319·4 (8)	319·0 (77)	321·2 (39)	—	302·9 (10)
S	363·7 (44)	367·0 (7)	367·4 (8)	365·7 (73)	370·3 (78)	—	363·5 (33)
S_1	128·6 (44)	124·9 (7)	127·9 (8)	—	126·6 (57)	127·0 (21)	127·1 (33)
S_2	124·0 (45)	124·4 (7)	122·5 (8)	—	126·0 (56)	—	126·5 (33)
S_3	111·2 (44)	117·7 (7)	117·0 (8)	—	116·1 (56)	—	110·1 (33)
S_1'	94·7 (44)	98·1 (7)	99·7 (8)	98·0 (73)	98·7 (19)	—	92·3 (9)
U	505·7 (44)	497·7 (7)	503·1 (8)	505·7 (77)	508·5 (82)	488·7 (38)	493·5 (69)
G′H ...	71·4 (39)	68·2 (8)	74·5 (7)	70·1 (73)	71·2 (49)	—	63·8 (9)
GB	101·9 (40)	100·9 (8)	94·3 (7)	99·3 (76)	99·9 (38)	—	95·2 (9)
J	134·0 (40)	131·7 (8)	126·7 (7)	133·2 (75)	131·8 (65)	124·4 (21)	126·8 (32)
NH ...	53·5 (41)	52·4 (8)	55·2 (7)	51·9 (75)	53·1 (54)	46·5 (38)	49·0 (45)
NB	28·1 (41)	26·6 (8)	25·5 (7)	26·1 (73)	25·5 (60)	24·0 (38)	24·2 (43)
DC	22·8 (43)	21·3 (8)	18·7 (7)	—	—	—	—
O_1R ...	44·2 (41)	44·3 (8)	44·4 (7)	—	—	—	42·6 (10)
O_1L ...	43·6 (41)	43·2 (8)	44·1 (7)				
O_1'	39·6 (41)	39·1 (8)	39·4 (7)	39·1 (73)	38·3 (54)	36·6 (17)	37·6 (46)
O_2R ...	35·0 (41)	33·6 (8)	35·6 (7)	33·5 (74)	33·8 (54)	31·7 (38)	32·4 (45)
O_2L ...	35·0 (41)	34·3 (8)	35·9 (7)				
$G'(G_1)$ ...	49·9 (37)	49·2 (8)	50·0 (7)	—	54·5 (4)	50·5 (21)	46·4 (5)
G_1'	45·4 (37)	44·8 (8)	46·2 (7)	47·3 (69)	45·8 (32)	—	45·2 (7)
G_2	39·6 (40)	40·1 (6)	40·6 (7)	41·2 (70)	39·2 (35)	—	35·9 (9)
GL	96·6 (39)	93·8 (8)	95·9 (7)	98·2 (76)	97·7 (58)	96·0 (36)	95·1 (39)
fml	36·7 (43)	35·4 (8)	37·3 (8)	35·5 (73)	36·0 (23)	33·9 (21)	35·5 (8)
fmb ...	30·7 (42)	30·6 (8)	30·8 (8)	30·0 (74)	30·9 (23)	28·2 (21)	27·4 (8)
100 *B*/*L′* ...	82·4 (44)	80·0 (7)	79·4 (8)	—	77·8 (13)	—	76·2 (9)
100 *B*/*L* ...	82·9 (44)	80·4 (7)	79·5 (8)	81·7 (77)	78·9 (73)	74·6 (21)	75·8 (33)
100 *H*/*L′* ...	78·2 (43)	76·3 (7)	78·7 (8)	—	—	—	75·5 (9)
100 *H*/*L* ...	78·5 (43)	76·7 (7)	78·9 (8)	78·2 (76)	77·4 (21)	75·2 (21)	75·6 (9)
100 *H′*/*L* ...	(78·4 (43))	(77·5 (7))	(78·7 (8))	—	77·4 (35)	—	75·8 (10)
100 *H″*/*L* ...	(78·8 (44))	(79·8 (8))	(78·8 (8))	—	78·5 (17)	—	78·3 (23)
100 *B*/*H* ...	105·8 (43)	104·9 (7)	101·0 (8)	104·6 (76)	103·2 (37)	(99·1 (21))	100·0 (43)
100 *B*/*H′* ...	(105·7 (43))	(104·8 (7))	(100·9 (8))	—	102·4 (46)	—	99·8 (10)
100 (*B* − *H*)/*L*	+4·0 (43)	+2·6 (7)	+0·1 (8)	+3·2 (76)	+1·3 (69)	(−0·6 (38))	−0·2 (44)
100 *G′H*/*GB*	69·8 (38)	67·8 (8)	79·0 (7)	70·7 (73)	71·1 (36)	—	68·0 (8)
100 *NB*/*NH*	52·7 (41)	51·0 (8)	46·3 (7)	50·4 (73)	48·9 (53)	51·7 (38)	49·7 (43)
100 O_2/O_1R	79·1 (41)	75·9 (8)	80·3 (7)	—	—	—	77·2 (9)
100 O_2/O_1L	80·0 (41)	79·4 (8)	81·5 (7)				
100 O_2/O_1' ...	(88·4 (41))	(87·7 (8))	(91·1 (7))	86·0 (73)	88·4 (54)	84·4 (17)	86·3 (45)
100 G_2/G_1' ...	79·8 (37)	82·2 (6)	81·3 (7)	—	78·4 (4)	—	77·8 (5)
100 *fmb*/*fml*	83·8 (42)	86·6 (8)	82·9 (8)	84·6 (73)	85·9 (23)	83·3 (21)	77·3 (8)
P ∠ ...	86°·0 (39)	85°·9 (8)	84°·0 (7)	—	84°·9 (17)	—	86°·1 (9)

* Square brackets outside entries and frequencies indicate that the measurements have been taken by possibly non-comparable methods. Round brackets outside entries and frequencies indicate that the values may be only approximate, not having been obtained directly from individual crania; for example in the case of indices the figures give the ratio of the means of two measurements.

† *H″* is here used to denote Barnard Davis' measure of cranial height, namely the greatest height above the plane of the foramen magnum. For the Burmese I have merely read the values off from the type contours.

TABLE VIII*. *Comparative Table.* (*Females.*)

Types:	Burmese ♀			Malayan ♀	Chinese ♀	Hindu ♀
	A	B	C			
Capacity ...	1267·9 (27)	1232·4 (11)	1231·4 (7)	1288·9 (16)]	[1265·0 (4)]	[1135·1 (8)]
F	166·5 (39)	163·2 (17)	169·4 (18)	160·0 (1)!	170·3 (3)	167·4 (14)
L'	167·4 (38)	164·7 (17)	170·7 (18)	—	—	169·2 (6)
L	166·5 (38)	163·8 (17)	170·2 (18)	167·6 (55)	172·3 (8)	174·2 (31)
B	138·0 (39)	135·6 (17)	130·6 (18)	136·6 (55)	132·3 (11)	128·4 (39)
B'	90·7 (39)	88·5 (16)	88·7 (18)	93·0 (18)	92·0 (2)	87·9 (8)
H	131·4 (39)	131·9 (17)	129·9 (18)	129·0 (6)	130·2 (4)	126·3 (14)
H'	130·4 (39)	130·8 (17)	129·4 (18)	130·1 (38)	130·0 (3)	126·8 (7)
H''†	(132·1 (39))	(131·7 (17))	(129·8 (18))	133·5 (18)	136·1 (5)	130·6 (23)
OH	111·7 (38)	111·9 (17)	110·1 (17)	112·6 (12)	112·5 (2)	107·6 (7)
LB	93·9 (39)	94·7 (17)	95·7 (17)	94·6 (14)	97·3 (6)	92·8 (15)
Q'	312·0 (37)	308·8 (17)	302·1 (16)	312·1 (12)	300·5 (2)	297·0 (7)
S	350·2 (39)	344·0 (17)	350·9 (18)	351·8 (52)	356·6 (7)	359·1 (30)
S_1	121·9 (39)	119·5 (17)	122·6 (18)	122·9 (45)	120·5 (7)	124·6 (31)
S_2	120·3 (39)	119·6 (17)	119·6 (18)	123·4 (44)	126·5 (7)	124·6 (31)
S_3	107·9 (39)	104·9 (17)	108·7 (18)	107·9 (45)	112·4 (6)	109·8 (30)
S_3'	92·8 (39)	92·0 (17)	91·4 (18)	92·5 (6)	94·0 (1!)	92·8 (6)
U	485·1 (39)	476·9 (16)	479·6 (18)	489·1 (35)	488·4 (10)	482·3 (38)
G'H	66·0 (34)	66·1 (17)	63·9 (16)	65·1 (18)	66·0 (2)	60·8 (7)
GB	96·7 (34)	93·6 (15)	91·8 (15)	95·7 (12)	85·0 (1!)	89·5 (6)
J	126·7 (35)	121·9 (15)	118·7 (13)	125·6 (35)	122·7 (7)	119·3 (28)
NH	50·6 (34)	49·4 (17)	47·3 (16)	47·3 (12)	49·4 (5)	44·9 (14)
NB	26·8 (34)	25·0 (17)	24·0 (14)	25·5 (15)	23·6 (5)	22·6 (14)
DC	20·9 (37)	19·7 (17)	18·8 (15)	—	—	18·4 (6)
O_1R	42·0 (34)	42·2 (17)	42·2 (15)	41·0 (1)!	—	38·6 (5)
O_1L	41·8 (34)	41·5 (14)	42·2 (15)		—	39·3 (6)
O_1'	38·0 (34)	37·0 (17)	38·0 (16)	37·2 (11)	36·4 (5)	36·1 (15)
O_2R	34·5 (34)	34·0 (16)	34·2 (15)	31·9 (12)	35·4 (5)	32·0 (15)
O_2L	34·6 (34)	34·6 (15)	33·2 (16)			
G_1	46·7 (26)	47·0 (14)	46·4 (13)	48·6 (5)	—	44·3 (5)
G_1'	42·7 (29)	43·1 (16)	42·8 (13)	45·2 (6)	43·0 (1)!	41·1 (5)
G_2	38·0 (32)	36·4 (15)	38·1 (14)	39·0 (12)	35·5 (2)	34·8 (6)
GL	90·7 (34)	91·5 (17)	89·2 (16)	95·1 (20)	96·0 (5)	89·6 (13)
fml	34·7 (39)	35·1 (16)	35·1 (18)	33·0 (6)	35·0 (1)!	34·2 (6)
fmb	29·4 (39)	30·0 (16)	29·8 (17)	28·5 (6)	28·0 (1)!	28·7 (6)
100 *B*/*L'* ...	83·1 (38)	83·0 (17)	76·6 (18)	—	—	76·3 (6)
100 *B*/*L* ...	83·1 (38)	82·9 (17)	76·9 (18)	81·8 (54)	76·4 (8)	73·8 (31)
100 *H*/*L'* ...	78·0 (38)	79·5 (17)	75·9 (18)	—	—	74·7 (6)
100 *H*/*L* ...	78·5 (38)	79·9 (17)	76·1 (18)	80·1 (6)	74·4 (1)!	74·9 (6)
100 *H'*/*L* ...	(78·3 (39))	(79·8 (17))	(76·0 (18))	78·2 (38)	76·1 (2)	75·4 (7)
100 *H''*/*L* ...	(79·3 (39))	(80·4 (17))	(76·3 (18))	79·1 (17)	78·9 (5)	74·3 (23)
100 *B*/*H* ...	105·8 (39)	103·7 (17)	101·0 (18)	107·5 (6)	101·8 (4)	102·2 (14)
100 *B*/*H'* ...	(105·8 (39))	(103·7 (17))	(100·9 (18))	104·1 (38)	101·3 (3)	101·4 (7)
100 (*B* − *H*)/*L*	+3·9 (38)	+2·4 (17)	+0·9 (18)	+4·9 (37)	+1·1 (6)	+1·5 (15)
100 *G'H*/*GB*	68·6 (34)	69·8 (15)	70·2 (15)	67·2 (12)	68·3 (1)!	67·5 (6)
100 *NB*/*NH*	52·7 (34)	50·7 (17)	50·5 (14)	53·6 (12)	48·1 (5)	50·4 (14)
100 O_2/O_1R ...	82·4 (34)	80·4 (16)	81·2 (15)	78·0 (1)!	—	83·3 (5)
100 O_2/O_1L ...	82·8 (34)	83·0 (14)	78·8 (15)		—	83·5 (6)
100 O_2/O_1' ...	(91·1 (34))	(93·0 (16))	(87·4 (16))	86·0 (11)	97·3 (5)	88·7 (15)
100 G_2/G_1 ...	80·5 (25)	76·8 (12)	83·0 (13)	78·6 (5)	—	78·9 (5)
100 *fmb*/*fml*	84·8 (39)	85·9 (16)	85·0 (17)	86·5 (6)	80·0 (1)!	84·1 (6)
P ∠	85°·8 (33)	84°·0 (17)	86°·4 (16)	81°·2 (6)	—	83°·6 (6)

* See note * p. 239.

† See note † p. 239.

Burman; he has also a smaller orbital index (O_2/O_1'), and an examination of the means of the absolute measurements on which this index is based shows that this is due to the greater height of the Burman orbit, the width being about the same in the two races. The nose again is larger in the Burman.

Both sexes exhibit greater height of nose (NH); this is accompanied by greater width (NB) a feature however which is much more marked in the male than in the female, so that while the resultant index for female Burmans is not markedly different from that for female Malayans, this is not true of the male Burman whose nasal index (NB/NH) is considerably higher than the male Malayan's. Both sexes agree in giving a longer palate to the Malayan (G_1'), and as this is accompanied by greater face length (GL), i.e. greater projection of alveolar point from basion, we are led to infer that the Malayan is the more prognathous. Unfortunately, our data supply no measurement of the profile angle ($P\angle$) for male Malayans; for females we have it in six cases only, but these give the Malayan a considerably smaller angle, the difference being about six times the probable error. It will be remembered that the Burmese are frequently characterised by sub-nasal prognathism: of true prognathism the examples are rare among them.

It has frequently been remarked in the study of other races, that racial characteristics are more emphasised in the male than in the female. We shall accordingly be quite prepared for the discovery of characters in which the females of our two races manifest no significant differences, while males do so. Thus we find in the male Burman a considerably greater transverse arc (Q'—measured from auricular point to auricular point vertically over the top of the head) than in the Malayan. The difference is about five times the probable error. An examination of the figures shows that this is not due to greater height of skull above the auricular axis (OH). It may be due in part to greater width at this point, but our two width measurements, namely, that for maximum width (B) and for width of forehead (B'), show no difference that can be regarded as significant. We have, however, noted in the Burman the heavy ridges carried back from the zygomatic arches above the auricular passages. It is possible that these have contributed somewhat to the length of the curve which crosses over them. Or, what is perhaps still more likely, the room for personal equation in determining the "auricular point" according to Frankfurt definition may have caused Dr Emil Schmidt, who measured the Malayan ♂ series, to select his terminals for this arc rather higher than I have done, thus shortening his transverse arc. Again, in the male Burman, we find a shorter occipital chord (S_3') than in the Malayan. Whether the corresponding arc from lambda to opisthion (S_3) is also less, and, accordingly, how the occipital indices of the two races compare together, we are unable to discover, since the arc measurement was one that Dr Schmidt did not take. Another racial difference is to be found in the male in the greater Burman breadth of face (GB) as measured by the width of the upper maxillary bone at the lower end of the malar-maxillary suture; associated with this is the greater nose-breadth which has already been noted. The palate, on the other hand, is wider in the Malayan, and we have already seen that it is longer and more projecting. The heavier development about the nose

in the Burman, and about the mouth in the Malayan, would serve to mark a distinct difference between the appearance of the two races.

Finally, we find that the foramen magnum is significantly larger in our male Burman skull, both longer and broader.

In one point only do the females of the two races show a significant difference that is not confirmed by the data for the males. The Burman women have rather narrower heads than the Malayan, both as to maximum breadth (B) and breadth of forehead (B').

Burmans and Chinese. The slenderness of our data for ♀ Chinese makes it idle to use them for detailed comparison with the Burmans. I will therefore confine myself to a consideration of racial differences as shown by the males of the two series, pausing, however, to remark that the ♀ figures, for the little they are worth, are in quite good agreement as to the main racial characters.

As in the case of the Malayans, so with the Chinese, one is struck by the number of characters in which no significant difference is indicated between the two races. In the main proportions of the skull, however, the Chinaman is more markedly differentiated from the Burman: his skull is both longer (L) and narrower (B), the effect of both differences being to bring the Chinaman within the mesocephalic category (B/L) and to make his skull capacity somewhat greater. In height of skull (H) as also in auricular height (OH) there is no significant difference between the two. Taking these main linear measurements into consideration, we are not surprised to find a shorter transverse arc (Q') in the Chinaman, and with this a longer sagittal arc (S), whose greater length is due to greater occipital development: the distance from nasion to lambda ($S_1 + S_2$) is identical in the two means; from lambda to opisthion is considerably greater in the Chinese, whether arc-length (S_3) or chord-length (S_3') be taken. Unfortunately, we have not the data for calculating the occipital indices for the Chinese, to compare with those of the Burmans.

Turning now to the facial characters, we find the Chinese have definitely smaller orbits, both as to width (O_1') and height (O_2), the proportions, however, (O_2/O_1'), remaining approximately the same.

The great zygomatic breadth which was found among the Burmans is here reduced, though not to a very large extent, the degree of difference being about three times its probable error. The breadth of the upper maxillary (GB) is reduced in about the same proportion; and, height of face remaining practically unchanged, the upper-face index ($G'H/GB$) undergoes a similar reduction.

The width of nose (NB) is, however, reduced to a much greater extent, and since the height again is not changed significantly, the proportions of the nose (NB/NH) bring the Chinese into the middle nasal category, in contrast to the platyrhine Burmans. In the remaining features of which our data supply us with a measure—palate, foramen, etc.—we find no racial divergence indicated that may not be accounted for by random sampling.

Burmans and Hindus. We next compare our Burmans with the Hindu material that has been gathered together. Here we find more characters exhibiting differences which cannot be other than racial. The major measurements reveal at once the comparative smallness of the Hindu skull, which, though not losing anything in length* (L), shows a markedly smaller maximum breadth (B) especially for males, and less height (H), and, as we should expect, smaller capacity (C). The height above the auricular points (OH), as also the arc (Q') connecting these two in the vertical plane, are correspondingly reduced; it may be noted, however, that the breadth of forehead (B'), unlike the maximum breadth, remains practically the same. These figures bring out the fact that the cranial box of the Hindu differs from that of the Burman not only in size but in shape. Thus the Hindu skull is mesocephalic (B/L) against the brachycephaly of the Burman; its relative height and length (H/L) bring it to the border-line between hypsicephaly and orthocephaly, and its mean maximum breadth (B) is exactly equal to its mean height (H) while the Burman breadth exceeds the height.

When we examine the features of the Hindu face, we find again a considerable diminution in size. It is less high ($G'H$) than that of the Burman, less broad, both as regards zygomatic (J) and maxillary (GB) breadth; the nose is both shorter (NH) and narrower (NB); so also are the orbits (O_1' and O_2). Its palate is rather narrower, but is approximately as long as that of the Burman; the difference, however, is not so great as to put them in different palate categories.

The finer proportions of the Hindu nose classify it as mesorhine, one class higher than that of the Burman nose; and the orbit is less round.

All these racial characteristics are exhibited by both male and female, in their mean values.

As before, we find some racial differences which can be considered significant in the males alone: a smaller horizontal circumference (U) in the Hindu, and a

* It will be noted that this inference is based upon the data for (L) from Hindu skulls alone. The bulk of the data for this character for Hindu females was obtained from the catalogue of Barnard Davis' collection, and a comparison of this mean measurement both with the corresponding male value, and with that of the females in our Hindu group at the Biometric Laboratory, suggests some doubt as to the accuracy of these particular figures.

Mean maximum length of Hindu skulls		
	♂	♀
B. Davis' Collection	176·1 (23)	175·7 (23)
Biometric Laboratory Collection	172·9 (9)	168·6 (6)

Our own series is too small for any real stress to be laid upon the means obtained from them alone; but that the mean length of the male and female head should be practically the same in any one race, as the figures from the Barnard Davis' catalogue would indicate, is a condition paralleled by the constants of no other race that has been studied: the average reduction for the female mean is about one twenty-fourth of the male value (mean of 20 races).

narrower foramen magnum both in its absolute measurement (*fmb*) and in relation to its length (*fmb*/*fml*).

Burmans and Dravidians (*Maravars*). The number of characters which are available for comparison are somewhat fewer in number in the case of the Maravar, and, as already remarked, we have only data for male skulls at our disposal.

As in the case of the Hindus, we are confronted with considerable divergencies from our Burman constants, and frequently in the same direction.

Again we find a smaller capacity (*C*) and height (*H*) and a much smaller maximum breadth (*B*), while the maximum length (*L*) and minimum forehead breadth (*B'*) are approximately the same; again a reduction in all the cephalic indices (*B*/*L*, *H*/*L*, *B*/*H*); also in the horizontal circumference (*U*), zygomatic breadth (*J*) and height of nose (*NH*), though to a greater extent than in the Hindu; the width of the nose (*NB*) is also reduced, but not quite so much as in the Hindu, so that the nasal index of the small nose of the Maravar does not differ significantly from that of the Burman's large one. The orbit (O_1', O_2) too is small in comparison with the Burman's and flatter; still flatter than the Hindu orbit. Unlike the Hindu, the Maravar has a foramen magnum which is shorter (*fml*) than the Burman's as well as being narrower (*fmb*), so that its proportions do not differ materially from those of the latter.

In examining columns of means such as are supplied by the comparative tables here discussed, it is possible to gain some idea of the extent to which a cranial character of one race is modified in another race. But is it possible to answer from them the much larger and more important question of how nearly the various races characterised by the multiplicity of racial constants are related together? One may certainly gain some vague appreciation of relative affinities between the various races, as here one gathers that the Burman seems more nearly related to the Malayan and Chinaman, than either to the Hindu or the Maravar.

But to which of the first is he more closely akin? to which of the last?

Would it be possible to get a measure of the relationship by taking a large number of characters in combination instead of considering each individually?

And if so, could we by this means test whether the larger group and the two smaller groups of our Burmese series belong indeed to one and the same population? or whether we were justified in considering them distinct?

These are such problems as must often have troubled other craniologists, and I have to thank Professor Pearson for providing a provisional solution*, in the form of a coefficient which shall express numerically to a first approximation the degree of relationship existing between any two racial groups.

Before passing, however, to the consideration of this coefficient, there is one feature in our comparative tables of which I have reserved mention until now;

* Provisional in the sense that a more complete definition will only be possible when wider study of the type constants of races provides material for the determination of inter-racial variabilities and correlations.

this is the Index $(B - H)/L$, which measures the excess of breadth over height in proportion to length, and gives us the following suggestive order:

Values of 100 $(B - H)/L$ *for Burmese and neighbouring Races.*

Malayan Women	+4·9	(37)
Burmese Group A, Men	+4·0	(43)
„ „ Women ...	+3·9	(38)
Malayan Men	+3·2	(76)
Burmese Group B, Men	+2·6	(7)
„ „ Women ...	+2·4	(17)
Hindu (Bengal) Women	+1·5	(15)
Chinese Men	+1·3	(69)
Chinese Women	+1·1	(6)
Burmese Group C, Women ...	+0·9	(18)
„ „ Men	+0·1	(8)
Hindu (Bengal) Men	−0·2	(44)
Maravar (Dravidians) Men ...	[−0·6]	(38)*

In this table it will be noted that the Hindu women are placed considerably higher than the Hindu men, but having already seen that the data obtained for Hindu females may not be altogether reliable, it may be well if we reserve any conclusions that might be drawn from this until their place in the list has been confirmed or altered by further evidence.

With this exception, the table gives a not unreasonable order: the Burman proper (Group A) is nearest to the Malayan, between that and the Chinese; Group C (Karens?) lies between Chinese and Hindus; and Group B is between Groups A and C in order, which would be expected if the surmise that it is hybrid be correct.

The Dravidian type is the most remote of any, from the Burman type.

Inserting these various values into the table of this index for a long series of races given by Thomson in the Moriori paper† we get the following:

Values of 100 $(B - H)/L$.

Würtemberg Women	+9·6	Congo Men	+2·6
„ Men	+9·5	Burmese (Group B) Men	+2·6
Bavarian Women	+9·2	„ „ Women	+2·4
„ Men	+9·0	Australian Women	+2·4
English Women (Moorfields)	+7·9	Long Barrow Women	+2·3
French Men	+7·2	„ „ Men	+2·2
English Men (Moorfields)	+7·1	Fuegian Men	+2·0
„ Women (Whitechapel) ...	+5·6	Congo Women	+1·9
Malayan Women	+4·9	Theban Mummies, Women	+1·8
English Men (Whitechapel)	+4·2	Hindu (Bengal) Women	+1·5‡
Burmese (Group A) Men	+4·0	Chinese Men	+1·3
„ „ Women	+3·9	„ Women	+1·1
Moriori Men	+3·5	Prehistoric Egyptian (Naqada) Women	+1·0
„ Women	+3·2	Aino Women	+0·9
Malayan Men	+3·2	„ Men	+0·9

* As the individual measurements were not given for the 21 skulls measured by Callamand, this index for the Maravars has had to be calculated from their mean value and from the individual measurements of Flower's seventeen skulls.

† *Biometrika*, Vol. XI. p. 95.

‡ See remark higher upon this page.

Values of 100 $(B - H)/L$—*(continued)*

Gaboon Men (1880)	+0·9	Hindu (Bengal) Men	−0·2
Burmese (Group C) Women	+0·9	Coptic Men	−0·2
Maori Men	+0·7	Australian Men	−0·2
„ Women	+0·5	Prehistoric Egyptian (Naqada) Men	−0·3
Gaboon Women (1864)	+0·5	Gaboon Women (1880)	−0·4
Coptic Women	+0·5	Maravar (Dravidian) Men	−0·6
Theban Mummies, Men	+0·4	Zulu Men	−0·7
Burmese (Group C) Men	+0·1	Angoni Men	−1·2
Gaboon Men (1864)	+0·0	Negro Men (N. Africa)	−1·3
Kaffir Men	+0·0		

Here we find the Malayan women, highest of our new series, intruding into the European group, which previously remained exclusive, at the top of the list: that they really are higher up in this respect than English Men (Whitechapel) is not absolutely proved, since the difference between them, 0·7, would not be equal to three times its probable error*. In any case, both Malayan series and both Burman series approach more nearly to the Modern European values than any of the non-European races cited with the exception of the Moriori; while the Burmese hybrids do not lag far behind. The Dravidian series finds its place towards the bottom of the list.

Another feature which deserves special notice is the group of nasal measurements and indices. The comparative study of the nasal bridge to which Benington† devoted some attention in our own Laboratory was later made the subject of extensive research there by Kathleen V. Ryley and Julia Bell‡.

To the list of races arranged in order for values of the Simotic and Dacryal Indices§ I can now add my three Burmese Groups.

Simotic and Mesodacryal Indices. Mean Values.

Simotic Index *SS/SC*		Mesodacryal Index *DS/DC*	
♂	♀	♂	♀
Congo 26	Nubians 19	Congo 39	Congo 37
Moluccas 27		Philippinos 42	Nubians 40
Gaboon (1880) 29	Gaboon (1880) 23	Borneo 42	Borneo 41
Sumatra 29	Congo 26	Moluccas 43	Javanese 43
Celebes 30	Gaboon (1864) 28	Gaboon (1864) 43	Gaboon (1880) 44
Borneo 30		Gaboon (1880) 43	Gaboon (1864) 44
Philippinos 30	Philippinos 29	Malays 44	Philippinos 45
Gaboon (1864) 31		Celebes 45	
Malays 32		Sumatra 45	
Burmans 33	Burmans 30	Javanese 45	
Javanese 34	Javanese 33	Burmans 45	Burmans 45
Nubians 34	Borneo 35	Nubians 48	
Ainos 43		Ainos 50	
Veddahs 44	Veddahs 37	Burmese Group B	
Egyptians 44	Egyptians 37	(? Hybrids) 50	Burmese Group B
Burmese Group B	Burmese Group B	Hindus 55	(? Hybrids) 49
(? Hybrids) 45	(? Hybrids) 39	English 57	Hindus 49
Hindus 45	Hindus 39	Veddahs 57	English 53
English 51	English 47	Egyptians 58	Egyptians 53
Burmese Group C	Burmese Group C	Burmese Group C	Veddahs 59
(? Karens) 54	(? Karens) 49	(? Karens) 62	Burmese Group C
			(? Karens) 60

* Compare Table II, p. 221.

† *Biometrika*, Vol. VIII. pp. 315–320.

‡ *Ibid.* Vol. IX. pp. 391–445.

§ *Ibid.* Vol. IX. p. 404.

Here again we find our Burmans in close proximity to the Malayan Groups of the West Indian Archipelago. For each index of the nasal bridge the Burmese Group C gives a higher value than any other race for which it has been obtained, English and Egyptian included. It is regrettable that at present we have not these indices for the bridge of the nose in the case of more Caucasian races, nor for Chinese or Dravidians. It is clear that a complete table—especially with rather longer series for some of the races here provided—would be of the greatest suggestiveness. We need also to discover, if possible, the values for palaeolithic and neolithic man. The nasal bridge index and the differential cephalic index $[(B-H)/L]$ seem to me perhaps the more capable than any other cranial indices of providing adequate racial characterisations.

12. COEFFICIENT OF RACIAL LIKENESS.

We turn now to the problem already stated (p. 244) of obtaining a measure of the relationship between different races. For the general solution of this problem, an extensive knowledge of the degree of correlation between different cranial characters is required. We have two types of correlation in craniometry.

First, the intra-racial, that is, the correlation of characters in the individual, the measure of this relation being obtainable from a group of individuals to which he belongs. A good deal is known about such correlations already; more will be known when the long series of Egyptian skulls—1800 of one period—now in hand in the Biometric Laboratory, is completely reduced. But we do know quite enough to assert that the correlation is never very high between cranial characters which do not have any portion in common, and which are not right and left measurements of homologous characters. It is indeed often wholly negligible.

The *second* form of correlation is that between the *means of characters* in different races, and to obtain a measure of this we must have a large number of races whose means have been established. This may be conveniently termed inter-racial correlation to distinguish it from intra-racial.

There are at present, however, so few races for which we have a knowledge of the mean values of forty or more characters, based upon a series of adequate dimensions, that the principal inter-racial correlations remain largely unknown. We do know that inter-racial and intra-racial correlations differ considerably in a number of cases: thus, for example, the value of the relation between orbital and nasal index in the individuals of one race is considerably different from that between their mean values in a group of races. There is, however, on the basis of our present knowledge, no reason to believe that inter-racial correlations are likely to be at all intense. Accordingly, when we consider the difference in cephalic index of two races, and then proceed to consider the difference in zygomatic breadth, any inference of significant difference in the first will not be weakened by the fact that we find a significant difference in the second: in other words, the two judgments may be considered independent in the present state of our knowledge of inter-racial correlation. That is to say, we shall obtain at least a first approximation to an

appreciation of racial resemblance or racial difference by treating each character as an independent judgment, i.e. omitting the influence of correlation.

Accepting this principle as a *temporary* working condition, we can reach for every pair of races, characterised by m measured characters, a single numerical coefficient; and can at the same time give its probable error. This coefficient will measure the probability that the two groups of crania under consideration are samples of one and the same population: it may be termed the *Coefficient of Racial Likeness* (C.R.L.). Such a coefficient will enable us to compensate for the smallness of a cranial series by measuring a large number of characters. The advantage to the craniologist will be considerable. This coefficient is defined as follows:

Let m characters be measured, and let the sth character in the first race have M_s for mean and σ_s for standard deviation, these two constants being based on n_s individual measurements. Let the corresponding characters for the second race be M_s', σ_s' and n_s'.

Then if the two races were really samples of the same population, the following expression should be sensibly zero:

$$\frac{1}{m}\left\{\frac{S\,(M_s - M_s')^2}{\frac{\sigma_s^2}{n_s} + \frac{\sigma_s'^2}{n_s'}}\right\} - 1,$$

and the probable error of its deviation from zero will be $\cdot 67449/\sqrt{2m}$.

Now, for most cranial samples we have not adequate numbers to enable us to determine σ_s and σ_s': but there is not a very great difference in variability between one race and another. It will therefore suffice, for a first approximation, to put $\sigma_s = \sigma_s' =$ the value found for the standard deviation of this character in the longest series of homogeneous crania available. It is only needful to use the appropriate σ for the appropriate sex.

Thus our coefficient of racial likeness becomes:

$$\text{C.R.L.} = \frac{1}{m}\,S\left\{\frac{n_s n_s'}{n_s + n_s'}\left(\frac{M_s - M_s'}{\sigma_s}\right)^2\right\} - 1 \pm \frac{\cdot 67449}{\sqrt{2m}}.$$

I have calculated the C.R.L. for the Whitechapel Plague Pit Crania of the Biometric Laboratory which were measured by Macdonnell*; also for the Long Barrow British of the Oxford University Museum as measured by Schuster†. I have also compared the Whitechapel Crania with the measurements of the skulls of French prisoners dying in Germany in 1871*. I have compared Bavarians* with Würtembergers* and Prehistoric Egyptians as measured by Fawcett‡ with Congo negroes measured by Crewdson Benington§, using for these comparisons male skulls only.

The table on p. 249 gives us at once a much better appreciation of the degree of resemblance between the seven races they represent than could have been obtained from a comparison of seven columns of means for twenty to thirty individual characters.

* *Biometrika*, Vol. III. p. 208.
† *Ibid.* Vol. V. p. 104.
‡ *Ibid.* Vol. I. p. 426.
§ *Ibid.* Vol. VIII. p. 298.

The following are the results:

Races ♂	No. of Characters	C.R.L.	Probable Error
17th Century Londoners and Long Barrow British ...	23	3·8	±·10
Bavarians and Würtembergers	24	12·1	±·10
17th Century Londoners and Modern French	28	24·5	±·09
Prehistoric Egyptians and Congo Negroes	27	27·3	±·09

Here we find the justification of Macdonnell's statement that the Long Barrow British have a close resemblance to seventeenth century Londoners*.

It is also clear that the critics who asserted a large negroid element in our Prehistoric Egyptians had little basis for their criticism. French and English are shown to be almost as far apart racially as Egyptians and Negroes. And if Bavarians and Würtembergers are not as racially distinct as French and seventeenth century Londoners, they are much further apart than are the latter from the Long Barrow British.

I turn now to the question with which this paper is more immediately concerned, and see how far this method will throw light upon my Burmese mixed population, which mere *appreciative* analysis of characters led us to differentiate into three types: Type A, the Burman proper; C, another type—possibly Karen; and Type B, a supposed hybrid.

Races	Sex	No. of characters	C.R.L.	Probable error
A and C	♂♀	30	3·12	±·09
,,	♂	30	6·44	±·09
A and B	♂♀	30	0·47	±·09
,,	♂	30	1·16	±·09
C and B	♂♀	30	1·30	±·09
,,	♂	30	2·87	±·09

These results indicate that A and C are as racially distinct as Long Barrow British and seventeenth century Londoners, but not as distinct as Bavarians and Würtembergers; they also show that both A and C are closer to the supposed hybrid than to each other.

The method therefore provides a numerical evaluation of a purely appreciative judgment, a judgment, however, which anyone who had examined a large number of cranial groups from the racial standpoint would be likely to make.

Before making use of this coefficient to elucidate the problem of the relative affinities between the Burmese groups and the adjacent racial series with which they have been compared, it will be well to measure the relation of these groups to one another.

* *Biometrika*, Vol. III. pp. 243–4; Vol. V. p. 104.

Race	Sex	No. of characters	C.R.L.	Probable error
Chinese and Malayan	♂	30	3·15	±·09
" "	♀	30	2·15	±·09
Chinese and Hindu	♂	30	15·10	±·09
" "	♀	30	1·50*	±·09
Malayan and Hindu	♂	30	19·50	±·09
" "	♀	30	11·85	±·09

* See remark on p. 245.

These results show several interesting points:

1. The Chinese are closely related to the Malayan.

2. Neither race is closely related to the Hindu, but the Chinese is nearer to a Caucasian type than the Malayan.

3. In so far as we can trust our rather slender data for women, the women are rather more alike than the men, that is, characteristic racial features are more pronounced in man than in woman.

Finally, the coefficient gives an answer to the fundamental problems of our Burmese series. The first is: Do either A or C or the assumed hybrid B approach the Caucasian or Dravidian types?

Race	Sex	No. of characters	C.R.L.	Probable error
Burmese A and Hindu	♂	30	23·94	±·09
" "	♀	30	16·36	±·09
Burmese A and Maravar	♂	30	31·15	±·10
Burmese C and Hindu	♂	30	6·30	±·09
" "	♀	30	1·44	±·09
Burmese C and Maravar	♂	30	7·92	±·10
Burmese B and Hindu	♂	30	3·79	±·09
" "	♀	30	8·56	±·09
Burmese B and Maravar	♂	30	5·59	±·10

This table shows that Type A, the Burman proper, is widely divergent from both Caucasian and Dravidian races. The Burmese Type C (possibly Karens) is less markedly distinguished, though quite adequately distinguished, from the Caucasian and Dravidian types in the male, and may come closer to the Hindu in the female*. But on the whole, I think, we can safely say that our Type C is not a Hindu or Dravidian type. Except in the case of the females of Burmese Type B and Hindu* we see that the rule remains that the male shows the more marked racial characters.

From the more reliable male figures we see that Hybrid B stands closer in order of resemblance than either of the purer types A and C.

Lastly, we turn, after rejecting the Caucasian and Dravidian as direct sources of C, to the inter-relation of the Burmese series to Malayans and Chinese. Here

* It would not be wise to attach too much importance to exceptional results obtained from the Hindu female data on the ground already stated on p. 245.

it is seen at once that much higher orders of racial resemblance are reached. The following table presents the closest results on the whole that have yet been obtained:

Race	Sex	No. of characters	C.R.L.	Probable error
Burmese A and Malayans	♂	30	2·69	±·09
" "	♀	30	1·71	±·09
Burmese A and Chinese	♂	30	6·38	±·09
" "	♀	30	2·66	±·09
Burmese B and Malayans	♂	30	2·35	±·09
" "	♀	30	4·47	±·09
Burmese B and Chinese	♂	30	0·16	±·09
" "	♀	30	0·53	±·09
Burmese C and Malayans	♂	30	0·03	±·09
" "	♀	30	2·39	±·09
Burmese C and Chinese	♂	30	0·29	±·09
" "	♀	30	1·54	±·09

Here the main results are at once apparent. Burmese A are closer for both sexes to the Malayans than to the Chinese. Burmese C are closer to the Chinese than to the Malayans. And again we find Burmese B (Hybrids) closer to both Malayans and Chinese than either of the purer types is. Their greater resemblance to both prototypes emphasises the probability that they are hybrids: it is a case of throwing back after mixture.

It seems reasonable accordingly to conclude as follows:

1. The Burmese population in the neighbourhood of Moulmein consists of at least two distinct races and a probable hybrid race.

2. One of these races, the pure Burman, is more closely allied to the Malayan than to the Chinese.

3. The second of these, probably the Karen, is more closely allied to the Chinese than to the Malayan; and as the Chinese is considerably closer to the Caucasian than is the Malayan, this section of the population, although quite distinct from the Caucasian, is somewhat closer to that type than the Burman proper.

4. The group we have spoken of as hybrid is closer to *both* Malayan and Chinese than either purer element of the population is to Malayan or Chinese. This is a not improbable result of hybridisation.

I have to thank Miss Noel Karn of the Biometric Laboratory for her help in working out the troublesome calculations on which these conclusions are based.

13. THE ASYMMETRY OF THE SKULL.

The fact of cranial asymmetry is of course a commonplace to anyone who has handled skulls, and reference has already been made* to the pitfalls it prepares if we assume that the "horizontal plane" as determined by certain features of the skull will, in the individual, be perpendicular to the "sagittal plane" as determined by others, and both of them perpendicular to the "vertical plane."

* See p. 202.

I have made an attempt to examine in some degree the frequency with which certain kinds of asymmetry occur, and the extent to which they tend to be associated with one another.

Dr D. E. Derry, whom I have to thank for giving me the benefit of his anatomical experience in examining my series for anomalies, pointed out the frequent asymmetry in the occipital region, and suggested that this was usually accompanied by a slightly greater prominence of the frontal bone on the opposite side. I have, accordingly, made observations as to frontal and occipital asymmetry in this series of skulls, and submitted the data to mathematical processes, to see what association, if any, existed between the two.

Most of the observations made were merely appreciative: if some quantitative measure of the degree of asymmetry could be taken, the association could of course be determined with much greater accuracy.

The skull was held in such a manner that the frontal bosses and that portion of the squama occipitalis which lies above the region of the inion were silhouetted against the light, care being taken that what seemed to be asymmetry was actually so, and was not due to any fault of position. Three categories were made, classifying the material according to whether there was greater prominence on the left or right, or equally great on both. The following are the numbers which fell into the various classes.

	Greater on right	Equal	Greater on left	Total
Occipital prominence	38	34	69	141
Frontal „	41	63	37	141

These figures confirm the suggestion that the occipital region is usually asymmetrical, and that in this case it is more often the left side which manifests the greater development.

To test the degree of association between the two asymmetries two methods were open to me, neither, however, an ideal treatment for the data. One could throw the "Greater on Right" and "Equal" together, thus reducing the number of categories to two, and use the method of "tetrachoric r" to obtain a coefficient of correlation. This method, however, implies linearity of regression between the two variates. Or one could apply the method of mean squared contingency, which makes no such assumption, but which would give a much better result had there been five categories or more instead of three. I therefore used both methods.

The results were as follows:

Measure of association between greater occipital development on one side and greater frontal development on the opposite side:

Coefficient of correlation (Tetrachoric) $r = + \cdot 6169 \pm \cdot 0734$.

Coefficient of mean squared contingency corrected with class index corrections $C_2 = \cdot 6374$*.

* Before the class index correction is applied, the value obtained for uncorrected C_2 is ·4965.

Table for obtaining Tetrachoric r.

Greater Frontal Prominence

Greater Occipital Prominence	Right and equal	Left	Totals
Right and equal	17	21	38
Left	87	16	103
Totals	104	37	141

Contingency Table.

Greater Frontal Prominence

Greater Occipital Prominence	Right	Equal	Left	Totals
Right	3	14	21	38
Equal	3	20	11	34
Left	35	29	5	69
Totals	41	63	37	141

The mean value of C_2 (uncorrected) in 3×3 tables of samples of this size drawn from a population where there is *no* contingency will be $\cdot 1661 \pm \cdot 0568$. The value of C_2 obtained may be taken as significant as compared with this, and that of r is distinctly so. We can therefore conclude, on the basis of our material, that these two forms of asymmetry do tend to be associated together, though the association is by no means perfect.

A feature that has been frequently noted in the Cranial "Remarks" of previous workers, is the existence of a depression slightly above the region of the pterion, called the Sylvian depression. Noticing the frequency with which this occurred on one side only, or on one side more markedly than the other, it occurred to me to subject this depression to the same investigation as was applied to the occipital and frontal bones. For this purpose I have not concerned myself with the depth of the depression, but merely with the question whether it was equally represented (or equally unrepresented) on both sides. One hundred and thirty of my skulls were undamaged in that region and available for data. They gave the following results:

Sylvian depression greater on *R* in 90 skulls.
" " equal *R* and *L* in 33 "
" " greater on *L* in 7 "

In my series, therefore, there is a considerable preponderance in favour of the right side.

To test, by the method of mean squared contingency, whether this kind of asymmetry is associated with either of the other two I made out the following tables:

Greater Frontal Prominence

Greater Sylvian Depression	Right	Equal	Left	Totals
Right	29	41	20	90
Equal	10	15	8	33
Left	1	2	4	7
Totals	40	58	32	130

Greater Occipital Prominence

Greater Sylvian Depression	Right	Equal	Left	Totals
Right	23	17	50	90
Equal	7	11	15	33
Left	4	2	1	7
Totals	34	30	66	130

These gave the following results:

Greater Sylvian Depression on one side with Greater Frontal Prominence on same side: $C_2 = \cdot1794$.

Greater Sylvian Depression on one side with Greater Occipital Prominence on opposite side: $C_2 = \cdot2348$.

For 130 in a sample, with this number of categories, mean C_2 when there is no contingency is $\cdot1728 \pm \cdot0592$. As even the greater of our results differs from this by hardly more than the probable error, the results cannot be said to indicate any certain association between Sylvian asymmetry and asymmetry of the frontal and occipital bones.

Another point in which there is frequently a marked difference between the two sides of the cranium is to be found in the grooves on the inner side of the occipital bone which accommodate the lateral sinuses. Le Double* found that out of 200 European skulls, the superior longitudinal sinus was directly continued into the right lateral sinus in 137 cases, into the left lateral sinus in 29 cases, and in the rest neither could claim any advantage over the other.

One hundred and thirty-five of our Burmese crania were available for examination on this point. There were 94 in which the right lateral groove was the more obvious continuation of the superior longitudinal groove (and in this case usually

* *Traité des Variations des Os du Crâne*, p. 16 (1903).

larger than the left), 28 where it was the left lateral sinus, and 13 in which it was impossible to decide between them.

It seems reasonable to suppose that where the main stream of blood is directed down one side rather than the other, the outlet for the blood-vessel on that side will be larger than the one on the opposite side; and there was indeed a very noticeable inequality in the size of the two jugular foramina in a large proportion of skulls. Owing to the difference in shape of the two foramina, it was sometimes difficult to decide which was the greater when they approached each other in size. Our figures would, however, give a good approximation to the truth; they are: 82 skulls in which the right jugular foramen was the greater, 32 the left, 13 equal.

It will thus be seen that it is the right side which has much the greater number both of larger grooves and larger foramina. Can we rely upon the jugular foramina to tell us what is the arrangement of the sinuses? The answer is given by the coefficient of mean squared contingency, as obtained from the following table, which is ·6541 for uncorrected C_2 and ·7036 with the class index correction applied.

Greater Jugular Foramen

Greater Groove	Right	Equal	Left	Totals
Right	79	6	4	89
Equal	2	5	6	13
Left	1	2	22	25
Totals	82	13	32	127

This is for 127 skulls, and mean C_2 where no contingency exists will be of the same order as that for 130, which has been quoted already. The difference between this and the result obtained is roughly 8 times the probable error, and we may therefore conclude that there is a very close relation between the two (though not a perfect one). This means that we can accept the evidence of the foramina as to the arrangement of the sinuses, if the more direct evidence of the grooves be unobtainable.

In connection with this subject my attention was directed to an interesting paper by Professor Elliot Smith* in which a relation is suggested between exterior occipital asymmetry and the direction of the main sinus. The argument briefly is as follows: The occipital poles of the cerebral hemispheres are frequently asymmetrical. Professor Elliot Smith estimates that in roughly 80 per cent. of Egyptian brains, the area of cortex containing Gennari's stria (the area striata) extends much further on to the outer aspect of the *left* hemisphere than on to the corresponding surface of the right hemisphere: often as far as 3 or even 4 cms. from the median line on the left, whereas on the right side the area striata barely crosses the edge of the hemisphere. "As a rule, this lateral part of the area striata

* *Anatomischer Anzeiger*, Band xxx. pp. 574–578, Jena, 1907.

is on a different plane from the rest of the surface of the occipital region, as though it were a mushroom-like cake placed upon the surface. Hence the caudal pole of the left hemisphere projects much further backward in such cases than that of the right, and a distinct depression is formed within the superior (cerebral) fossa of the *squama occipitalis*. Corresponding to this inner depression there is in such cases a marked projection of the external surface of the left side of the occipital bone."

"The fulness of the left occipital pole seems to be the reason for the dextral bending of the superior longitudinal sinus, the flattening of the right pole allowing more room for the bigger sinus on that side. That this is really the case, is shewn by the fact that in those cases in which the *right* area striata spreads on to the caudal surface to a greater extent than it does on the left side, the *right* occipital pole and the right occipital squama become more prominent and the superior longitudinal sinus turns to the *left*. Again in those cases in which the occipital poles are symmetrical, the superior longitudinal sinus usually bifurcates, one half going to each lateral sinus."

It seemed possible that the data I had already obtained might serve to give some measure statistically to the interesting associations thus suggested. It was naturally impossible to obtain from our material any measure of the association of extended area striata on one side, with main lateral sinus on the other. It is unlikely that the size of the area striata will be perfectly represented by a depression in the inner surface of the occipital (fossa cor. striatae): as Professor Elliot Smith is careful to say, "*as a rule* this lateral part of the area striata is on a different plane from the rest of the surface of the occipital region," and where it does *not* "resemble a mushroom-like cake," there will be no corresponding fossa. Probably, however, there would be a correlation between the two. Similarly, the occipital grooves are not bound to be an exact reflex of the longitudinal and lateral sinuses, though perhaps manifesting correlation. Again, the occipital bone is of very varying thickness, and an interior depression is not necessarily accompanied by an exterior projection within limits of say 1 to 2 mms. though again there might be a certain degree of correlation between them over a series. Lacking perfect association between these different stages, it would be impossible to form unassailable conclusions as to the conditions obtaining in the brain and sinuses from an examination of the exterior of the skull. Again, assuming correlation, but not almost perfect correlation, to exist between these different variates, it is by no means needful, as the theory of correlation shows, when C is correlated with B and B again with A and when D is correlated with E, for a correlation between C and D to involve a correlation between A and E; but it is worth examining whether A and E (exterior occipital asymmetry and interior direction of grooves) which we can observe are really correlated. The presence or absence of such correlation unless it were almost perfect would not, however, enable us to predict a relationship of C and D (area striata to direction of superior longitudinal sinus).

It is with data concerning exterior occipital asymmetry, and the direction of the main inner groove that I was furnished, and on these based a contingency

table of the same kind (9-celled) as the previous ones. I also connected by a table the data relating to occipital asymmetry and size of jugular foramen, though one naturally expected the association to be rather less in this case.

Greater Occipital Prominence

Greater Groove	Right	Equal	Left	Totals
Right	24	18	52	94
Equal	6	1	6	13
Left	8	9	11	28
Totals	38	28	69	135

Greater Occipital Prominence

Greater Jugular Foramen	Right	Equal	Left	Totals
Right	23	17	42	82
Equal	5	3	5	13
Left	6	8	18	32
Totals	34	28	65	127

The results were as follows:

Association of greater occipital prominence on one side, with main lateral groove on the other side: coefficient of mean squared contingency $C_2 = \cdot 2035$. Association of greater occipital prominence on one side with greater jugular foramen on the other side: coefficient of mean squared contingency $C_2 = \cdot 1319$. The number of cases is 127, and the mean value of C_2 where there is no contingency in the sampled population is again of the order $\cdot 17 \pm \cdot 06$.

One has to conclude therefore that in this series there is no significant association between outer occipital asymmetry and the direction in which the main occipital sinus is turned when its course is changed from longitudinal to lateral.

The remaining point dealt with in this investigation concerned the asymmetry, not of the cranium, but of the face; and for this I was able to use actual measurements instead of merely appreciative methods to provide the data. It has been the custom among the workers of this school to take measurements of both orbits, not of one alone as did the older craniometricians (usually without specifying which), but in the case of the nose the nasal height has previously been taken to the lowest part of the edge of the pyriform aperture on one side only, the left, unless this was missing, in which case the measurement was taken on the right side. I have, however, taken the measurements on both sides.

It is obvious on comparing the means of the various series for height of orbit and height of nose, that symmetry is the normal condition: there is no tendency

for either side to exceed the other. Thus, taking all the adults of the collection together, we get 51·45 mms. as the mean nasal height on the right side against 51·36 mms. on the left; 34·51 mms. is mean height of right orbit, 34·59 mms. mean of left: in both cases a difference of only about ·085 mm. between right and left.

In the bones of the individual face, however, as also in the lines of the living face, exact symmetry is by no means a rule. Is the lack of symmetry in one feature usually associated with lack of symmetry in another? Is inequality in height of orbits generally accompanied by inequality in nose-height measured right and left? and, if so, is the higher orbital height on one side associated with greater height of nose on that same side or the other side?

This question it was possible to answer with considerable exactness as far as my own series was concerned, by means of a Correlation Table in which one variate was the excess or defect of left nasal height over right, divided by *right* nasal height; and the other was the corresponding measure of orbital inequality. The range of difference in absolute height was from + 2·7 mms. to − 2·1 mms. for the nose and from + 2·2 mms. to − 2·2 mms. for the orbit. This gave for the nasal index $100\left(\frac{NH(R)-NH(L)}{NH(R)}\right)$ a range of + 5·3 mms. to − 4·4 mms.; and for the orbital index $100\left(\frac{O_2(R)-O_2(L)}{O_2(R)}\right)$ a range of + 5·9 mms. to − 6·6 mms., these varieties being divided into 11 and 13 categories respectively.

Correlation Table between Nasal and Orbital Asymmetry

$\frac{NH(R)-NH(L)}{NH(R)} \times 100$

$\frac{O_2(R)-O_2(L)}{O_2(R)} \times 100$	−4·95	−3·95	−2·95	−1·95	·95 −	·05 +	+1·05	+2·05	+3·05	+4·05	+5·05	Totals
−6·95	—	—	1	1	2	—	—	—	—	—	—	4
−5·95	—	—	2	1	1	1	—	—	—	—	—	5
−4·95	—	—	—	—	3	2	—	—	—	—	1	6
−3·95	—	—	1	2	—	3	—	—	1	—	—	7
−2·95	1	—	1	2	7	4	3	—	—	—	—	18
−1·95	—	2	1	1	5	—	1	—	—	—	—	10
− ·95	—	—	3	3	8	5	4	—	—	—	—	23
+ ·05	—	—	—	1	—	4	2	1	—	—	—	8
+1·05	—	—	—	—	4	3	3	1	1	—	—	12
+2·05	—	—	—	2	8	1	1	—	—	—	—	12
+3·05	—	—	2	—	4	1	1	2	—	—	—	10
+4·05	—	—	—	—	—	—	—	—	—	—	—	—
+5·05	—	—	—	—	—	—	1	—	1	—	—	2
Totals	1	2	11	13	42	24	16	4	3	—	1	117

The standard deviation of the difference in nasal height R and L was found to be 1·53, that of the difference in orbital height 2·76; and we obtain a coefficient of correlation between greater orbital height on one side and greater nasal height on the same side of the value $r = + \cdot2160 \pm \cdot595$.

This is more than three times as great as the probable error, and may therefore be deemed significant, though the correlation is not high. Hence the conclusion that where the orbits are of unequal height, there is a significant but slight tendency for the pyriform aperture to descend lower on that side where the higher orbit exists.

My thanks are due to Miss Noel Karn and Miss Margaret Moul of the Biometric Laboratory for assistance in working out the above Contingency and Correlation Tables. I am also deeply indebted to Professor Arthur Keith for the permission to photograph Chinese, Karen, Maravar and Malay crania in the collection of the Royal College of Surgeons.

DESCRIPTION OF PLATES.

The skulls selected to illustrate the difference in type in our Burmese series are all male, since the characteristic features are more marked in the male skull. The supposed hybrids (Type B) are unrepresented: I have contented myself with illustrations of the two extreme groups.

Plates I–III show two skulls of Type A (Burman) from various aspects:

Plate I, Fig. 1, Burman ♂, No. 110, Norma lateralis.
„ Fig. 2 „ „ „ verticalis.
„ Fig. 3 „ „ „ facialis.
Plate II, Fig. 4, Burman ♂, No. 4, Norma occipitalis.
„ Fig. 5 „ „ „ verticalis.
„ Fig. 6 „ „ „ facialis.
„ Fig. 7 „ „ „ basalis.
Plate III, Fig. 8 „ „ „ lateralis.

Plate III also shows a skull of Type C (? Karen) from two points of view:

Plate III, Fig. 9, Burmese Type C, ♂, No. 80, Norma lateralis.
„ Fig. 10 „ „ „ „ facialis.

In Plates IV, V and VI we have typical Karen*, Maravar, Hindu, Chinese and Malay skulls, for comparison:

Plate IV, Fig. 11, Karen ♂, Norma facialis.
„ Fig. 12 „ „ lateralis.
„ Fig. 13, Maravar ♂, Norma facialis.
„ Fig. 14 „ „ lateralis.
Plate V, Fig. 15, Hindu ♂, Norma lateralis.
„ Fig. 16 „ „ verticalis.
„ Fig. 17 „ „ facialis.
Plate VI, Fig. 18, Chinese ♂, Norma facialis.
„ Fig. 19 „ „ lateralis.
„ Fig. 20, Malay ♂, Norma facialis.
„ Fig. 21 „ „ lateralis.

* The series from which the Karen skull was chosen was, perhaps, too slight (five in number) to enable me to assert that this one is "typical." It is however Karen.

The remaining plates illustrate anomalies of conformation found in the Burmese cranial series:

Plate VII,	Fig. 22, skull No. 23: right temporal bone divided into three separate parts.
„	Fig. 23, skull No. 136: peculiar ossification in temporal suture.
„	Fig. 24, skull No. 95: two pre-condyles fused.
„	Fig. 25, skull, No. 13: two pre-condyles.
Plate VIII,	Fig. 26, skull No. 113: very deep canine fossae.
„	Fig. 27, skull No. 82: os japonicum on L.
„	Fig. 28, skull No. 8: L. upper canine thrust in lateral direction.
„	Fig. 29, skull No. 92: fossa at anterior border of foramen magnum.
Plate IX,	Fig. 30, skull No. 121: R. os triangulare of interparietal, and ossicle at lambda.
„	Fig. 31, skull No. 74: long epipteric bone on R.
„	Fig. 32, skull No. 103: complete tri-partite interparietal.

APPENDIX I

TABLE OF THE OCCIPITAL INDEX

For the convenience of future craniometrists I have tabled the occipital index as obtained from the formula

$$Oc.\,I. = \frac{S_3}{S_3'}\sqrt{\frac{S_3}{24\,(S_3 - S_3')}}$$

for all values of S_3/S_3' from 1·050 to 1·500; S_3/S_3' being the ratio of the occipital arc from lambda to opisthion, to the chord connecting these two points.

This index measures the convexity of the occipital bone from lambda to opisthion, giving the ratio of the radius of curvature (supposing the curvature to be that of an arc of a circle, which is only roughly the case), to the chord S_3'. The index obviously decreases as greater convexity shortens the radius.

The value given by our formula is very nearly equal to the index required. Its error for a normal proportion like $S_3/S_3' = 1{\cdot}250$, say, is only 3 per cent. At $S_3/S_3' = 1{\cdot}500$ it reaches 5 per cent., and after this point would give a value which increases instead of decreasing, with increased convexity. I have therefore made $S_3/S_3' = 1{\cdot}500$ the limit of my table, and it would indeed be a very abnormal skull for which this limit did not suffice.

Table for Calculating Occipital Index (Oc. I.) from Occipital Arc (S_3) and Chord (S_3').

S_3/S_3	Oc. I.	S_3/S_3'	Oc. I.	S_3/S_3'	Oc. I.	S_3/S_3'	Oc. I.	S_3/S_3'	Oc. I.
1·050	98·22	1·100	74·47	1·150	65·00	1·200	60·00	1·250	57·05 +
1·051	97·39	1·101	74·20	1·151	64·87	1·201	59·92	1·251	57·01
1·052	96·59	1·102	73·94	1·152	64·74	1·202	59·85 +	1·252	56·96
1·053	95·81	1·103	73·68	1·153	64·61	1·203	59·78	1·253	56·92
1·054	95·05 +	1·104	73·42	1·154	64·48	1·204	59·71	1·254	56·88
1·055	94·32	1·105	73·17	1·155	64·36	1·205	59·63	1·255	56·83
1·056	93·60	1·106	72·92	1·156	64·23	1·206	59·56	1·256	56·79
1·057	92·91	1·107	72·68	1·157	64·11	1·207	59·49	1·257	56·75
1·058	92·24	1·108	72·44	1·158	63·99	1·208	59·42	1·258	56·70
1·059	91·58	1·109	72·21	1·159	63·87	1·209	59·36	1·259	56·66
1·060	90·95 −	1·110	71·98	1·160	63·76	1·210	59·29	1·260	56·62
1·061	90·32	1·111	71·74	1·161	63·64	1·211	59·22 +	1·261	56·58
1·062	89·72	1·112	71·52	1·162	63·52	1·212	59·15 +	1·262	56·54
1·063	89·13	1·113	71·30	1·163	63·41	1·213	59·09	1·263	56·50
1·064	88·56	1·114	71·08	1·164	63·30	1·214	59·02	1·264	56·46
1·065	88·00	1·115	70·87	1·165	63·19	1·215	58·96	1·265	56·42
1·066	87·45 −	1·116	70·66	1·166	63·08	1·216	58·89	1·266	56·38
1·067	86·92	1·117	70·45 −	1·167	62·97	1·217	58·83	1·267	56·34
1·068	86·40	1·118	70·25 −	1·168	62·86	1·218	58·77	1·268	56·30
1·069	85·89	1·119	70·04	1·169	62·76	1·219	58·71	1·269	56·26
1·070	85·39	1·120	69·84	1·170	62·65 +	1·220	58·64	1·270	56·22
1·071	84·91	1·121	69·65 −	1·171	62·55 +	1·221	58·58	1·271	56·19
1·072	84·43	1·122	69·46	1·172	62·45 −	1·222	58·52	1·272	56·15 −
1·073	83·97	1·123	69·26	1·173	62·35 −	1·223	58·46	1·273	56·11
1·074	83·52	1·124	69·08	1·174	62·25 −	1·224	58·40	1·274	56·08
1·075	83·08	1·125	68·89	1·175	62·15 −	1·225	58·35 +	1·275	56·04
1·076	82·64	1·126	68·71	1·176	62·05 +	1·226	58·29	1·276	56·00
1·077	82·22	1·127	68·53	1·177	61·95 +	1·227	58·23	1·277	55·97
1·078	81·80	1·128	68·35 +	1·178	61·86	1·228	58·17	1·278	55·93
1·079	81·40	1·129	68·18	1·179	61·77	1·229	58·12	1·279	55·90
1·080	81·00	1·130	68·01	1·180	61·67	1·230	58·06	1·280	55·86
1·081	80·61	1·131	67·84	1·181	61·58	1·231	58·01 −	1·281	55·83
1·082	80·23	1·132	67·67	1·182	61·49	1·232	57·95 −	1·282	55·80
1·083	79·85 +	1·133	67·50	1·183	61·40	1·233	57·90	1·283	55·76
1·084	79·49	1·134	67·34	1·184	61·31	1·234	57·84	1·284	55·73
1·085	79·13	1·135	67·18	1·185	61·22	1·235	57·79	1·285	55·70
1·086	78·78	1·136	67·02	1·186	61·13	1·236	57·74	1·286	55·67
1·087	78·43	1·137	66·86	1·187	61·05 −	1·237	57·69	1·287	55·64
1·088	78·09	1·138	66·71	1·188	60·96	1·238	57·64	1·288	55·60
1·089	77·76	1·139	66·55 +	1·189	60·87	1·239	57·58	1·289	55·57
1·090	77·43	1·140	66·40	1·190	60·79	1·240	57·53	1·290	55·54
1·091	77·11	1·141	66·25 +	1·191	60·71	1·241	57·48	1·291	55·51
1·092	76·80	1·142	66·11	1·192	60·63	1·242	57·43	1·292	55·48
1·093	76·49	1·143	65·96	1·193	60·55 −	1·243	57·38	1·293	55·44
1·094	76·18	1·144	65·82	1·194	60·46	1·244	57·34	1·294	55·41
1·095	75·88	1·145	65·68	1·195	60·38	1·245	57·29	1·295	55·38
1·096	75·59	1·146	65·54	1·196	60·31	1·246	57·24	1·296	55·35 +
1·097	75·30	1·147	65·40	1·197	60·23	1·247	57·19	1·297	55·32
1·098	75·02	1·148	65·26	1·198	60·15 +	1·248	57·15 −	1·298	55·30
1·099	74·74	1·149	65·13	1·199	60·07	1·249	57·10	1·299	55·27
1·100	74·47	1·150	65·00	1·200	60·00	1·250	57·05 +	1·300	55·24

MALES

					Lengths									
No.	Type	Sex	*C*	W. in grs.	*F*	*L'*	*L*	*B*	*B'*	*H*	*H'*	*OH*	*LB*	*Q*
3	A	♂	—	655	174	174	174	138	97	139	139·5	125·5	102	326
4	A	♂	1496	828	175	177	176	153	92·5	140·5	139	125·5	92	340
14	A	♂	1264	685	185	189·5	188·5	131	91·5	135	135·5	115·5	107	304·5
16	A	♂	1613·5	647	185·5	186·5	185·5	142	91	142	142	120·5	97·5	321
20	A	♂	1382	—	169	171	170	143	94·5	130	129	118	91·5	322
22	A	♂	—	716	173	175·5	174·5	148	92	139·5	137·5	125	97·5	331·5
43	A?	♂	—	—	—	—	—	142	—	—	—	—	—	—
44	A	♂	—	679	182	182	181·5	154	105	146	143·5	127	106·5	345
52	A	♂?	—	734	169·5	173	171·5	144	87·5	135	133	117·5	97	322·5
53	A	♂	1415·5	840	173·5	175·5	174·5	146	90·5	147·5	145·5	122	106	327
54	A	♂?	—	600	170	171	170·5	141·5	103	139	136	119	101·5	321·5
55	A	♂	—	568	166	168·5	167	140·5	85	140·5	137·5	122	92	327·5
56	A	♂	—	823	170·5	182·5	181·5	153	108·5	157·5	155·5	133	108·5	357
60	A	♂	—	478	168	167·5	168·5	142	98	137	138	115·5	100	315
61	A	♂	1359	669	172	172·5	171·5	137	97·5	138	142	120	108	322
65	A	♂	—	733	171·5	172	173	149	102	148	145·5	126	103·5	336·5
68	A	♂	1458·5	645	169·5	172	171	144·5	102	135·5	134·5	121	96	327
69	A (B?)	♂	1429·5	713	169	170·5	171	147	95·5	137	137·5	124	103	333·5
71	A	♂	1412·5	867	171	173	172·5	145·5	88	146	144·5	125	101	331·5
72	A	♂?	1421	566	163	164·5	163·5	148	90	135	135	122	98	330
74	A	♂	1380	555	166·5	167	166	142·5	94·5	135	135	120	92	319·5
76	A	♂	1329·5	584	171	169	170	144	97	140	140	123·5	97·5	327
78	A	♂??	—	—	170·5	171·5	170·5	130?	94·5	130	130	115·5	100	302·5?
86	A	♂	1473·5	837	178·5	178·5	178	151·5	97·5	131	130·5	122	93	332
90	A	♂?	1338	551	170	170	170	141·5	95	132·5	130·5	115	95·5	313
91	A	♂?	1389	446	174	180	179	144	95	130·5	130	116·5	98·5	316
94	A (B?)	♂?	1379·5	604	166	168·5	168·5	142	90	138	136	120	98·5	321·5
96	A	♂?	1365·5	610	170	172·5	170·5	140·5	97·5	131	132	117·5	99·5	317·5
97	A	♂	1353·5	545	164	167·5	165·5	145	98·5	128·5	126	117·5	92	319
98	A	♂?	—	—	176·5	176	176	143·5	99	130	130·5	116·5	98	321·5
105	A	♂	1458·5	600	179	180	178·5	146	94	139	138·5	121·5	104·5	325
106	A	♂?	1291	649	169·5	175	174·5	142·5	93·5	125·5	126	111·5	94·5	306·5
109	A	♂	—	591	168	167·5	168	144	86·5	131·5	132·5	110·5	100·5	313·5?
110	A	♂	1492·5	785	166·5	170	168·5	149·5	100	146·5	146	123	99·5	333
112	A	♂	1488·5	653	179·5	182	182	142	95·5	137·5	136	122	103·5	325
113	A	♂	1406	707	162	167·5	165·5	151	96·5	137·5	137	118	96·5	323·5
117	A	♂	—	—	175·5	180·5	178·5	148	91	—	—	120	—	326·5?
119	A	♂	—	562	160·5	163·5	162	142	87	130	128·5	115	90	313·5
121	A	♂	1481·5	640	183·5	183·5	183	143	90·5	133	130	118·5	96·5	322·5
123	A?	♂	—	—	178·5	180·5	179	138	90	126	126	115	93	312
124	A	♂?	1211	605	166·5	166·5	167	136	92·5	135	133	112·5	95·5	304
130	A	♂	—	—	173	177	174·5	154	94	136	135	121·5	101·5	335
136	A	♂	1593	701	185	184·5	183·5	137·5	91	142	141	129	96	337·5
139	A	♂?	—	685	183	184	184	139	88·5	135·5	134·5	120·5	99	316
142	A	♂	1305·5	602	164	166·5	166·5	141·5	90	134·5	132·5	120·5	94	317·5

	Arcs								Face				Nose							
LB	*Q*	*Q'*	*S*	S_1	S_2	S_3	S_4'	*U*	*PH*	*G'H*	*GB*	*J*	*NH R*	*NH L*	*NB*	*DS*	*DC*	*DA*	*SS*	*SC*
102	326	327·5	375	137	124	114	97	503	18·0?	73	99	127	57·2	56·5	24·4	10·5	19·0	31	4·7	11·0
92	340	340	383	137	128	118	97	525	20·0	75·5	107·5	141	57·7	58·0	29·5	9·0	20·8	29	2·0	6·3
107	304·5	307	370	130	128	112	92	506·5	19·5	66·5	105·5	133	48·3	48·3	31·4	14·5	27·5	43	6·1	14·0
97·5	321	323	387	137	130	120	102·5	526	20·2	70	105	136	52·5	53·9	28·0	9·0	20·8	28·5	3·2	7·5
91·5	322	319	351?	120	124	107?	88?	503	—	—	—	—	—	—	—	—	26·0	—	—	—
97·5	331·5	332	373	125	130	118	99	508	21·0?	69·5	104	133	52·0	52·0	28·2	11·6	24·5	35·5	3·9	11·3
—	—	—	—	—	133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
106·5	345	349	380	131·5	129·5	119	98	539	23·3	87	107·5	149	63·5	63·1	28·4	10·0	23·0	32·5	2·6	10·2
97	322·5	323	357	125	124	108	95	503·5	21·0	75·5	102	138	56·3	57·2	28·9	10·6	23·2	34	2·4	10·1
106	327	326	364	128	113	123	107	509	20·1	73·5	104·5	135·5	54·5	55·0	27·0	10·8	22·8	33	3·6	9·6
101·5	321·5	320	353	122·5	125·5	105	93·5	505	20·3	74	103	136·5	55·0	54·7	27·3	8·4	25·5	31·5	—	9·5
92	327·5	329	356·5	128	125·5	103	90	493	15·3	64	101	127	51·9	51·4	26·6	10·2	23·9	31	2·5	10·0
108·5	357	358	387	140	139	108	95·5	535·5	24·6	77	114·5	146	57·0	56·0	27·2	10·2	25·6	35	3·2	10·1
100	315	315·5	354	124	112	118	103	497	16·3	68	103	137	55·4	56·2	29·9	9·9	23·0	33	3·7	13·6
108	322	326	360	124	128	108	89	496	22·0?	75	99·5	134	55·6	56·0	27·0	11·9	24·0	38	3·6	9·6
103·5	336·5	339	362	125	132	105	88·5	511	18·6	75	115	146·5	57·5	58·6	29·0	10·6	25·0	35	1·5	7·0
96	327	329	366	134	120·5	111·5	93	510	22·1?	72	107	139	51·0	52·2	27·4	14·5	24·5	41	2·5	9·0
103	333·5	336	358	128	125	105	95	506	21·5?	73·5	106	131	53·5	53·0	26·0	12·1	22·0	35	3·6	8·5
101	331·5	334	362	132·5	127	102·5	89·5	501·5	20·0	73	104·5	136	55·2	54·5	28·6	9·2	20·2	31	2·6	5·9
98	330	331	354	128	116	110	94	494	—	—	95	126·5?	51·5	51·0	27·3	8·2	19·6	28	1·5	7·0
92	319·5	320	358	129·5	120·5	108	93·5	489·5	21·4?	69	95·5	132·5	49·5	49·2	29·8	8·0	22·0	30	1·6	9·1
97·5	327	331	364	132	123	109	95	497	19·3	69	96·5	124	52·3	53·1	28·5	9·4	24·3	32·5	2·9	9·5
100	302·5?	—	349·5	119	122·5	108	91	492?	19·7	68	97	130·5?	48·0	49·1	25·0	10·8	23·8	34	4·0	12·0
93	332	333	383	132·5	125	125·5	99	526	23·0?	71?	105	129·5	51·3	50·4	28·3	9·1	22·5	31·5	2·5	10·6
95·5	313	315	363	120	131	112	92·5	499	20·0	76	101	135·5	57·0	57·2	29·0	6·7	23·0	29·5	1·4	9·9
98·5	316	318	368	135·5	116·5	116	97	518	18·0?	68·5?	98	132	51·7	51·7	29·2	10·2	24·0	33·5	5·2	11·0
98·5	321·5	322·5	350	128	126·5	95·5	86·5	488	25·4?	76·5	99	128	56·8	55·8	25·8	8·8	20·5	31·5	1·8	6·0
99·5	317·5	319	353	132	118	103	88	500	16·6	65	95·5	131·5	51·0	50·9	26·7	10·4	22·0	31·5	2·8	8·4
92	319	321	352·5	119	121	112·5	96	499	21·0?	64	97	125	46·0	46·0	28·5	10·1	23·0	34	2·1	10·0
98	321·5	322	367	136	119	112	92	508·5	28·2	80	—	—	57·3	57·3	29·0	12·0	21·5	34·5	2·2	7·6
104·5	325	328	373	134·5	123	115·5	99	514	14·5	67	102	142	53·7	54·0	26·4	9·8	21·0	30	1·9	8·1
94·5	306·5	311	350	125·5	113·5	111	97	499	13·2?	68?	97·5	129·5	54·7	54·7	26·9	11·0	22·0	29	3·6	8·9
100·5	313·5?	314	338	127	95	116	97·5	490	19·2	68	100	134?	52·0	51·3	28·0	12·7	21·0	35	2·9	7·6
99·5	333	333·5	365	138	124	103	91	510	17·6?	72?	108	143	56·3	56·7	31·3	8·6	25·0	31	3·3	7·4
103·5	325	330	369	121	115	133	113	513	—	64·5?	103·5	132	52·7	52·9	30·6	8·9	24·0	30	2·8	7·2
96·5	323·5	326·5	349	131	112	106	94·5	503	20·0	78	98	141	58·3	56·3	26·3	10·9	20·6	31·5	4·4	10·2
—	326·5?	320?	376	136·5	126	113·5	92	498	—	—	—	—	—	—	—	11·4	19·5?	—	4·7	9·6
90	313·5	313	339·5	120·5	115	104	93	481	16·0?	64	94·5	127	48·0	48·7	28·1	8·0	19·7	26	2·5	5·1
96·5	322·5	325	384·5	125	140·5	119	95·5	521	16·8	65	101	128	48·8	49·2	31·5	11·6	27·0	37	3·5	10·1
93	312	312	368·5	134	129·5	105	86	503	—	—	—	—	—	—	—	—	—	—	—	—
95·5	304	307	341·5	118	121	102·5	89	489·5	26·0	76	99·5	129·5	51·2	51·0	29·0	9·1	22·5	31	1·6	6·9
101·5	335	341	360·5	124·5	119	117	100·5	516	17·3?	71·5	96·5?	132·5	56·2	55·8	27·7?	12·4?	23·0?	36·5?	3·2	9·3
96	337·5	343	392·5	132	147	113·5	96	517	—	—	99	127	49·0	48·7	25·0	10·7	23·5	33	3·2	9·8
99	316	316	381	129·5	136·5	115	95	512	19·8?	74	103	134·5	53·9	55·0	29·1	10·2	20·5	29	2·7	6·9
94	317·5	324	352·5	120·5	128	104	90	495	22·0?	68·5	104	139	49·5	49·5	29·1	9·3	23·0	32	1·2	7·1

HE ABSOLUTE AND INDICIAL MEASUREMENTS OF THE INDIVIDUAL CRANIA

		Orbits				Palate and Profile				Foramen										
	SC	O_1R	O_1L	O_2R	O_2L	G_1	G_1'	G_2	*GL*	*fml*	*fmb*	*Oc. I*	*B/L'*	*H/L'*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH R*	
	11·0	44·5	44·9	36·8	35·7	—	—	—	96·5	34·0	30·5	62·4	79·2	80·1	79·3	80·2	98·9	74·0	42·7	
	6·3	45·1	44·9	36·2	37·2	47·6	43·7	42·3	89·5	37·0	33·4	58·9	86·4	78·5	86·8	78·9	110·0	70·1	51·1	
	14·0	43·3	43·0	33·0	33·0	53·1	48·0	41·0	103·5	37·0	31·7	58·8	69·2	71·4	69·6	71·9	96·9	62·7	65·0	
	7·5	43·5	43·0	34·2	34·6	49·0	44·0	40·0	98	41·0	34·0	62·6	76·0	76·0	76·5	76·5	99·9	66·7	53·3	
	—	—	—	—	—	—	—	—	—	38·5?	—	58·9	83·6	75·5	84·2	76·0	110·8	—	—	
	11·3	45·0	44·0	33·9	36·1	49·8	45·3	37·6	95·5	35 4	30·0	60·8	84·4	78·4	84·9	78·9	107·6	66·8	54·2	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	10·2	48·8	48·9	36·1	37·1	60·2	53·3	47·0	103	34·1	28·7	59·0	84·8	78·8	84·9	79·0	107·5	81·9	44·7	
	10·1	41·4	41·1	37·5	38·0	55·2	51·1	39·0	99	38·2	32·6	66·9	83·3	77·0	83·9	77·6	108·2	73·9	51·3	
	9·6	46·2	45·2	35·2	35·2	49·3	45·0	39·0?	96·5	36·8	30·3	65·3	83·1	83·0	83·7	83·5	100·2	70·2	49·5	
	9·5	45·0	43·5	34·0	32·7	51·2	47·7	40·1	97·5	39·0	31·8	69·6	72·7	79·5	83·1	79·9	104·0	71·5	49·6	
	10·0	39·6	38·8	33·9	34·2	47·2	44·2	43·2	89	37·2	30·0	65·8	83·5	81·8	84·1	82·4	102·6	63·6	51·3	
	10·1	47·2	47·3	36·2	36·0	46·7	43·0	46·0?	98·5	36·1	33·0	67·8	83·9	85·4	84·3	85·9	98·2	67·5	47·7	
	13·6	41·8	42·0	33·7	33·0	42·7	41·0	41·5	89·5	35·5	31·8	53·2	84·7	82·2	84·4	81·9	103·0	66·3	54·0	
	9·6	45·6	44·0	31·8	33·1	50·9	46·1	39·0	107	31·0	25·6	58·9	79·5	82·5	79·8	82·9	96·3	75·6	48·6	
	7·0	46·4	48·0	35·5	35·8	54·2	46·5	47·3	101·5	38·1	30·6	63·1	86·7	84·5	86·1	84·0	102·5	65·0	50·4	
	9·0	47·4	47·1	38·2	38·1	50·7	47·0	40·2	98	41·0	38·3	60·1	84·0	78·1	84·5	78·5	107·6	67·4	53·7	
	8·5	46·6	44·0	34·2	34·2	54·5	49·2	34·5	103·5	37·2	32·0	72·9	86·2	80·5	86·0	80·3	107·1	69·4	48·6	
	5·9	42·5	42·0	37·3	37·2	48·0	44·5	38·5	96·5	41·8	30·5	65·7	84·0	83·4	84·2	83·6	100·8	70·0	51·8	
	7·0	42·3	43·6	34·3	33·5	—	—	38·0?	89·5?	32·2	28·5	62·7	90·0	82·2	90·5	82·6	109·5	—	53·0	
	9·1	43·6	42·0	33·3	32·8	50·7?	45·8	38·4	95	36·2	29·0	64·6	85·2	80·8	85·6	81·2	105·4	72·3	60·2	
	9·5	43·0	42·8	36·6	38·0	48·7	46·1	40·7	95·5	32·7	27·8	65·4	85·3	82·8	84·9	82·4	103·0	71·4	54·5	
	12·0	45·0	44·6	34·8	34·5	45·5	42·0	39·1	98	35·1	29·5	60·9	75·8	75·8	75·9?	76·3	100·0?	69·9	52·1	
	10·6	45·0	44·3	38·0	38·8	53·4	48·8	37·0?	100·5?	34·0	28·3	56·3	84·7	73·1	85·0	73·4	115·8	67·6?	55·2	
	9·9	46·0	45·2	36·2	36·8	50·0	46·0	38·2	94	31·4	31·4	59·2	83·2	76·8	83·3	76·9	108·4	75·1	50·9	
	11·0	44·8	45·6	34·8	35·2	50·1?	45·5?	38·5?	92·5?	33·3	25·1	60·3	79·9	72·3	80·4	72·8	110·4	69·8?	56·5	
	6·0	43·8	43·6	37·2	36·0	51·0	46·4	41·3	96·5	37·0	30·8	73·7	84·1	80·7	84·2	80·8	104·2	77·2	45·4	
	8·4	42·7	43·2	33·0	33·0	51·0	47·3	37·7	97·5	34·6	31·0	62·9	81·5	76·6	82·4	77·4	106·4	68·1	52·4	
	10·0	45·0	44·1	33·6	33·3	49·5	45·1	33·4	91	33·6	29·0	62·4	86·5	75·2	87·6	76·1	115·0	65·8	62·0	
	7·6	43·2	—	37·5	—	48·0	45·2	37·0?	—	34·2	30·4	59·0	81·5	74·0	81·6	74·1	110·1	—	50·6	
	8·1	45·6	43·0	32·0	34·1	46·8	42·5	41·4?	99·5	38·0	32·0	63·0	81·0	77·0	81·7	77·7	105·2	65·8	49·2	
	8·9	45·0	43·7	34·0	33·0	46·7	37·8?	41·4	89?	40·8	29·8	65·4	81·3	72·1	81·6	72·4	112·8	69·8?	49·2	
	7·6	43·0	43·0	32·3	31·9	49·0	43·7	36·8	88	37·7	32·2	60·8	87·0	79·0	85·7	78·8	108·8	68·3	53·8	
	7·4	44·4	43·8	37·8	37·2	51·5	47·0	41·9?	96·5?	41·5	34·0	67·7	88·0	86·0	88·6	86·5	102·4	66·6	55·6	
	7·2	45·0	43·5	34·7	33·5	47·5?	42·5?	41·5?	95?	41·5	31·7	62·0	77·9	74·6	78·1	74·8	104·4	62·2	58·1	
	10·2	44·7	43·7	38·1	37·4	52·4	47·6	34·2	96	39·6?	34·0?	69·5	90·2	81·9	91·3	82·9	110·2	79·7	45·1	
	9·6	—	40·2	—	35·0	—	—	—	—	—	—	57·8	82·1	—	82·9	—	—	—	—	
	5·1	43·8	42·1	36·5	37·0	41·3?	39·7?	35·5	88	37·4	34·0	70·2	86·8	78·6	87·0	79·2	110·5	67·6	58·5	
	10·1	42·7	41·2	32·5	32·8	55·5	49·1	38·9	101·5	33·0	24·9	57·1	78·0	70·8	78·3	71·1	110·1	64·1	64·5	
	—	—	—	—	—	—	—	—	—	36·6	29·5	58·8	76·3	69·9	77·1	70·5	109·4	—	—	
	6·9	42·6	42·4	35·6	35·5	49·3	45·9	39·1	95·5	38·0	27·8	65·0	81·9	80·0	81·6	79·7	102·3	76·5	56·6	
	9·3	43·1	42·5?	35·1	34·4?	—	—	40·5	96·5?	37·0	30·2	63·3	87·2	76·4	88·2	77·3	114·1	73·7?	49·3?	
	9·8	42·0	43·3	32·5	33·2	—	—	40·3	—	40·8	34·6	61·2	74·5	76·5	74·9	76·9	97·4	—	51·0	
	6·5	43·9	43·5	32·5	32·8	51·6	45·6	38·9?	108·5	36·1	29·0	59·2	75·4	73·0	75·5	73·1	103·3	71·8	54·0	
	7·1	44·0	42·5	33·2	34·0	46·3?	42·0	39·2	98·5	37·0	29·1	64·6	85·0	79·5	85·1	79·6	106·9	65·7	58·8	

CRANIA (ABSOLUTE LENGTHS IN MMS.)

	Indices								Angles						
H/GB	*NB/NH R*	*NB/NH L*	O_2/O_1 R	O_2/O_1 L	G_2/G_1	*fmb/fml*	*DS/DC*	*SS/SC*	*N∠*	*A∠*	*B∠*	θ_2	θ_1	*P∠*	
74·0	42·7	43·2	82·7	78·2	—	89·7	55·3	42·7	64°·2	72°·6	43°·2	17°·4?	25°·8?	90°·0?	P.B. sloping forward a
70·1	51·1	50·9	80·3	82·9	88·9	90·3	43·3	31·8	63°·6	67°·4	49°·0	21°·6	27°·4	89°·0	Sutures becoming obli mastoid ridges; wo
62·7	65·0	65·0	76·2	76·7	77·2	85·7	52·7	43·6	68°·7	74°·8	36°·5	7°·7	28°·8	82°·5	P.B. rounded; sub-na
66·7	53·3	51·9	78·6	80·5	81·6	82·9	43·3	42·7	69°·3	68°·6	42°·1	8°·5	33°·6	77°·1	P.B. rounded; sub-nas
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Marked depression of oc
66·8	54·2	54·2	75·3	82·0	75·5	84·7	47·3	34·5	67°·1	70°·8	42°·1	14°·2	27°·9	85°·0	Sutures open; teeth w along lambdoid; fro
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Skull cap only; healed
81·9	44·7	45·0	74·0	75·9	78·1	84·2	43·5	25·5	63°·2	67°·6	49°·2	17°·4	31°·8	85°·0	Teeth worn; large alve epipteric on R.; fa
73·9	51·3	50·5	90·6	92·5	70·7	85·3	45·7	23·8	68°·8	66°·0	45°·2	17°·5	27°·7	83°·5	Molars much worn; le L. occipital, R. fron angle L. and R.; ma
70·2	49·5	49·1	76·2	77·9	79·1?	82·3	47·4	37·5	61°·8	76°·1	42°·1	9°·9	32°·2	86°·0	Ageing; sutures becom rounded edge; sub-
71·5	49·6	49·9	75·6	75·2	78·3	81·5	33·0	—	65°·2	71°·1	43°·7	12°·5	30°·2	84°·6	P.B. flat with rounded
63·6	51·3	51·8	85·6	88·1	91·5	80·7	42·7	25·0	66°·9	71°·7	41°·4	13°·3	28°·1	85°·0	Young adult; P.B. flat
67·5	47·7	48·6	76·7	76·1	98·5?	91·5	39·8	31·7	61°·2	75°·4	43°·4	13°·6	29°·8	89°·0	Old; sutures partly obli parietal, L. occipita
66·3	54·0	53·2	80·6	78·6	97·2	89·6	43·0	27·2	60°·7	77°·4	41°·9	14°·6	27°·3	92°·0	P.B. with outward-up small epipteric on R.
75·6	48·6	48·2	69·7	75·2	76·6	82·6	49·6	37·5	69°·0	70°·0	41°·0	15°·4	25°·6	85°·4	P.B. with outward-up
95·0	50·4	49·5	76·5	74·6	87·1	80·3	42·4	21·5	67°·1	69°·9	43°·0	16°·1	26°·9	86°·0	P.B. flat with very rou
67·4	53·7	52·4	80·6	80·9	79·3	93·4	59·2	27·8	69°·7	66°·8	43°·5	15°·9	27°·6	82°·7	Old; sutures becoming japonicum on both
69·4	48·6	49·1	73·4	77·7	63·3	86·0	55·0	42·4	69°·4	68°·9	41°·7	18°·1?	23°·6?	87°·0?	Aged; 3rd molars never
70·0	51·8	52·5	87·8	88·6	80·2	73·0	45·5	44·1	65°·0	71°·6	43°·4	13°·1	30°·3	84°·7	Sutures open; teeth wo mastoid.
—	53·0	53·5	80·4	76·8	—	88·5	41·8	21·5	—	—	—	—	—	—	Old; sutures partly obli
72·3	60·2	60·6	76·4	78·1	75·7?	80·1	36·4	17·6	70°·7	66°·0	43°·3	16°·0	27°·3	82°·0	Teeth much worn; 3rd epipteric on R.
71·4	54·5	53·7	85·1	88·8	83·6	85·0	38·7	30·5	67°·6	70°·4	42°·0	11°·6	30°·4	82°·0	Teeth much worn; P.B. on L.; epipteric on
69·9	52·1	50·9	77·3	77·4	85·9	84·0	45·4	33·3	68°·5	71°·4	40°·1	14°·6	25°·5	86°·0	Ageing; sutures becomi
67·6?	55·2	56·2	84·4	87·6	69·3?	83·2	40·4	23·6	74°·0?	63°·0?	43°·0?	17°·0?	26°·0?	80°·0?	Alveolus almost comple
75·1	50·9	50·7	78·7	81·4	76·4	100·0	29·1	14·1	65°·1	67°·6	47°·3	14°·5	32°·8	82°·1	Ageing; sutures nearly to R. of sagittal su
69·8?	56·5	56·5	77·7	77·2	76·8?	75·4	42·5	47·3	64°·1?	73°·9?	42°·0?	26°·1?	15°·9?	100°·0?	Senile; sutures oblitera large mastoids; pro
77·2	45·4	46·2	84·9	82·6	81·0	83·2	43·0	30·0	65°·5	68°·3	46°·2	16°·9	29°·3	85°·2	P.B. flat with rounded
68·1	52·4	52·5	77·3	76·4	74·0	89·6	47·3	33·3	69°·0	72°·5	38°·5	13°·7	24°·8	86°·2	Sutures partly obliter metopic ridge; epip
65·8	62·0	62·0	74·7	65·5	67·5	86·3	43·9	21·0	68°·6	70°·6	40°·8	12°·4	28°·4	83°·0	Basilar synchondrosis;
—	50·6	50·6	86·8	—	77·1?	88·9	55·8	29·0	—	—	—	—	—	—	Ageing; sutures partly plates R. and L.; L
65·8	49·2	48·9	70·2	79·3	88·5?	84·2	46·7	23·5	66°·6	75°·2	38°·2	15°·8	22°·4	91°·0	Ageing; sutures begin bulging obelion; bi asterion; heavy su
69·8?	49·2	49·2	75·6	75·5	88·7	73·0	50·0	40·4	64°·1?	72°·4?	43°·5?	18°·6?	24°·9?	91°·0	Teeth much worn; P. foramina.
68·3	53·8	54·6	75·1	74·2	75·1	85·4	60·5	38·2	59°·0	79°·2	41°·8	16°·8	25°·0	96°·0	Ageing; sutures becom obelion; perforated
66·6	55·6	55·2	85·1	84·9	81·4?	81·9	34·4	44·6	66°·0?	71°·0?	43°·0?	11°·7?	31°·3?	82°·7?	Teeth much worn on in mastoid angle; epip
62·2	58·1	57·8	77·1	77·0	87·4?	76·4	37·1	38·9	64°·0?	78°·2?	37°·8?	12°·5?	25°·3?	90°·7?	Old; sutures nearly ob
79·7	45·1	46·7	85·2	85·1	65·3	85·9?	52·9	43·1	63°·9	67°·7	48°·4	18°·3	30°·1	86°·0	Ageing; sagittal sutur prognathism; very
—	—	—	—	87·1	—	—	58·5	49·0	—	—	—	—	—	—	Fairly well marked sup
67·6	58·5	57·7	83·3	87·9	86·0?	90·9?	40·6	49·0	67°·1	70°·8	42°·1	17°·2	24°·9	88°·0	P.B. flat with rounded R. and L.; L. tymp
64·1	64·5	64·0	76·1	79·6	70·1	75·5	43·0	34·7	75°·0	67°·0	38°·0	16°·0	22°·0	83°·0	Flattening round lamb with R. frontal and R. and L.; P.B. fl
—	—	—	—	—	—	80·6	—	—	—	—	—	—	—	—	Ageing; coronal and s
76·5	56·6	56·9	83·6	83·7	79·3	73·2	40·4	23·2	66°·4	66°·6	47°·0	18°·0	29°·0	84°·6	Teeth much worn; alv
73·7?	49·3?	49·6?	81·4	81·0?	—	81·6	54·0?	34·4	65°·1?	72°·7?	42°·2?	16°·0?	26°·2?	88°·7?	Teeth much worn; P. L. tympanic plate
—	51·0	51·3	77·4	76·7	—	84·8	45·5	32·7	—	—	—	—	—	87°·5?	3rd molars undevelop lambdoid compli over foramen ma
71·8	54·0	52·9	74·0	75·4	75·4?	80·3	49·8	41·5	76°·1	62°·4	41°·5	18°·4	23°·1	80°·8	Old; sutures partly o ossicles at R. and
65·7	58·8	58·8	75·5	80·0	84·7?	78·6	40·5	16·9	72°·9	65°·6	41°·5	18°·1	23°·4	83°·7	Old; sutures becomin sub-nasal progn

ngles

θ_2	θ_1	$P\angle$	Remarks*
17°·4?	25°·8?	90°·0?	P.B. sloping forward and downward with very rounded double edge.
21°·6	27°·4	89°·0	Sutures becoming obliterated; P.B. very rounded and double with large depressions between; very large mast mastoid ridges; wormians in R. lambdoid and at R. asterion; lipping of margin of L. occipital condyle.
7°·7	28°·8	82°·5	P.B. rounded; sub-nasal prognathism.
8°·5	33°·6	77°·1	P.B. rounded; sub-nasal prognathism; wormians in L. lambdoid; fair malar marginal processes.
—	—	—	Marked depression of occipital bone behind foramen magnum; wormians R. and L. lambdoid; epipteric on R.; foram
14°·2	27°·9	85°·0	Sutures open; teeth worn; torus palatinus; very asymmetric incisive foramen; receding forehead; P.B. rounded along lambdoid; fronto-squamous process on L.; small pre-condyle on R.
—	—	—	Skull cap only; healed wound on L. lambdoid, leaving depression 3 cm. × 2½ cm. × 8 mm. deep; wormian in R. lar
17°·4	31°·8	85°·0	Teeth worn; large alveolar abscesses at base of wisdom teeth R. and L.; P.B. flat with blunt edge; slight flatte epipteric on R.; face asymmetrical; alveolar point 4·2 mm. to R. of sagittal plane.
17°·5	27°·7	83°·5	Molars much worn; left inner palate bridge; P.B. flat with rounded edge; sub-nasal prognathism; very asymm L. occipital, R. frontal bulges; 3 large ossicles and depressions about lambda and wormians in L. lambdoid; ossic angle L. and R.; marked malar marginal prominence on L.
9°·9	32°·2	86°·0	Ageing; sutures becoming obliterated; remaining tooth much worn; alveolar rim partly absorbed; receding for rounded edge; sub-nasal prognathism; flattened obelion; 2 pre-condyles, the larger on L.
12°·5	30°·2	84°·6	P.B. flat with rounded edge; metopic; ossicle at R. squamo-mastoid angle; ossicle at bregma; suggestion of os jap
13°·3	28°·1	85°·0	Young adult; P.B. flat with rounded edge; very prominent malar marginal process on R.
13°·6	29°·8	89°·0	Old; sutures partly obliterated; surface of teeth much worn to inward-upward slant; ossicle at R. asterion; very parietal, L. occipital and R. frontal bulges; P.B. flat with sharp edge on L., rounded on R.
14°·6	27°·3	92°·0	P.B. with outward-upward slant and rounded edge; flattened obelion; ossicle at obelion and at lambda; ossicles i small epipteric on R.
15°·4	25°·6	85°·4	P.B. with outward-upward slant and very rounded edge; wormians R. and L. lambdoid.
16°·1	26°·9	86°·0	P.B. flat with very rounded edge, ageing; sutures partly obliterated.
15°·9	27°·6	82°·7	Old; sutures becoming obliterated; P.B. with steep upward-outward slope and rounded edge; sub-nasal prognati japonicum on both sides; very large foramen magnum.
18°·1?	23°·6?	87°·0?	Aged; 3rd molars never cut; sutures closed; P.B. flat with sharp edge; sub-nasal prognathism; very asymmetric.
13°·1	30°·3	84°·7	Sutures open; teeth worn; P.B. flat with rounded edge; flattened obelion; ossicle R. and L. lambdoid and at R. asteri mastoid.
—	—	—	Old; sutures partly obliterated, alveolar rim absorbed; P.B. flat with rounded edge; flattened obelion; ossicle R.
16°·0	27°·3	82°·0	Teeth much worn; 3rd molars small; P.B. flat with rounded edge; marked sub-nasal prognathism; ossicles R. epipteric on R.
11°·6	30°·4	82°·0	Teeth much worn; P.B. sloping upward and outward with very rounded edge; wormians R. and L. lambdoid; fro on L.; epipteric on R.; prognathous; ivory exostosis just above R. mastoid.
14°·6	25°·5	86°·0	Ageing; sutures becoming obliterated; teeth much worn; P.B. flat with sharp edge; sub-nasal prognathism.
17°·0?	26°·0?	80°·0?	Alveolus almost completely absorbed; P.B. flat with rounded edge; prognathous; flattened obelion; epipterics R. and
14°·5	32°·8	82°·1	Ageing; sutures nearly obliterated; teeth much worn; P.B. flat with very rounded edge; sub-nasal prognathism to R. of sagittal suture.
26°·1?	15°·9?	100°·0?	Senile; sutures obliterated; alveolus almost completely absorbed; P.B. flat with blunt edge; old injury to L. su large mastoids; pronounced inion.
16°·9	29°·3	85°·2	P.B. flat with rounded edge; flattened obelion.
13°·7	24°·8	86°·2	Sutures partly obliterated; P.B. flat with very rounded edge; sub-nasal prognathism; ossicles in occipito-masto metopic ridge; epipteric on L.
12°·4	28°·4	83°·0	Basilar synchondrosis; teeth worn; P.B. flat with very rounded edge; beginnings of suture across occipital bone fro
—	—	—	Ageing; sutures partly obliterated; teeth worn; P.B. with outward-upward slope and very rounded double edge; p plates R. and L.; L. outer palate bridge; abscess cavity in 1st L. bi-cuspid.
15°·8	22°·4	91°·0	Ageing; sutures beginning to be obliterated; alveolus partly absorbed; P.B. flat with blunt edge; whole base of bulging obelion; bi-parietal depressions; wormians R. and L. lambdoid; beginning of suture across occipital asterion; heavy supra-orbital ridges; receding forehead, marked malar marginal processes.
18°·6?	24°·9?	91°·0	Teeth much worn; P.B. flat with very rounded edge; flattened obelion; perforated tympanic plate on L.; very foramina.
16°·8	25°·0	96°·0	Ageing; sutures becoming obliterated; P.B. with upward-outward slope and double edge, inner edge rounded, obelion; perforated tympanic plates R. and L.; deep symmetrical depressions behind foramen magnum.
11°·7?	31°·3?	82°·7?	Teeth much worn on inside; P.B. with outward-upward slope and double edge, both edges sharp; ossicles R. lambd mastoid angle; epipterics R. and L.; fairly heavy inion; marked supra-orbital ridges.
12°·5?	25°·3?	90°·7?	Old; sutures nearly obliterated; alveolar rim absorbed; ossicle at lambda; P.B. flat with blunt edge.
18°·3	30°·1	86°·0	Ageing; sagittal suture nearly obliterated; abscess cavity in L. 3rd molar; P.B. with outward-upward slant and prognathism; very asymmetrical with R. frontal, L. occipital, L. parietal bulges; very deep canine fossae; wo
—	—	—	Fairly well marked supra-orbital ridges.
17°·2	24°·9	88°·0	P.B. flat with rounded edge; sub-nasal prognathism; flattened obelion; small ossicle at R. asterion and in L. occip R. and L.; L. tympanic plate perforated; large round foramen magnum.
16°·0	22°·0	83°·0	Flattening round lambda and at obelion; large ossicles at lambda and R. inter-parietal associated with bathrocepha with R. frontal and L. parietal and L. occipital bulges; wormian at obelion and in L. lambdoid; beginning of ho R. and L.; P.B. flat with rounded edge; very small asymmetrical foramen magnum.
—	—	—	Ageing; coronal and sagittal sutures nearly obliterated.
18°·0	29°·0	84°·6	Teeth much worn; abscess cavity in 1st molar on L.; P.B. flat with blunt edge; prognathous; R. inner palate bri
16°·0?	26°·2?	88°·7?	Teeth much worn; P.B. flat with rounded edge; very asymmetrical with R. parietal and R. occipital bulges; flat L. tympanic plates perforated.
—	—	87°·5?	3rd molars undeveloped; P.B. flat with sharp edge; flattened lambdoid region with 2 ossicles at lambda; bulgi lambdoid complicated with numerous large ossicles continuing right round temporals to asterions; flat circul over foramen magnum.
18°·4	23°·1	80°·8	Old; sutures partly obliterated; P.B. flat with blunt edge; prognathous; metopic ridge and sagittal ridge; w ossicles at R. and L. asterion; R. epipteric.
18°·1	23°·4	83°·7	Old; sutures becoming obliterated; teeth worn; receding forehead; sagittal crest and heavy inionic ridge; P.B. fl sub-nasal prognathism; os japonicum on R.; pre-condyle on R.; marked temporal and supra-orbital ridges.

very large mastoids and heavy supra
al condyle.

teric on R.; foramen magnum cut round.
ad; P.B. rounded; wormians nearly all

wormian in R. lambdoid.
edge; slight flattening at obelion; small

sm; very asymmetric, with L. parietal,
L. lambdoid; ossicles in squamo-mastoid

bed; receding forehead; P.B. flat with

ggestion of os japonicum R. and L.

R. asterion; very asymmetric, with L.

lambda; ossicles in R. and L. lambdoid;

ub-nasal prognathism; suggestion of os

very asymmetric.
d and at R. asterion, also in L. occipito-

belion; ossicle R. lambdoid.
ism; ossicles R. and L. lambdoid, long

L. lambdoid; fronto-squamous process

rognathism.
epipterics R. and L.; heavy inionic ridge.
asal prognathism; large dint near apex

ld injury to L. superciliary ridge; very

in occipito-mastoid sutures R. and L.;

occipital bone from R. and L. asterion.
ed double edge; perforation of tympanic

ge; whole base of skull much inflamed;
e across occipital bone from R. and L.

plate on L.; very large post-condyloid

er edge rounded, outer sharp; flattened
n magnum.
ossicles R. lambdoid and at R. squamo-

dge.
upward slant and sharp edge; sub-nasal
canine fossae; wormians R. lambdoid.

on and in L. occipito-mastoid; epipterics

with bathrocephaly; very asymmetrical
d; beginning of horizontal malar sutures

R. inner palate bridge.
cipital bulges; flattened obelion; R. and

at lambda; bulging occiput; lower half
terions; flat circular condyles projecting

sagittal ridge; wormians R. lambdoid;

onic ridge; P.B. flat with rounded edge;
a-orbital ridges.

139	A	♂?	—	685	183	184	184	139	88·5	135·5	134·5	120·5	99	316	316
142	A	♂	1305·5	602	164	166·5	166·5	141·5	90	134·5	132·5	120·5	94	317·5	324
45	B	♂	1416·5	662	182·5	185·5	184·5	136	89	142	140	121	106	320	322
49	B	♂?	—	—	162?	—	160?	—	95·5	—	—	—	100·5	—	—
50	B	♂	1354	569	168·5	171·5	170·5	137·5	87	142·5	141	119	100·5	320	318
63	B?	♂	—	563	165	169·5	169	148·5	82	132·5	131·5	116·5	99·5	321·5	321
66	B	♂?	—	537	168·5	172	170·5	136	84	132	130·5	116	96	308	309
75	B	♂	1596·5	508	175	177·5	176·5	148	99	142·5	139	126	99	335·5	336
79	B?	♂	—	689	177·5	181	179·5	142·5	95·5	132	131	117	99	315·5	316
88	B	♂?	1293	641	180	180	180	139·5	86	132·5	130	119	90	314	314
24	C	♂?	1450	580	163·5	166	165	144	87·5	147	145	122·5	100	326·5	333
28	C	♂?	1578·5	560	185	184·5	184·5	144	91	141·5	140·5	118	102	317·5	319
31	C or outlier	♂?	1470·5	556	179	181	180	130·5	89·5	143·5	142·5	120	100	309	309
51	C	♂	—	526	178	183	182	141·5	88	136·5	134·5	120·5	97	320	323
67	C	♂	1322·5	607	172	170	171	133·5	90	140	139·5	117	101·5	308·5	309
80	C	♂	1389·5	639	169·5	171·5	171	142·5	91	142·5	141	124	100	327	329
111	C	♂	—	—	171	174·5	173·5	152	90	142·5	142	118	103	320?	320
135	C	♂?	—	—	186	187·5	187	135	96	127	128	115·5	—	314	313

99	316	316	381	129·5	136·5	115	95	512	19·8?	74	103	134·5	53·9	55·0	29·1	10·2	20·5	29	2·7	6·5	43·9
94	317·5	324	352·5	120·5	128	104	90	495	22·0?	68·5	104	139	49·5	49·5	29·1	9·3	23·0	32	1·2	7·1	44·0
96	320	322	375	123	125·5	126·5	103	512	16·6?	68·5?	101	130	54·0	53·2	26·2	13·4	23·1	37	4·0	7·8	44·5
00·5	—	—	—	—	—	—	—	475·5?	—	65·5?	103·5?	136·5	49·3	49·3	27·9?	10·2	21·8	31·5	2·8	8·3	44·2
00·5	320	318	359	126	121	112	97·5	483	20·0	69	98	133	54·9	54·9	26·0	11·5	21·5	33	3·1	7·1	43·9
99·5	321·5	321	350	124·5	120	105·5	95·5	497	17·1	71·5	91	128·5	55·6	55·6	23·5	9·4	16·9	21·5	4·4	7·9	42·5
96	308	309	361	119·5	134·5	107	87	484·5	15·9	64	104·5	133	50·0	50·0	29·3	9·0	21·8	29	3·4	8·0	43·2
99	335·5	336·5	375	130	131	114	97·5	520	18·0?	71·5?	106·5	133?	55·9	57·0	27·4	11·5	22·9	33·5	3·2	8·8	44·7
99	315·5	316	366	126	119·5	120·5	95·5	512	14·0	68·5	107	139	52·4	52·4	26·1	11·6	23·0	35·5	4·2	7·9	43·3
00	314	314	383	125	119·5	138·5	110·5	498	19·0	67	96	120·5	46·2	47·2	26·5	8·3	19·6	26	2·9	6·4	44·1
00	326·5	333	357	128	115	114	103	486	19·2?	72	95	133·5	54·7	54·5	26·6	11·3	19·2	31	3·9	7·5	45·1
02	317·5	319	380	133	130	117	95·5	524	14·4	73	99	136·5	58·0	58·0	26·5	12·2	18·8	32	3·6	6·5	43·1
00	309	309	383·5	132	133	118·5	100	500	22·0	72	91	117·5	49·2	49·0	23·4	14·4	20·7	36	3·7	7·8	46·0
97	320	323·5	377·5	128·5	117	132	113	509·5	21·0	76·5	98·5	126·5	56·4	57·1	27·6	12·1	18·5	30·5	3·1	9·5	44·1
01·5	308·5	309	351·5	120	117	114·5	92·5	491	20·5	76	90·5	130·5?	54·9	54·6	23·5	9·8	19·0	28·5	3·3	7·4	45·1
00	327	329	362	119·5	128·5	114	101	492	24·0?	77	98	127·5	55·0	55·0	25·5	10·5	18·5	29	3·6	7·8	42·8
03	320?	320?	360	130	119	111	98·5	510	19·4	75	88	—	57·5	58·3	25·6	10·7	16·2	28	4·2	4·2	43·3
—	314	313	367·5	132	120·5	115	94·5	512	—	—	—	115?	—	—	—	—	—	—	—	—	—

6·5	43·9	43·5	32·5	32·8	51·6	45·6	38·9?	108·5	36·1	29·0	59·2	75·4	73·0	75·5	73·1	103·3	71·8	54·0
7·1	44·0	42·5	33·2	34·0	46·3?	42·0	39·2	98·5	37·0	29·1	64·6	85·0	79·5	85·1	79·6	106·9	65·7	58·8
7·8	44·5	43·0	34·6	34·2	53·2	47·2	—	102?	35·0	30·7	58·2	73·5	75·6	73·8	75·9	97·2	67·8	48·5
8·3	44·2	43·5	34·1	34·1	48·0?	43·8?	—	89·5	35·5	30·3	—	—	—	—	—	—	63·2?	56·6?
7·1	45·0	44·0	31·0	32·6	47·7	44·0	41·6	92	36·1	33·2	65·0	80·2	82·4	80·5	82·8	97·2	70·4	47·4
7·9	42·5	41·5	35·8	36·2	46·1	41·3	36·2	94	33·6	30·9	72·7	87·4	77·6	87·8	77·9	112·6	78·4	42·3
8·0	43·2	42·1	32·1	31·7	49·5	44·7	42·0	90·5	34·2	29·9	58·0	79·2	75·8	79·7	76·4	104·4	61·2	58·6
8·8	44·7	45·0	35·0	37·0	50·2	47·5	40·8?	92	37·3	30·8	62·8	83·5	78·3	84·1	78·8	106·6	67·2	49·0
7·9	45·5	43·3	33·3	34·2	51·1	47·1	41·0	98·5	32·6	29·5	56·5	78·7	72·4	79·5	73·1	108·7	64·1	49·8
6·4	44·6	43·5	32·6	34·6	48·0	43·0	39·0?	92	39·0	29·2	57·0	77·5	72·1	77·6	72·2	107·5	69·8	57·4
7·5	45·0	44·5	36·6	37·4	49·4	44·0	39·0	92	41·5	33·5	73·2	86·7	87·4	87·2	87·9	99·2	75·8	48·6
6·5	43·6	43·0	35·8	34·4	52·7	47·3	45·0	95	38·7	32·0	58·4	78·0	76·3	78·0	76·0	102·3	73·5	45·7
7·8	46·0	46·4	33·6	34·7	48·7	48·0	36·3	95·5	35·3	29·0	61·4	72·2	78·6	72·5	79·0	91·8	78·9	47·6
9·5	44·2	44·1	34·2	34·0	50·0	49·0	44·0	101	36·9	29·4	62·9	77·3	73·5	77·7	73·9	105·2	77·7	48·9
7·4	45·1	45·6	35·9	37·0	48·7	43·9	39·3	93	38·8	30·0	57·5	78·6	81·9	78·0	81·4	95·9	83·9	42·8
7·8	42·0	41·4	37·1	37·1	53·0	47·2	40·0	99	33·6	29·2	67·8	83·1	82·4	83·2	82·5	100·9	78·5	46·4
4·2	45·0	43·6	36·3	36·5	47·4	43·7	40·6	95·5	37·5	32·8	62·9	87·1	81·4	87·6	81·8	107·0	85·0	44·5
—	—	—	—	—	—	—	—	—	35·8	30·8	58·6	71·9	67·9	72·2	68·4	105·5	—	—

·8	54·0	52·9	74·0	75·4	75·4?	80·3	49·8	41·5	76°·1	62°·4	41°·5	18°·4	23°·1	80°·8	Old; sutures partly obli… ossicles at R. and L. …
·7	58·8	58·8	75·5	80·0	84·7?	78·6	40·5	16·9	72°·9	65°·6	41°·5	18°·1	23°·4	83°·7	Old; sutures becoming … sub-nasal prognathism …
·8	48·5	49·2	77·8	79·5	—	87·8	58·0	51·3	67°·6	74°·0	38°·4	11°·0	27°·4	85°·0	Alveolar rim absorbed …
·2?	56·6?	56·6?	77·1	78·4	—	85·4	46·8	33·7	60°·9?	79°·5?	39°·6?	8°·5?	31°·1?	88°·0	Crown sawn off and m…
·4	47·4	47·4	68·9	74·1	87·2	92·0	53·5	43·7	62°·3	76°·0	41°·7	10°·7	31°·0	86°·7	Old; sutures becoming … ridges; flattening a…
·4	42·3	42·3	84·2	87·2	78·5	92·0	55·6	55·7	64°·0	72°·5	43°·5	11°·2	32°·3	83°·7	Teeth worn; P.B. flat wi…
·2	58·6	58·6	74·3	75·3	84·8	87·4	41·3	42·5	65°·1	74°·8	40°·1	17°·2	22°·9	92°·0	Teeth worn; P.B. flat w…
·2	49·0	48·1	78·3	82·2	81·3?	82·6	50·2	36·4	63°·0	73°·1	43°·9	14°·9?	32°·0?	85°·0?	Old; sutures nearly obli… surface of skull, pro… tympanic plates R. a…
·1	49·8	49·8	73·2	79·0	80·2	90·5	50·5	53·2	69°·1	70°·2	40°·7	13°·3	27°·4	83°·5	Old; teeth worn; sutur…
·8	57·4	56·1	73·1	79·5	81·3?	74·9	42·3	45·3	69°·9	67°·0	43°·1	16°·6	26°·5	83°·6	Old; sutures nearly obli… epipteric on L.
·8	48·6	48·8	81·3	84·0	79·0	80·7	59·0	52·0	62°·1	74°·1	43°·8	8°·9	34°·9	83°·0	Remaining tooth worn; … depression on L.; an…
·5	45·7	45·7	82·1	80·0	85·4	82·7	64·9	55·4	63°·4	73°·5	43°·1	12°·0	31°·1	85°·5	Abscess cavity in outer … P.B. flat with sharp …
·9	47·6	47·8	73·0	74·8	74·5	82·2	69·6	47·4	65°·0	71°·9	43°·1	9°·1	34°·0	81°·0	Sutures partly obliter… malar marginal pro…
·7	48·9	48·3	77·4	77·1	88·0	79·7	65·4	32·6	70°·0	64°·5	45°·5	18°·5	27°·0	83°·0	Old; teeth badly worn, … large ossicle in R. la… edge.
·9	42·8	43·0	79·6	81·1	80·7	77·3	51·6	44·6	61°·2	73°·0	45°·8	12°·2	33°·6	85°·2	Hole through frontal p… outward-upward sl…
·5	46·4	46·4	88·3	89·6	75·5	86·9	56·8	46·2	66°·6	67°·9	45°·5	14°·1	31°·4	82°·0	P.B. flat with roundish …
·0	44·5	43·9	80·7	83·7	85·7	87·5	66·0	100·0	62°·4	73°·2	44°·4	14°·8	29°·6	88°·0	Teeth worn; hooked in…
—	—	—	—	—	—	86·0	—	—	—	—	—	—	—	—	Old; sutures nearly obl…

			over foramen magnum.
18°·4	23°·1	80°·8	Old; sutures partly obliterated; P.B. flat with blunt edge; prognathous; met ossicles at R. and L. asterion; R. epipteric.
18°·1	23°·4	83°·7	Old; sutures becoming obliterated; teeth worn; receding forehead; sagittal cres sub-nasal prognathism; os japonicum on R.; pre-condyle on R.; marked ten
11°·0	27°·4	85°·0	Alveolar rim absorbed except near middle incisors; P.B. flat with blunt edge.
8°·5?	31°·1?	88°·0	Crown sawn off and missing; senile; alveolus almost entirely absorbed; P.B. slo
10°·7	31°·0	86°·7	Old; sutures becoming obliterated; teeth worn; P.B. flat with double edge; up ridges; flattening at obelion; wormians at L. asterion; tympanic plates perfo
11°·2	32°·3	83°·7	Teeth worn; P.B. flat with rounded edge; L. tympanic plate perforated.
17°·2	22°·9	92°·0	Teeth worn; P.B. flat with rounded edge R. and sharp edge L.; small ossicle R
14°·9?	32°·0?	85°·0?	Old; sutures nearly obliterated; alveolus partly absorbed; parietal protrusions surface of skull, probably Pacchionian depressions; flattened obelion and l tympanic plates R. and L.; P.B. flat with sharp edge.
13°·3	27°·4	83°·5	Old; teeth worn; sutures mostly obliterated; P.B. flat with sharp edge.
16°·6	26°·5	83°·6	Old; sutures nearly obliterated; alveolus about molars absorbed; P.B. flat wit epipteric on L.
8°·9	34°·9	83°·0	Remaining tooth worn; receding forehead; P.B. flat with blunt edge; ossicles in depression on L.; small pre-condyles R. and L.
12°·0	31°·1	85°·5	Abscess cavity in outer alveolus L., and in 2nd L. molar; inflammation along P.B. flat with sharp edge; wormians R. and L. lambdoid; epipteric on R.
9°·1	34°·0	81°·0	Sutures partly obliterated; P.B. has steep upward-outward slant with sharp malar marginal process on L.
18°·5	27°·0	83°·0	Old; teeth badly worn, with surface slanting inwards and upwards; big absces large ossicle in R. lambdoid; very large post-condyloid foramina especially o edge.
12°·2	33°·6	85°·2	Hole through frontal near L. stephanion probably due to abscess; 1st molars outward-upward slant and sharp edge; flattened obelion; ossicles R. and L.
14°·1	31°·4	82°·0	P.B. flat with roundish edge; small ossicle R. lambdoid; perforation of R. tym
14°·8	29°·6	88°·0	Teeth worn; hooked inion; sutures beginning to be obliterated; P.B. flat with
—	—	—	Old; sutures nearly obliterated; depressed obelion.

* See Section 9 of

prognathous; metopic ridge and sagittal ridge; wormians R. lambdoid;

ead; sagittal crest and heavy inionic ridge; P.B. flat with rounded edge;
n R.; marked temporal and supra-orbital ridges.

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ith blunt edge.
bsorbed; P.B. slopes upward and outward and has sharp edge.
double edge; upper edge blunt, lower edge rounded; heavy superciliary
panic plates perforated R. and L.
erforated.
.; small ossicle R. lambdoid; perforated tympanic plates R. and L.
ietal protrusions near vertex corresponding to twin depressions on inner
ed obelion and lambda; ossicles in squamo-mastoid angles; perforated

arp edge.
ed; P.B. flat with sharp edge; flattened obelion; R. inner palate bridge;

---

t edge; ossicles in occipito-mastoid sutures R. and L.; large post-condyloid

ammation along alveolar margin; slight torus palatinus; bulging occiput;
epipteric on R.
slant with sharp edge; sub-nasal prognathism; sagittal crest; prominent

wards; big abscess cavity in 1st R. molar; marked depression at obelion;
mina especially on R.; suggestion of os japonicum; P.B. flat with rounded

scess; 1st molars worn; abscess cavity 1st and 2nd R. molars; P.B. with
ssicles R. and L. lambdoid, and at R. asterion.
ration of R. tympanic plate.
l; P.B. flat with slightly rounded edge; prognathous.

See Section 9 of Text.

FEMALES

					Lengths									
No.	Type	Sex	*C*	W. in grs.	*F*	*L'*	*L*	*B*	*B'*	*H*	*H'*	*OH*	*LB*	*Q*
1	A (B?)	♀	1276·5	550	168	168·5	169	142·5	87	127	127	111	93·5	312
2	A	♀	1248	643	159·5	159	159	147·5	90	139·5	137·5	119·5	95	325·5
5	A	♀	—	—	160	161	159·5	142	84	130	128	114·5	86·5	315
7	A	♀	1085·5	574	159·5	161	160	132	85·5	126·5	126·5	110·5	92·5	295·5
8	A	♀	—	497	173	176	175	139·5	91·5	136	132·5	116	96	308·5
10	A	♀	1090·5	500	163	165·5	164	133	97	130	129·5	111	97·5	296·5
11	A	♀	—	555	175	175	174	133·5	88·5	139	139	118·5	96	311?
17	A	♀	1215·5	472	166	167	166	132·5	87·5	127	126·5	110	95	302·5
21	A	♀?	1442·5	487	166	167·5	166·5	142	94	143·5	140·5	118	97	315
26	A	♀	1417·5	527	172	172	172	135·5	96·5	141	140	117·5	100·5	311·5
29	A	♀	1342·5	578	168	171	170	141	91·5	133	131	114	102·5	309·5
32	A	♀	1411·5	515	170·5	171·5	171	142	93·5	133·5	134·5	112	99·5	310·5
33	A	♀	—	—	162	—	162·5	145	92·5	133·5	131	—	85	—
35	A	♀	1322·5	560	179·5	181·5	181	131	93	128·5	126·5	111·5	98	300
37	A (B?)	♀	1254	554	172·5	172	171·5	129	88·5	125	124	110	89	297
42	A	♀?	—	478	180·5	181	—	134	95·5	122	123	108	96·5	294·5
64	A	♀	—	455	164	167·5	166	142	92	141·5	139	115·5	91	310·5
70	A	♀	1335·5	570	176	176·5	176	136	95·5	132	130	119	97	316
73	A	♀?	1404·5	570	165	169	168·5	145	92·5	136	135	118	98	318
81	A	♀	—	—	161	162·5	163	136	86	125·5	123·5	111·5	88·5	—
83	A	♀	—	473	168	170	169	134·5	88	120·5	120	109·5	94	302?
84	A	♀	1169·5	588	153	152	152·5	143	92·5	131	129	118·5	92·5	320
85	A	♀	—	527	155	155·5	154	146·5	90	129	128	115	91	321
87	A (B?)	♀	1239	503	161·5	162	161·5	139·5	87·5	133·5	131·5	115	90	308·5
89	A	♀	1345·5	527	171·5	172	171·5	132·5	91	134·5	134	115	96·5	302·5
93	A	♀	1111·5	256	158·5	158·5	158	134	89	121	120	107	83	289·5
99	A	♀	1481·5	549	169	171	170	147	91	129	127·5	116	92	317·5
103	A	♀	1184	621	171	167·5	169·5	133	95	131·5	132·5	114	97	305
107	A	♀	1373	420	174	175	174	140·5	93·5	135	134	118·5	97·5	318·5
115	A	♀	—	—	181·5	182·5	182	133	91	124	127	108	100	—
118	A?	♀	1061·5	—	163	165·5?	165	132·5	91·5	125·5?	125	111?	89·5	301·5
120	A	♀	1194·5	—	168	165·5	167	134	93·5	132?	132	116?	93	314
125	A	♀	1355·5	697	170	169·5	169·5	143·5	93	134	134	117	92	318
126	A	♀	1130	538	154·5	154	154·5	138·5	86	126·5	126	111	91	302·5
129	A	♀	1126	640	164·5	164·5	164·5	136·5	88	132·5	132	112·5	92·5	308
131	A	♀	—	—	160·5	159?	160	145	84	133	132	118·5?	97?	314?
132	A?	♀?	1295·5	—	163	163·5	163	135·5	86·5	135·5	135	117·5	94·5	311·5
137	A	♀	1320·5	471	163	164	163	139·5	93	134·5	133	122	89·5	324·5
141	A	♀	—	555	164	163·5	164	133·5	89·5	131	130	111·5	96·5	300
12	B	♀	—	274	156·5	158	157·5	126·5	70·5	123	121	104·5	87·5	281
13	B?	♀	1320	518	166·5	171·5	170·5	138	93·5	139·5	138	123	102	321·5
15	B?	♀	1333	561	179	179	179·5	131·5	99	138	138	119·5	104·5	311·5
19	B?	♀	1149	418	164	166·5	166	127	91	133	131·5	112	96	294
34	B?	♀	1112·5	523	156·5	158·5	157	134	81	130	128	113·5	94	307
36	B	♀	—	—	171·5	170·5	169	125·5	—	125·5	125·5	107	89·5	286
41	B (A?)	♀	1369·5	368	164	165·5	164·5	139·5	90·5	137	137	116	93	315
47	B?	♀	1289·5	537	167	168	166·5	134·5	90	136·5	136	117	95	308·5
58	B	♀?	—	—	161	161·5	161	144	94	133·5	132·5	114	100	315
62	B	♀?	—	422	157·5	159·5	159	148	92·5	134·5	134	114	96	318·5
82	B	♀?	1257·5	576	164·5	166·5	165·5	140	90	130	127	116	93·5	315
92	B	♀	—	555	163	166·5	164	139·5	86	132?	130·5	118?	96	313·5

		Arcs								Face				Nose							
I	*LB*	*Q*	*Q'*	*S*	S_1	S_2	S_3	S_2'	*U*	*PH*	*G'H*	*GB*	*J*	*NH R*	*NH L*	*NB*	*DS*	*DC*	*DA*	*SS*	*SO*
	93·5	312	312·5	350	124	118	108	93	495	16·1?	65·5	91	125	49·7	49·2	25·0	8·0	17·9	25	1·7	5
5	95	325·5	324	344	121	123	100	90·5	485	20·0?	71?	95·5	130·5	53·1	52·5	24·9	10·1?	22·6?	32?	3·3	9
5	86·5	315	310·5	359	127?	129	103	91	480	—	—	—	117?	—	—	—	—	—	—	—	—
5	92·5	295·5	295·5	334	115	115	104	89·5	465	22·1	70·5	92	119	51·5	51·5	24·3	11·7	21·5	33	2·4	7
	96	308·5	312	360	117	129	114	94	502	18·7	73·5	99	134·5	56·4	57·5	27·2	8·5	21·0	29	2·9	10
	97·5	296·5	298	339	125·5	118	95·5	85	474	18·0?	65	102	129	51·8	51·8	30·2	12·4	22·0	35	2·6	9
5	96	311?	310	372	125	130	117	100	489	16·3?	65·5	95·5	125?	49·5	49·6	25·8	8·0	18·8	26·5	2·1	6
	95	302·5	302·5	339	125	112	102	88	477	11·6?	58	92	122·5	46·9	46·9	26·5	10·1	20·6	30	3·2	11
	97	315	311·5	360	121	125	114	98·5	493	13·2	69·5	99	136	57·9	56·8	28·0	10·7	19·2	31	3·3	9
5	100·5	311·5	312·5	357·5	121·5	127	109	93·5	494	14·7	65·5	105	139·5?	53·1	54·8	29·0	7·6	23·0	28·5	2·6	8
	102·5	309·5	309	343	117	108	118	103	491	21·0	71	108·5	132	52·0	51·0	29·5	9·3	24·4	34·5	1·5	11
	99·5	310·5	312	354	130·5	122	101·5	89	498	18·8	70	104	135	52·2	52·9	26·3	—	20·4	—	—	6
	85	—	—	370	132·5	133	104·5	91·5	491	17·2?	63	95	—	48·0	47·9	25·5	11·8	20·8	33	3·9	8
5	98	300	299	365	124	125	116	95	504	21·9	69·5	99	126·5	49·9	49·9	26·4	7·0	20·8	26	1·4	5
	89	297	295·5	365	123	134	108	91·5	486	14·2?	59	98	123	45·0	46·0	24·5	11·1	21·1	32	2·6	8
	96·5	294·5	293	364	121	126	117	89·5	508	24·0?	73	105·5	142	51·5	51·5	27·6	10·7	27·5	36	2·9	9
5	91	310·5	313	365	127	126	112	102	490	—	65?	100·5	137	51·9	51·4	24·3	7·8	18·1	25·5	2·2	7
	97	316	321	362	125·5	121	115·5	99	499	15·0	64·5	97·5	124?	51·2	52·0	28·9	10·4	24·2	36	2·5	8
	98	318	319	356	131·5	121·5	103	94	495·5	18·4	69·5	99·5	135	54·7	53·7	26·0	12·1	18·2	28	5·1	9
5	88·5	—	—	339	119	113	107	95·5	467	18·8	68	92	118·5?	52·0	52·0	27·4	8·9	18·0	28	2·5	9
5	94	302?	300	344·5	119	115	110·5	91	479·5?	17·0?	60	88	112?	45·0	45·0	25·6	8·9	17·8	26·5	2·1	7
5	92·5	320	318	332	116·5	122	93·5	81	469	14·0	59	97·5	129	48·2	48·0	27·9	9·3	19·8	28·5	2·4	7
	91	321	325	328	116·5	108	103·5	90·5	479	13·0?	64·5	90·5	128·5	51·2	51·8	24·4	6·5	17·7	23·5	1·6	7
	90	308·5	306	345	116	124	105	91·5	474	22·0?	70	98·5	127	49·9	50·8	29·0	8·1	20·2	29	2·4	9
	96·5	302·5	302	353	120·5	122·5	110	93	486	18·5?	71?	96	124	53·2	53·2	27·5?	7·6	23·6	30	2·3	9
	83	289·5	292	332	115	106	111	93·5	467	17·0?	63?	92	120·5	50·5	47·8	30·7	6·8	22·2	29	1·6	12
	92	317·5	320	352	124·5	118·5	109	96	501·5	16·3?	65·5	95·5	131	50·2	50·0	26·2	8·4	22·0	29	0·9	4
	97	305	306	348·5	117·5	119	112	93	479	15·7	63·5	96	122	49·0	48·0	28·3	8·9	22·0	30·5	1·4	9
5	97·5	318·5	318	354·5	127	117	110·5	96	501·5	18·6?	64·5	94	130	50·2	48·8	27·5	10·1	20·5	30	2·6	5
	100	—	398?	359·5	124·5	123	112	91·5	502?	13·5?	60·5	93?	—	47·4	47·8	28·0	11·6	23·0	36·5	2·8	8
?	89·5	301·5?	299?	339	122	106	111	96	472	—	—	—	—	—	—	—	—	—	—	3·3?	9
?	93	314	313	352	123	119·5	109·5	91·5	477·5	—	—	—	123·5?	—	—	—	10·1	24·3	33	2·0	11
	92	318	320	363	126	127	110	95	493·5	16·2?	66·5	99	123·5	51·8	50·6	26·3	9·9	18·3	29	2·7	8
	91	302·5	301·5	321·5	112	111	98·5	86	462	19·3	69	96	116	51·5	53·0	23·8	7·4	20·3	27	2·6	9
5	92·5	308	310	350·5	121·5	118·5	110·5	96	475·5	13·5	59	95·5	120·5	48·0	47·5	26·8	7·0	19·7	24·5	1·4	6
5?	97?	314?	316?	342·5	116	121	105·5	91	483·5	—	—	—	—	—	—	—	11·9	21·0	33	2·9	9
5	94·5	311·5	320	348·5	126·5	124	98	84·5	473	—	—	—	128?	—	—	—	9·8?	21·9?	—	2·3	6
	89·5	324·5	327	353	124	124	105	92·5	485·5	17·7	65	92	128	49·6	49·7	24·0	8·7	20·1	29	2·4	8
5	96·5	300	301	340	112·5	111·5	116	98	474·5	17·0	67	94?	119	52·0	50·8	27·2	10·1	19·0	27	3·8	8
5	87·5	281	279	326	114·5	120·5	91	81·5	451	14·2	59·5	87·5	111·5	46·4	46·4	21·5	6·9	14·7	21	1·8	4
	102	321·5	321	354	123	122	109	98·5	490	18·1	69	97	—	52·4	52·0	27·0	10·0	24·8	34·5	3·5	11
5	104·5	311·5	313	363	127	113	123	105	498	17·0?	65?	94	126	51·0	51·0	28·0	13·1	23·1	37	4·7	12
	96	294	293	344	110·5	136·5	97	84·5?	465	8·5?	59·5?	87	120·5	47·5	47·5	23·0	13·4	19·5	36	3·7	9
5	94	307	308	330·5	111	106	113·5	98	458	17·9?	68	96	116·5	51·0	51·0	25·7	9·5	19·4	29·5	2·6	7
	89·5	286	292	355	120	127·5	107·5	90·5	—	21·4?	67·5	90·5	120·5	46·5	46·8	27·6	10·0	19·0	31	4·8	5
	93	315	315	347	124	123	100	90	482	20·0?	70·5	94	125	51·0	49·2	23·9	8·5	20·0	28	3·0	6
	95	308·5	310	353	128	117	108	94·5	486	16·8?	67?	93	124·5	49·9	50·5	25·0	9·5	23·0	31·5	2·3	7
	100	315	316	334	119	116·5	98·5	89·5	488	22·8	72·5	—	—	50·3	49·5	26·1	9·8	23·2	31	3·6	8
	96	318·5	321	335	123·5	92·5	119	104·5	480	19·3?	68·5?	97·5	131	49·5	49·5	24·4	8·1	16·8	24	2·4	6
	93·5	315	315	349	118·5	120	110·5	94	492·5	20·4?	71·5	96	123·5?	52·0	52·7	23·3	8·5	18·0	27	2·8	7
?	96	313·5?	312·5?	345	117·5	121	106·5	94	482	16·4?	66·5?	94	125·5	51·0	50·9	24·0	11·2	18·5	31	2·6	8

ntinued). TABLES OF THE ABSOLUTE AND INDICIAL ME

Nose						Orbits				Palate and Profile				F
NB	*DS*	*DC*	*DA*	*SS*	*SC*	O_1R	O_1L	O_2R	O_2L	G_1	G_1'	G_2	*GL*	*fm*
25·0	8·0	17·9	25	1·7	5·2	40·0	39·7	36·6	36·5	46·0	41·6	37·0	90	37·
24·9	10·1?	22·6?	32?	3·3	9·8	37·3?	39·2	34·0	35·0	50·0	44·1	39·5	91?	33·
—	—	—	—	—	—	—	—	—	—	—	—	—	—	32·
24·3	11·7	21·5	33	2·4	7·9	38·8	37·9	32·5	31·4	50·0	47·4	38·2	93·5	32·
27·2	8·5	21·0	29	2·9	10·8	41 3	41·8	36·2	35·0	44·0	39·5	39·2	88·5	37·
30·2	12·4	22·0	35	2·6	9·0	44·8	43·7	30·8	30·8	52·5	50·5	40·0?	99	34·
25·8	8·0	18·8	26·5	2·1	6·5	40·0	41·0	31·0	30·5	46·0	43·5	41·0	93·5	34·
26·5	10·1	20·6	30	3·2	11·0	41·4	42·3	33·5	32·4	42·5	40·0	29·0	87	34·
28·0	10·7	19·2	31	3·3	9·2	43·5	42·8	36·0	36·6	47·0	42·5	40·8	87?	36·
29·0	7·6	23·0	28·5	2·6	8·5	44·1	45·2	38·4	37·8	48·8	45·0	40·0	95	35·
29·5	9·3	24·4	34·5	1·5	11·4	43·4	43·5	33·2	33·5	52·5	48·9	42·5	105·5	35·
26·3	—	20·4	—	—	6·6	44·4	44·0	39·0	38·1	—	—	41·5?	91	37·
25·5	11·8	20·8	33	3·9	8·7	41·6	41·1	30·8	31·8	43·8	42·3	38·2	91	34·
26·4	7·0	20·8	26	1·4	5·5	42·5	42·7	34·0	33·0	46·2	42·0	39·4	91	33·
24·5	11·1	21·1	32	2·6	8·5	42·0	40·5	32·3	34·1	—	37·2?	37·8	82·5	33·
27·6	10·7	27·5	36	2·9	9·3	46·3	45·5	36·3	36·8	52·1	45·6	39·0	95	32·
24·3	7·8	18·1	25·5	2·2	7·0	42·7	41·4	36·1	35·9	43·2	38·5	34·8	75?	38·
28·9	10·4	24·2	36	2·5	8·8	44·0	46·5	33·8	34·6	—	—	37·0	90·5	32·
26·0	12·1	18·2	28	5·1	9·5	45·5	45·0	35·0	34·6	43·2	38·2	41·2	90	36·
27·4	8·9	18·0	28	2·5	9·0	39·0	39·9	33·1	34·0	48·1	44·6	34·2	93	34·
25·6	8·9	17·8	26·5	2·1	7·0	40·0	40·5	32·7	32·0	43·2?	39·5	33·8	97·5	30·
27·9	9·3	19·8	28·5	2·4	7·9	42·5	40·9	33·1	33·2	43·8	39·0	35·3	86	32·
24·4	6·5	17·7	23·5	1·6	7·6	42·1	41·1	36·7	37·5	44·1	40·1	35·8	86	37·
29·0	8·1	20·2	29	2·4	9·0	43·3	42·0	37·3	37·5	49·5	45·2	36·7?	89	33·
27·5?	7·6	23·6	30	2·3	9·4	42·2	42·8	35·5	34·5	—	—	41·8	90·5?	40·
30·7	6·8	22·2	29	1·6	11·5	39·2	39·0	31·4	32·8	43·5	40·6	36·5	78·5?	34·
26·2	8·4	22·0	29	0·9	4·8	43·8	43·0	35·6	37·0	48·6	45·6	36·8	95	40·
28·3	8·9	22·0	30·5	1·4	9·5	43·8	43·0	35·0	35·2	46·7?	42·5	36·5	95	35·
27·5	10·1	20·5	30	2·6	5·9	42·1	40·6	35·5	35·0	42·4	40·1	—	90	35·
28·0	11·6	23·0	36·5	2·8	8·2	41·0	41·1	34·0	34·0	—	45·0?	39·6	94	35·
—	—	—	—	3·3?	9·0?	—	—	—	—	—	—	—	—	34·
—	10·1	24·3	33	2·0	11·2	—	—	—	—	—	—	—	—	33·
26·3	9·9	18·3	29	2·7	8·0	39·8	39·2	35·0	34·8	49·0	45·5	38·8	94	33·
23·8	7·4	20·3	27	2·6	9·0	40·9	39·2	35·0	35·0	—	42·0	41·0	92	33·
26·8	7·0	19·7	24·5	1·4	6·2	40·0	41·8	36·0	37·0	—	—	—	89·5	33·
—	11·9	21·0	33	2·9	9·0	—	—	—	—	—	—	—	—	33·
—	9·8?	21·9?	—	2·3	6·8	—	—	—	—	—	—	—	—	34·
24·0	8·7	20·1	29	2·4	8·9	41·9	41·2	33·5	33·6	48·0	43·2	36·8	89	36·
27·2	10·1	19·0	27	3·8	8·6	41·9	42·5	35·5	34·2	—	—	35·0	88·5	34·
21·5	6·9	14·7	21	1·8	4·0	38·7	37·4	34·0	34·3	41·2	36·8	32·9	82·5	38·
27·0	10·0	24·8	34·5	3·5	10·3	42·3	—	36·2	36·5	48·4	44·3	39·8	99·5	35·
28·0	13·1	23·1	37	4·7	11·5	45·8	44·5	35·0	34·0	—	—	41·0?	100?	39·
23·0	13·4	19·5	36	3·7	9·6	43·8	43·8	29·0	30·2	46·1	42·4	34·7	87?	—
25·7	9·5	19·4	29·5	2·6	7·7	40·0	40·4	32·0	32·8	50·8	46·6	34·8	93·5	32·
27·6	10·0	19·0	31	4·8	9·0	40·6	41·9	—	34·5	—	46·3	41·0	86	37·
23·9	8·5	20·0	28	3·0	8·1	42·9	41·8	37·5	35·3	48·0	44·3	—	91	38·
25·0	9·5	23·0	31·5	2·3	8·2	40·7	40·0	31·0	32·0	48·7	45·0	32·8	91·5	35·
26·1	9·8	23·2	31	3·6	8·7	43·9	—	34·4	—	51·1	47·2	41·2	100·5	34·
24·4	8·1	16·8	24	2·4	5·1	42·0	41·8	34·5	35·4	46·8	41·0	35·0	91?	35·
23·3	8·5	18·0	27	2·8	7·0	43·5	43·6	33·2	35·2	44·0	41·0	38·0	93	36·
24·0	11·2	18·5	31	2·6	9·3	44·9	41·0	34·5	35·5	48·5	45·0	—	92·5?	35·

ICIAL MEASUREMENTS OF THE INDIVIDUAL CRANIA (ABSOLUTE LENGTHS IN MMS.)

Profile		Foramen		Indices															
G_2	*GL*	*fml*	*fmb*	*Oc I*	*B/L'*	*H/L'*	*B/L*	*H/L*	*B/H*	*G'H/GB*	*NB/NH R*	*NB/NH L*	O_2/O_1 *R*	O_2/O_1 *L*	G_2/G_1	*fmb/fml*	*DS/DC*	*SS/SC*	*N*
7·0	90	37·0	31·0	63·8	84·6	75·5	84·3	75·3	112·0	72·2	50·3	50·8	91·5	92·0	80·4	83·8	44·7	32·7	66°
9·5	91?	33·9	29·0	72·7	92·7	86·1	92·8	86·4	107·4	74·3?	46·9	47·4	91·2?	89·3	80·6	85·3	45·0?	33·7	64°
—	—	32·0	29·9	67·7	88·4	79·7	88·4	80·5	110·8	—	—	—	—	—	—	93·8	—	—	—
8·2	93·5	32·5	28·2	63·9	81·9	78·5	82·4	78·9	104·4	76·6	47·2	47·2	83·8	82·9	76·4	86·8	54·4	30·4	68°
9·2	88·5	37·0	31·2	59·1	79·3	75·3	79·7	75·8	105·2	74·2	48·2	47·3	87·7	83·7	89·1	84·3	40·5	26·9	60°
0·0?	99	34·8	25·2	69·6	80·6	78·3	81·1	78·9	102·9	63·6	58·3	58·3	68·8	70·5	76·2?	72·4	56·4	28·9	72°
1·0	93·5	34·4	27·6	62·7	76·1	79·3	76·6	79·9	95·9	68·7	52·1	52·0	77·5	74·4	89·1	80·2	42·6	32·3	67°
9·0	87	34·6	29·0	64·0	79·3	75·7	79·7	76·1	97·1	63·0	56·5	56·5	80·9	76·6	68·2	83·8	49·0	29·1	64°
0·8	87?	36·4	31·0	63·8	84·9	83·8	85·5	84·4	101·3	70·3	48·4	49·3	82·8	85·5	86·8	85·2	55·7	35·9	60°
0·0	95	35·0	30·6	63·3	78·6	81·3	78·7	81·4	96·6	62·3	54·6	52·9	87·1	83·6	82·0	87·4	33·0	30·6	66°
2·5	105·5	35·0	29·0	65·5	82·5	76·6	82·9	77·1	107·6	65·6	56·7	57·8	76·5	77·0	81·0	82·9	38·1	13·2	72°
1·5?	91	37·2	33·0	66·4	82·8	78·4	83·0	78·6	105·7	67·3	50·4	49·7	87·8	86·6	—	88·7	—	—	62°
8·2	91	34·1	27·0	66·1	—	—	89·2	80·6	110·8	66·2	53·1	53·3	74·0	77·4	87·2	79·2	56·7	44·8	74°
9·4	91	33·5	30·0	58·6	72·2	69·5	72·5	69·8	103·9	70·4	52·9	52·9	80·0	77·3	85·3	89·6	33·7	25·5	63°
7·8	82·5	33·5	30·0	61·5	75·1	72·1	75·4	72·4	104·1	60·2	54·4	53·3	76·9	84·2	—	89·6	52·6	30·6	63°
9·0	95	32·0	28·7	55·0	74·0	68·0	—	—	108·9	69·0	54·0	54·0	78·4	80·9	74·9	89·7	38·9	31·2	66°
4·8	75?	38·9	33·1	75·1	84·9	83·2	85·5	83·0	102·0	64·7?	46·8	47·3	84·5	86·7	80·6	85·1	43 1	31·5	54°
7·0	90·5	32·8	29·5	63·0	77·0	77·3	77·3	73·0	104·6	66·1	56·4	55·6	76·8	74·4	—	89·9	43·0	28·2	64°
1·2	90	36·0	32·5	76·3	86·0	80·0	86·2	80·2	107·5	69·8	47·5	48·4	76·9	76·9	95·4	90·3	66·5	53·7	62°
4·2	93	34·0	30·1	70·2	83·6	76·1	83·4	75·9	109·9	73·6	52·5	52·5	84·9	85·2	71·1	88·5	49·4	27·8	71°
3·8	97·5	30·9	27·0	59·0	79·1	70·6	79·5	70·9	112·1	68·0	56·9	56·9	81·8	79·0	78·2?	87·4	49·9	30·0	71°
5·3	86	32·2	31·0	64·5	94·1	84·9	93·9	84·7	110·9	60·6	57·9	58·1	77·9	81·2	80·6	96·3	47·0	30·8	65°
5·8	86	37·5	29·0	65·8	94·1	82·3	94·9	83·0	114·4	71·3	47·7	47·1	87·2	91·2	81·2	77·3	36·7	21·1	64°
6·7?	89	33·5	31·0	65·5	86·0	81·2	86·4	81·6	105·9	70·9	58·1	57·1	86·1	89·3	74·1?	92·5	40·1	26·7	66°
1·8	90·5?	40·4	30·2	61·4	77·4	77·8	77·4	78·2	99·0	84·0?	51·7?	51·7?	84·1	80·6	—	74·8	32·2	24·5	63°
6·5	78·5?	34·6	27·5	61·2	84·6	75·7	85·0	76·0	111·8	68·3?	60·8	64·2	80·1	84·1	83·9	79·5	30·6	14·0	63°
6·8	95	40·5	33·5	67·5	86·1	74·6	86·6	75·0	115·5	68·5	52·2	52·4	81·3	86·0	75·7	82·7	38·2	18·8	71°
6·5	95	35·8	29·4	59·8	79·5	79·1	78·6	78·1	100·5	66·1	57·8	59·0	79·9	81·9	78·2?	82·1	40·5	14·7	68°
—	90	35·0	29·5	64·9	80·3	76·5	80·7	76·9	105·0	68·8	54·8	56·4	84·3	86·2	—	84·3	49·3	44·1	63°
9·6	94	35·9	27·2	58·4	93·0	69·7	73·1	69·7	104·8	64·8?	59·1	58·6	82·9	82·7	—	75·8	50·4	34·1	66°
—	—	34·0	27·2	64·2	79·8?	75·3?	80·2	75·7	106·0	—	—	—	—	—	—	80·0	—	36·7?	—
—	—	33·0	27·3	60·4	80·9	79·6	80·2	79·0	101·6	—	—	—	—	—	—	82·7	41·6	17·9	—
8·8	94	33·0	27·1	64·1	84·7	79·0	84·7	79·1	107·1	67·2	50·8	52·0	87·9	88·8	79·2	82·1	54·1	33·8	70°
1·0	92	33·8	28·5	53·2	89·9	81·7	89·8	81·7	110·0	71·5	46·2	44·9	85·6	89·3	—	84·3	36·5	26·9	68°
—	89·5	33·2	26·4	65·0	83·0	80·1	83·1	80·3	103·6	61·9	55·8	56·4	90·0	88·5	—	79·5	35·5	22·6	68°
—	—	33·0?	30·0	53·9	91·3	83·0	90·8	82·4	110·1	—	—	—	—	—	—	90·9?	56·7	32·2	—
—	—	34·0	28·0	63·8	82·8	82·5	83·2	82·8	100·4	—	—	—	—	—	—	82·4	44·7?	33·8	—
6·8	89	36·0	30·5	66·7	85·0	81·0	85·5	81·5	105·0	70·5	48·4	48·3	80·0	81·6	76·7	84·7	43·3	27·0	68°
5·0	88·5	34·3	32·0	61·3	81·8	79·5	81·5	78·3	102·8	71·1?	52·3	53·5	84·7	80·5	—	93·3	53·2	44·2	62°
2·9	82·5	38·5	30·0	71·1	80·1	76·6	80·3	76·8	104·5	67·8	46·3	46·3	87·9	91·7	79·9	77·9	47·0	45·0	62°
9·8	99·5	35·3	33·4	72·2	80·5	80·5	80·9	80·9	100·0	71·1	51·5	51·9	85·6	—	82·2	94·6	40·3	34·0	66°
1·0?	100?	39·0	34·0	62·2	73·4	77·1	73·1	76·9	95·1	69·1?	54·9	54·9	76·4	76·4	—	87·2	56·7	40·9	65°
4·7	87?	—	—	64·9	76·1	79·1	76·5	79·4	96·3	68·3?	48·4	48·4	66·2	69·0	75·3	—	68·7	38·5	55°
4·8	93·5	32·2	25·5	63·6	84·8	80·9	85·5	81·5	104·8	70·6	50·4	50·4	80·0	81·2	68·5	79·2	49·0	33·8	67°
1·0	86	37·3	30·2	61·0	73·6	73·5	74·2	74·1	100·1	74·4	59·3	59·0	—	82·3	—	81·0	52·6	53·3	67°
—	91	38·1	32·8	71·7	84·4	82·8	84·9	83·3	101·9	65·0	46·9	48·6	87·4	84·4	—	86·1	42·5	37·0	67°
2·8	91·5	35·1	29·9	65·8	80·0	80·9	80·9	81·8	99·0	72·0?	50·1	49·5	76·2	80·0	67·4	85·2	41·3	28·0	67°
1·2	100·5	34·7	31·3	74·2	89·1	82·1	89·4	82·4	108·5	—	51·9	52·7	78·4	—	80·6	90·2	42·2	41·4	67°
5·0	91?	35·4	31·1	66·6	92·8	84·1	93·1	84·3	110·4	70·2?	49·3	49·3	82·1	84·7	74·8	87·9	48·2	47·0	67°
8·0	93	36·3	30·5	62·1	84·0	76·1	84·5	76·6	110·3	74·5	44·8	44·2	76·3	80·7	86·4	84·0	47·2	40·0	67°
—	92·5?	35·0	25·0	67·2	83·7	78·2	85·0	79·4	107·0	71·0?	47·1	47·2	76·8	86·6	—	71·4	60·5	28·0	67°

(MS.)

	Angles						
SS/SC	*N ∠*	*A ∠*	*B ∠*	θ_2	θ_1	*P ∠*	
32·7	66°·2	71°·9	41°·9	16°·1?	25°·8?	88°·0	P.B. flat with blunt edge.
33·7	64°·6?	70°·6?	44°·8?	12°·9?	31°·9?	83°·5	Flattened obelion; P.B. flat with sharp edge; small triangul of tympanic plates R. and L.; right interior palate brid
—	—	—	—	—	—	—	Flattening in lambdoid region; ossicles in lambdoid suture
30·4	68°·5	66°·9	44°·6	15°·1	29°·5	82°·0	Young adult; sutures open; P.B. rounded; prognathous; fr condyle on L.
26·9	60°·8	72°·4	46°·8	14°·1	32°·7	86°·5	Sutures closed; L. canine never cut, but grown in lateral dir on L.; epipteric on L.; glenoid fossa perforated on R.
28·9	72°·0	69°·4	38°·6	10°·6	28°·0	80°·0	Ageing; sutures round lambda nearly obliterated; P.B. very
32·3	67°·6	72°·0	40°·4	12°·0	28°·4	84°·0	Adolescent; sutures open; P.B. flat with blunt edge; ossicle
29·1	64°·0	79°·0	37°·0	9°·7	27°·3	88°·7	Old; coronal and sagittal sutures becoming obliterated; os j of tympanic plates R. and L.
35·9	60°·2?	75°·8?	44°·0?	8°·3?	35°·7?	84°·1	Aged; sagittal suture partly obliterated; abscess cavity in 1s nasal prognathism; sagittal crest and slight metopic cre
30·6	66°·1	74°·9	39°·0	3°·1	35°·9	78°·0	P.B. flat with blunt edge; bony spicule for pulley of superic
13·2	72°·1	67°·9	40°·0	10°·8	29°·2	78°·7	P.B. flat with blunt edge; tympanic plates perforated R. ar
—	62°·2	74°·8	43°·0	10°·8	32°·2	85°·6	Old; sagittal suture nearly obliterated; 3rd molars never cut
44·8	74°·0	64°·2	41°·8	—	—	—	Sagittal suture open; molars much worn; slight torus pals japonicum R. and L.
25·5	63°·0	73°·9	43°·1	16°·0	27°·1	89°·9	Aged; sagittal suture nearly obliterated; wormian at R. squ
30·6	63°·8	76°·3	39°·9	13°·2	26°·7	89°·5	P.B. flat with sharp edge; triangular epipteric on L.; R. tym near epipteric.
31·2	66°·3	69°·0	44°·7	17°·0	27°·7	86°·0	Large hole in 1st L. molar; P.B. having outward-upward sl
31·5	54°·4?	80°·6?	45°·0?	13°·4?	31°·6?	94°·0?	P.B. flat with blunt edge; hooked inion; ossicle in R. lambo
28·2	64°·4	75°·5	40°·1	13°·5?	26°·6?	89°·0?	P.B. flat with rounded edge.
53·7	62°·0	74°·8	43°·2	13°·7	29°·5	88°·5	Old; sutures nearly all obliterated; teeth worn and the wh only and are shaped like incisors; P.B. flat with very ro
27·8	71°·5	64°·5	44°·0	21°·0	23°·0	85°·5	Young adult; L. 3rd molar coming through; sutures open; P.
30·0	74°·8	68°·8	36°·4	16°·8	19°·6	85°·6	P.B. flat with sharp edge; marked sub-nasal prognathism;
30·8	65°·2	76°·4	38°·4	15°·1	23°·3	91°·5	P.B. flat with blunt edge; wormians R. and L. lambdoid; I
21·1	64°·6	73°·1	42°·3	11°·4	30°·9	84°·5	P.B. flat with blunt edge; large anterior alveolar abscess suture R. and L.; L. tympanic plate perforated; progna
26·7	66°·3	67°·6	46°·1	13°·4	32°·7	81°·0	P.B. flat with sharp edge; prognathous; posterior half of sa wormians and ossicles; ossicle R. and L. asterion and tympanic plate perforated; pre-condyle on L.; parieto-a
24·5	63°·2?	72°·1?	44°·7?	14°·4?	30°·3?	86°·5?	P.B. flat with blunt edge; wormian L. lambdoid; ossicle in
14·0	63°·3?	71°·0?	45°·7?	14°·0?	31°·7?	85°·0?	Senile; sutures nearly all obliterated; alveolus absorbed n edge; sub-nasal prognathism; depressed obelion and pa and L. tympanic platea.
18·8	71°·5	67°·5	41°·0	14°·3	26°·7	81°·8	L. 3rd molar coming through; P.B. flat with sharp edge; worm
14·7	68°·9	72°·6	38°·5	8°·4	30°·1	81°·0	Bathrocephalic with tri-partite inter-parietal; flattened obel
44·1	63°·5	76°·5	40°·0	16°·0	24°·0	92°·5	Alveolus partly absorbed; P.B. flat with blunt edge; epipte
34·1	66°·5	77°·5	36°·0	10°·5	25°·5	88°·0	Sagittal suture becoming obliterated; surface of teeth worn in 2 parts, R. and L. divided by large groove; ossicle in
36·7?	—	—	—	—	—	—	Flattened obelion; small foramen magnum.
17·9	—	—	—	—	—	—	Oldish; depressed obelion; tympanic plate perforated R. ar
33·8	70°·4	67°·6	42°·0	15°·4	26°·6	83°·0	Teeth worn; P.B. flat with rounded edge; sub-nasal prognath
26·9	68°·4	67°·4	44°·2	17°·1	27°·1	84°·5	Beginning of cleft palate; depressed obelion; P.B. flat wit R. tympanic plate perforated.
22·6	68°·7	73°·4	37°·9	11°·6	26°·3	85°·0	Alveolus absorbed in molar region; P.B. flat with blunt ed
32·2	—	—	—	—	—		Old; sutures becoming obliterated; sagittal suture from verte
33·8	—	—	—	—	—	—	Young adult; flattened obelion; small ossicles at R. and L. plate perforated.
27·0	68°·3	68°·9	42°·8	19°·1	23°·7	88°·0	Remaining tooth worn; P.B. flat with blunt edge; sub-nasa plate perforated; 3rd L. molar single-fanged.
44·2	62°·3	77°·5	42°·2	15°·3	26°·9	90°·8	P.B. slanting upward and outward with sharp edge; wormi R. and L.
45·0	64°·7	74°·6	40°·7	8°·4	32°·3	83°·0	Adolescent; basilar synchondrosis; no 3rd molars and no ro and in L. occipito-mastoid suture.
34·0	68°·0	72°·0	40°·0	11°·0	29°·0	83°·0	Old sutures closing; teeth worn; 3rd molars not cut; R. in obelion; very marked pre-condyles nearly meeting to fo
40·9	67°·8?	75°·2?	37°·0?	8°·8?	28°·2?	84°·0?	Old; sutures closed and partly obliterated; alveolar margin at lambda; malar marginal processes; R. tympanic plat sagittal suture depressed about vertex.
38·5	62°·9?	79°·5?	37°·6?	6°·3?	31°·3?	85°·8?	P.B. rounded; alveolar abscess near 1st R. molar; inflamm L. temporal; mutilation of R. nasal bone (due to syphil
33·8	68°·3	69°·7	42°·0	11°·6	30°·4	81°·3	P.B. rounded.
53·3	65°·0	69°·9	45°·1	12°·6	32°·5	82°·5	Old; sutures closing; teeth much worn; P.B. flat with shar
37·0	63°·8	69°·2	45°·0	10°·8	34°·2	80°·0	Adolescent; basilar synchondrosis; 3rd molars undevelope
28·0	66°·1?	71°·8?	42°·1?	11°·9?	30°·2?	83°·7	Metopic; teeth worn; P.B. flat with sharp edge; flattened
41·4	69°·0	68°·7	42°·3	12°·3	30°·0	81°·0	Teeth much worn giving surface an inward-upward slope;
47·0	64°·6?	72°·7?	42°·7?	11°·8?	30°·9?	84°·5?	P.B. flat with sharp edge.
40·0	67°·2	67°·6	45°·2	16°·8	28°·4	84°·4	Young adult; P.B. flat with rounded edge; sub-nasal pro L.; suggestion of same on R.
28·0	66°·2?	72°·4?	41°·4?	14°·6?	26°·8?	87°·0	Alveolus absorbed except about R. incisors; P.B. flat wit plate perforated; glenoid cavity on L. flattened out by in by a ridge.

	Remarks*
	ge; small triangular epipteric on R.; alveolus absorbed in molar region R. and L.; perforation erior palate bridge.
	lambdoid suture R. and L.; squamo-mastoid ossicle on R.; small epipteric on R. ; prognathous; frontal process to squamous on R.; squamo-mastoid ossicle on L.; small pre-
	own in lateral direction across 2nd incisor; P.B. flat with blunt edge; squamo-mastoid ossicle rforated on R.
	rated; P.B. very rounded and double; sub-nasal prognathism; slight pre-condyles on R. and L. lunt edge; ossicle in R. occipito-mastoid suture; small pre-condyle on R. obliterated; os japonicum both sides but suture partly obliterated; P.B. rounded; perforation
	oscess cavity in 1st R. molar and in alveolus; sloping forehead; P.B. flat with blunt edge; sub- light metopic crest. pulley of superior oblique muscle in R. orbit; R. inner palate bridge. perforated R. and L. molars never cut; P.B. flat with blunt edge; slight flattening at obelion; ossicle at R. asterion. slight torus palatinus; P.B. flat with blunt edge; sub-nasal prognathism; suggestion of os
	vormian at R. squamo-mastoid angle; P.B. having outward-upward slope with blunt edge. eric on L.; R. tympanic plate perforated; 2 marked pre-condyles; slight exostosis on L. parietal
	utward-upward slant and rounded edge; suggestion of os japonicum R. and L. ssicle in R. lambdoid.
	worn and the whole crown of 1st L. molar destroyed by abscess; 3rd molars have one fang flat with very rounded edge; sub-nasal prognathism. ; sutures open; P.B. flat with very rounded edge; sub-nasal prognathism; small pre-condyle on R. al prognathism; ossicles R. and L. lambdoid; small epipterics R. and L. d L. lambdoid; L. tympanic plate perforated. alveolar abscess above 2nd L. molar; large epipteric on R.; beginning of horizontal malar erforated; prognathous. osterior half of sagittal suture depressed; R. and L. lambdoid almost entirely formed of large L. asterion and R. and L. squamo-mastoid angle; fronto-squamous process R. and L.; R. e on L.; parieto-squamous suture straight and nearly horizontal. bdoid; ossicle in L. occipito-mastoid; perforated tympanic plate on R. eolus absorbed near L. incisors; P.B. with outward-upward slope, steeper on L., with blunt d obelion and parietal depressions to R. and L. of obelion; epipteric on L.; perforation of R.
	sharp edge; wormians in L. lambdoid; ossicles in R. and L. occipito-mastoid; large epipteric on R. tal; flattened obelion; large perforations of R. and L. tympanic plates; P.B. flat with sharp edge. blunt edge; epipterics R. and L.; symmetrical parietal depressions. face of teeth worn down, with inward-upward slant; P.B. flat with rounded edge; nasal spine groove; ossicle in sagittal near lambda. n. perforated R. and L. ub-nasal prognathism; ossicle at L. asterion; fronto-squamous process on L.; pterion in K. on R. on; P.B. flat with slightly rounded edge; sub-nasal prognathism; small ossicle R. lambdoid;
	lat with blunt edge. l suture from vertex to obelion depressed between bi-parietal ridges; L. tympanic plate perforated. icles at R. and L. asterion, and in L. masto-parietal suture; small epipteric on R; L. tympanic
	nt edge; sub-nasal prognathism; depressed sagittal; ossicles R. and L. lambdoid; R. tympanic anged. harp edge; wormian near obelion, and at R. squamo-mastoid angle; large tympanic perforation
	molars and no room for them; P.B. flat with blunt edge; flattened obelion; ossicle at lambda,
	ars not cut; R. inner palate bridge; sloping forehead; P.B. flattish with sharp edge; flattened arly meeting to form bridge; symmetrical anterior parietal flattening. ; alveolar margin absorbed except at L. canine; P.B. flat with blunt double edge; large wormian R. tympanic plate perforated; healed fracture of L. superciliary ridge and L. zygomatic arch; R. molar; inflammatory condition about L. foramen ovale of sphenoid and petrous portion of one (due to syphilis?); foramen magnum cut; asymmetric palate.
	P.B. flat with sharp edge; sub-nasal prognathism. olars undeveloped; P.B. flat with blunt edge; wormian at R. squamo-mastoid angle. o edge; flattened obelion. rd-upward slope; P.B. flat with sharp edge; face asymmetrical; cranium very asymmetrical.
	ge; sub-nasal prognathism; wormians R. and L. lambdoid; epipteric on R.; os japonicum on
	ors; P.B. flat with blunt edge; epipteric on L.; fronto-squamous process on R.; R. tympanic flattened out by disease; long narrow foramen magnum with deep depression at fore-end walled

15	B?	♀	1333	561	179	179	179·5	131·5	99	138	138	119·5	104·5	311·5
19	B?	♀	1149	418	164	166·5	166	127	91	133	131·5	112	96	294
34	B?	♀	1112·5	523	156·5	158·5	157	134	81	130	128	113·5	94	307
36	B	♀	—	—	171·5	170·5	169	125·5	—	125·5	125·5	107	89·5	286
41	B (A?)	♀	1369·5	368	164	165·5	164·5	139·5	90·5	137	137	116	93	315
47	B?	♀	1289·5	537	167	168	166·5	134·5	90	136·5	136	117	95	308·5
58	B	♀?	—	—	161	161·5	161	144	94	133·5	132·5	114	100	315
62	B	♀?	—	422	157·5	159·5	159	148	92·5	134·5	134	114	96	318·5
82	B	♀?	1257·5	576	164·5	166·5	165·5	140	90	130	127	116	93·5	315
92	B	♀	—	555	163	166·5	164	139·5	86	132?	130·5	118?	96	313·5?
95	B	♀?	1233	544	157	157	157	138	89	137	134	118	91	313·5
100	B	♀	1161·5	557	159	160	159	137	89	127	127	111	93·5	301
102	B	♀	1134	—	160	160	158·5	132·5	84·5	129·5	130	117	96	314
116	B	♀	—	—	167·5	169	168	134	87	124·5	124·5	108	94	297·5
128	B	♀	1192	666	160·5	162·5	162	136	88	131	129	116	88	313
9	C	♀	1128·5	261	166·5	166	166·5	126	88·5	119	119	105·5	91·5	279
18	C	♀	—	375	167	171·5	169	122	88	131·5	131	112	97?	295·5
23	C	♀	—	348	177	178	178	125	87	127·5	127·5	111	96	291·5
25	C	♀	—	—	165	165?	166·5	127	88	128	127·5	—	98	—
27	C	♀?	—	400	177·5	180	180	127·5	84	129·5	128·5	112	98	294
30	C	♀	—	—	162·5	164	163·5	129·5	93	128	128	107	100	—
38	C	♀	1199·5	477	165	164·5	164·5	134·5	86·5	125·5	126	108·5	90·5	292·5
39	C	♀	—	—	180·5	181·5	181	128·5	89·5	137	138·5	111	103	295
40	C	♀	1266	554	175	176·5	175·5	130·5	91·5	130·5	129	114	96	303
46	C	♀	1207·5	505	155·5	156·5	155·5	135·5	89·5	140	138·5	121	95·5	320·5
48	C	♀	—	—	170·5	173·5	173	128·5	88	134	132·5	114	93	302
57	C	♀	—	—	162	162·5	161	136·5	86·5	135	132	116·5	91·5	313·5
59	C	♀	1392·5	474	179	179	179	135	93	137·5	138	113	97	305
77	C?	♀	—	395	169·5	172·5	171·5	139	90	126	124·5	117·5	91	312
108	C?	♀?	—	—	176	180?	179	137·5	91·5	132?	130·5	120?	98	320?
122	C	♀	1243	501	169·5	170·5	169·5	133·5	91	133	133	115	94·5	306·5
133	C	♀	1184·5	387	168·5	169	168·5	123·5	88·5	126	124·5	108	96	286
138	C	♀	—	—	162	162·5	162	130·5?	83·5	119	120	109	—	295?

NON-ADULTS

6	A	?	—	—	153·5	—	153·5	136·5	86·5	117?	116·5	—	78·5	—
101	A	?	1242·5	279	153·5	154·5	153·5	141	85	119·5	118·5	113·5	84	318
104	A??	?	—	—	157	156?	157	139	83·5?	132?	132	117?	85·5	—
114	A	?	—	—	153·5	153·5	152·5	140	85	122?	120·5	105·5?	89	303?
127	A	?	—	220	140·5	141	140·5	128·5	79	118·5	116·5	108·5	75	298·5
134	A	?	1286	295	149·5	149·5	149	149	88·5	125·5	125·5	112	87	320
140	A	?	1294·5	316	150·5	151	151	141	84	137	136·5	118·5	90·5	322·5

104·5	311·5	313	363	127	113	123	105	498	17·0?	65?	94	126	51·0	51·0	28·0	13·1	23·1	37	4·7	11·5
96	294	293	344	110·5	136·5	97	84·5?	465	8·5?	59·5?	87	120·5	47·5	47·5	23·0	13·4	19·5	36	3·7	9·6
94	307	308	330·5	111	106	113·5	98	458	17·9?	68	96	116·5	51·0	51·0	25·7	9·5	19·4	29·5	2·6	7·7
89·5	286	292	355	120	127·5	107·5	90·5	—	21·4?	67·5	90·5	120·5	46·5	46·8	27·6	10·0	19·0	31	4·8	9·0
93	315	315	347	124	123	100	90	482	20·0?	70·5	94	125	51·0	49·2	23·9	8·5	20·0	28	3·0	8·1
95	308·5	310	353	128	117	108	94·5	486	16·8?	67?	93	124·5	49·9	50·5	25·0	9·5	23·0	31·5	2·3	8·2
100	315	316	334	119	116·5	98·5	89·5	488	22·8	72·5	—	—	50·3	49·5	26·1	9·8	23·2	31	3·6	8·7
96	318·5	321	335	123·5	92·5	119	104·5	480	19·3?	68·5?	97·5	131	49·5	49·5	24·4	8·1	16·8	24	2·4	5·1
93·5	315	315	349	118·5	120	110·5	94	492·5	20·4?	71·5	96	123·5?	52·0	52·7	23·3	8·5	18·0	27	2·8	7·0
96	313·5?	312·5?	345	117·5	121	106·5	94	482	16·4?	66·5?	94	125·5	51·0	50·9	24·0	11·2	18·5	31	2·6	9·3
91	313·5	320	344	115	124	105	90	468	21·5?	70·5	94·5	125·5	52·5	52·5	25·8	9·8	19·5	30	2·8	9·6
93·5	301	305	334	120	118	96	85	467	17·0?	65	94	116	48·1	50·2	25·6	7·5	19·0	26	2·9	7·0
96	314	315	345	123	118	104	90·5	472·5	—	62?	—	120	47·5	48·0	24·0	9·0	19·6	28	2·7	6·5
94	297·5	299·5	344	124	127	93	82·5	480	13·0	58	93·5	123?	45·8	45·6	25·0	9·6	18·2	26	1·9	4·1
88	313	314	345·5	113	130·5	102	92	470	17·0	64	95	120	46·9	46·9	25·8	8·3	19·4	26	2·0	6·0
91·5	279	281	343	122	114	107	90	469	17·5?	64?	92	112·5	47·0	48·7	26·1	10·2	17·0	27	3·0	4·9
97?	295·5	295	345·5	122?	121	102·5	87	469	21·0?	67?	91·5	—	47·0?	47·0?	24·0?	—	21·8	—	—	8·0
96	291·5	293·5	357	127	122	108	85	489	17·0	64·5	89	120	47·5	48·2	22·5	—	17·0	—	—	—
98	—	—	331	116	112	103	86·5	465?	15·0	62	92	—	46·9	46·0	22·8	9·0	19·0	27	2·8	8·7
98	294	294	359·5	121·5	127	111	90·5	486	23·1	71	97·5	127?	50·0	49·0	27·3	12·6	20·5	35	4·2	9·1
100	—	—	327	115	117	95	82·5	468	16·8	62	91	—	45·8	45·8	20·6?	10·4	17·8	28·5	4·6	8·0
90·5	292·5	297	347	120	118	109	88	471	16·7	65	87·5	116	46·6	46·5	20·5	10·8	17·3	29	3·4	7·5
103	295	294	360	135	106	119	96	492·5	15·0	61·5	95?	120	—	47·7	—	10·3	20·1	30	3·7	7·8
96	303	305	357	124	123·5	109·5	92·5	497	21·8	70·5	101	119	49·6	50·8	24·1	15·4	17·8	36	4·4	8·9
95·5	320·5	324	348·5	121·5	124	103	91	465	7·7	63	88	118·5	50·7	50·7	25·0	10·8	18·3	29	2·9	6·5
93	302	297	357	118	123	116	96·5	478	12·3?	55·5	—	121	—	44·1	—	—	—	—	—	—
91·5	313·5	313	344·5	121	117·5	106	92	470	19·4	68	94	123	50·3	50·0	28·9	9·2	19·6	28	3·3	8·0
97	305	308	370	130	126	114	95·5	499	16·9	58·5	92	119	42·0	41·6	25·0	13·0	22·1	38	4·9	7·5
91	312	315	357	117	119	121	99·5	492	17·2?	67·5?	88	111·5	51·4	51·0	26·0	9·7	17·5	27·5	1·6	7·5
98	320?	320?	372	132	128	112	96·5	502	—	—	—	—	—	—	—	—	—	—	—	—
94·5	306·5	308	356	124	126·5	105·5	91·5	482	15·7	63·5	93	119	47·3	47·0	21·9	13·4	20·0	34·5	5·4	7·0
96	286	291	343	120	123·5	99·5	84·5	474	15·2?	59	86	117	43·7	43·0	20·8	11·1	16·0	30	3·2	7·8
—	295?	297·5?	341	120·5	104	116·5	101	464?	—	—	—	—	—	—	—	—	—	—	—	—

78·5	—	—	331	119	111	101	85·5	459	12·8?	57	83	—	43·7	43·7	23·0	8·0	18·3	25·5	3·4	8·
84	318	322	327·5	120	105	102·5	88·5	462	13·3	52	79	105	39·8	40·2	21·2	7·5	16·7	23	2·2	8·
85·5	—	—	339	125	115	99	89	468	—	—	—	—	—	—	—	—	—	—	—	—
89	303?	299	317·5	111	108	98·5	87	459	—	—	—	—	—	—	—	—	—	—	—	—
75	298·5	300	315·5	105	111	99·5	85·5	422	14·8?	51·5?	78·5	99	37·3	37·5	19·1	7·3	15·2	22	1·2	4
87	320	323	319	121	98·5	99·5	88	470	14·6	56·5	84	11;	42·5	42·5	21·9	7·1	18·9	24·5	1·9	5
90·5	322·5	327	336	121·5	122·5	92	85·5	459	15·2	60·5	84·5	113	46·9	46·9	23·8	—	17·0	—	—	—

28·0	13·1	23·1	37	4·7	11·5	45·8	44·5	35·0	34·0	—	—	41·0?	100?	39·0	34·0	62·2	73·4	77·1	73·1	76·9
23·0	13·4	19·5	36	3·7	9·6	43·8	43·8	29·0	30·2	46·1	42·4	34·7	87?	—	—	64·9	76·1	79·1	76·5	79·4
25·7	9·5	19·4	29·5	2·6	7·7	40·0	40·4	32·0	32·8	50·8	46·6	34·8	93·5	32·2	25·5	63·6	84·8	80·9	85·5	81·5
27·6	10·0	19·0	31	4·8	9·0	40·6	41·9	—	34·5	—	46·3	41·0	86	37·3	30·2	61·0	73·6	73·5	74·2	74·1
23·9	8·5	20·0	28	3·0	8·1	42·9	41·8	37·5	35·3	48·0	44·3	—	91	38·1	32·8	71·7	84·4	82·8	84·9	83·3
25·0	9·5	23·0	31·5	2·3	8·2	40·7	40·0	31·0	32·0	48·7	45·0	32·8	91·5	35·1	29·9	65·8	80·0	80·9	80·9	81·8
26·1	9·8	23·2	31	3·6	8·7	43·9	—	34·4	—	51·1	47·2	41·2	100·5	34·7	31·3	74·2	89·1	82·1	89·4	82·4
24·4	8·1	16·8	24	2·4	5·1	42·0	41·8	34·5	35·4	46·8	41·0	35·0	91?	35·4	31·1	66·6	92·8	84·1	93·1	84·3
23·3	8·5	18·0	27	2·8	7·0	43·5	43·6	33·2	35·2	44·0	41·0	38·0	93	36·3	30·5	62·1	84·0	76·1	84·5	76·6
24·0	11·2	18·5	31	2·6	9·3	44·9	41·0	34·5	35·5	48·5	45·0	—	92·5?	35·0	25·0	67·2	83·7	78·2	85·0	79·4
25·8	9·8	19·5	30	2·8	9·6	42·5	41·6	35·5	35·6	50·0	46·5	35·8	86	33·8	30·3	62·8	88·0	85·4	87·9	85·4
25·6	7·5	19·0	26	2·9	7·0	42·0	42·8	35·9	36·0	—	40·1	32·6	87	32·8	28·8	68·2	85·5	79·2	86·0	79·7
24·0	9·0	19·6	28	2·7	6·5	40·9	—	30·7	—	46·2?	41·0?	33·5?	93·5?	28·0	28·5	67·2	82·8	81·1	83·6	82·0
25·0	9·6	18·2	26	1·9	4·1	41·4	40·6	33·6	35·0	43·7	39·8	35·3	89	35·6	29·4	57·4	79·2	73·7	79·9	74·3
25·8	8·3	19·4	26	2·0	6·0	41·8	40·5	36·8	36·0	45·0	41·8	37·0	92	34·0	29·9	66·0	93·7	79·5	83·8	79·1
26·1	10·2	17·0	27	3·0	4·9	43·0	43·7	31·9	32·3	42·4	41·5	36·9	87?	37·3	30·8	60·6	75·8	71·7	75·6	71·1
? 24·0?	—	21·8	—	—	8·6	41·0	42·2	37·0	36·1	51·0	46·7	40·4	92·5?	37·0	28·0	61·9	71·2	76·6	72·0	77·5
22·5	—	17·0	—	—	—	43·6	44·3	37·2	37·3	50·2	46·0	38·5	92	34·8	32·0	56·3	70·2	71·6	70·2	71·6
22·8	9·0	19·0	27	2·8	8·7	44·1	43·6	37·0	35·0	48·0	44·0	38·0	93·5	35·1	31·0	61·0	76·8	77·3	77·3	76·
27·3	12·6	20·5	35	4·2	9·1	40·0	42·0	31·2	31·2	52·5	48·0	43·4	93·5	35·7	27·0	58·3	70·9	71·5	70·8	71·
20·6?	10·4	17·8	28·5	4·6	8·0	44·0	44·2	36·0	35·2	40·5	36·0	36·5	83	38·0	30·0	64·5	78·8	78·2	79·1	78·
20·5	10·8	17·3	29	3·4	7·5	39·4	39·4	33·1	33·0	47·2	44·0	36·3	81	35·0	28·8	57·6	81·7	76·6	81·7	76·
—	10·3	20·1	30	3·7	7·8	42·1	41·9	—	33·2	—	—	—	92	37·0	32·0	57·6	70·6	76·2	71·0	76·
24·1	15·4	17·8	36	4·4	8·9	41·0	40·3	33·3	34·0	45·5	42·0	39·0	89	36·5	29·5	61·4	74·1	73·2	74·5	73·
25·0	10·8	18·3	29	2·9	6·5	41·8	42·0	34·5	34·0	41·6?	38·6	37·1	86	31·2	26·5	67·7	86·7	88·5	87·3	89·
—	—	—	—	—	—	—	41·1	—	27·8	—	—	—	87	33·3	27·0	59·9	74·2	76·3	74·3	76·
28·9	9·2	19·6	28	3·3	8·0	41·1	41·0	35·3	34·8	46·1	41·6	36·3	89·5	33·3	30·0	64·7	83·9	81·3	84·6	81·
25·0	13·0	22·1	38	4·9	7·5	44·9	44·4	28·0	29·4	44·0	42·5	38·9	93	33·3	31·3	60·4	75·4	77·0	75·4	77·
26·0	9·7	17·5	27·5	1·6	7·5	43·8	44·0	35·2	35·0	—	—	33·5	87?	33·7	27·5	59·0	80·6	72·1	81·1	72·
—	—	—	—	—	—	—	—	—	—	—	—	—	—	36·2	31·0	63·9	76·4?	72·4?	76·9	72·
21·9	13·4	20·0	34·5	5·4	7·0	42·1	40·2	30·5	32·0	49·2	45·6	40·1	93	34·1	32·8	64·6	78·2	77·8	78·8	78·
20·8	11·1	16·0	30	3·2	7·8	40·8	40·4	31·0	31·0	45·0	40·5	38·9	89	35·0	31·3	61·4	73·1	73·6	73·3	73·
—	—	—	—	—	—	—	—	—	—	—	—	—	—	34·8	—	64·5	80·4?	73·9	80·6?	74·

23·0	8·0	18·3	25·5	3·4	8·9	40·0	39·6	32·0	32·0	—	37·2	31·8	77	34·7	27·2	61·4	—	—	88·9	75
21·2	7·5	16·7	23	2·2	8·1	36·6	36·6	32·2	32·4	38·0?	35·5	28·0	79·5	36·0	25·6	64·2	91·1	76·6	91·9	77
—	—	—	—	—	—	34·1	—	—	—	—	—	—	—	38·8	28·4	71·1	89·1?	84·6?	88·6	82
—	—	—	—	—	—	—	—	—	—	—	—	—	—	32·4	27·8	67·7	91·2	78·4	91·9	79
19·1	7·3	15·2	22	1·2	4·8	35·7	35·7	30·0	29·0	—	31·0	30·7?	73?	28·9	24·5	63·5	90·9	82·4	91·4	82
21·9	7·1	18·9	24·5	1·9	5·7	37·5	37·1	36·0	36·2	36·5	35·1	34·0	83·5	33·1	28·4	67·5	99·6	83·8	100·0	82
23·8	—	17·0	—	—	—	39·2	38·3	33·0	32·0	39·0	36·0	34·0	81·5	35·0	28·3	83·6	93·2	90·3	93·2	90

7·1	73·1	76·9	95·1	69·1?	54·9	54·9	76·4	76·4	—	87·2	36·7	40·9	67°·8?	75°·2?	37°·0?	8°·8?	28°·2?	8
9·1	76·5	79·4	96·3	68·3?	48·4	48·4	66·2	69·0	75·3	—	68·7	38·5	62°·9?	79°·5?	37°·6?	6°·3?	31°·3?	8
0·9	85·5	81·5	104·8	70·6	50·4	50·4	80·0	81·2	68·5	79·2	49·0	33·8	68°·3	69°·7	42°·0	11°·6	30°·4	8
3·5	74·2	74·1	100·1	74·4	59·3	59·0	—	82·3	—	81·0	52·6	53·3	65°·0	69°·9	45°·1	12°·6	32°·5	8
2·8	84·9	83·3	101·9	65·0	46·9	48·6	87·4	84·4	—	86·1	42·5	37·0	65°·8	69°·2	45°·0	10°·8	34°·2	8
0·9	80·9	81·8	99·0	72·0?	50·1	49·5	76·2	80·0	67·4	85·2	41·3	28·0	66°·1?	71°·8?	42°·1?	11°·9?	30°·2?	8
2·1	89·4	82·4	108·5	—	51·9	52·7	78·4	—	80·6	90·2	42·2	41·4	69°·0	68°·7	42°·3	12°·3	30°·0	8
4·1	93·1	84·3	110·4	70·2?	49·3	49·3	82·1	84·7	74·8	87·9	48·2	47·0	64°·6?	72°·7?	42°·7?	11°·8?	30°·9?	8
6·1	84·5	76·6	110·3	74·5	44·8	44·2	76·3	80·7	86·4	84·0	47·2	40·0	67°·2	67°·6	45°·2	16°·8	28°·4	8
8·2	85·0	79·4	107·0	71·0?	47·1	47·2	76·8	86·6	—	71·4	60·5	28·0	66°·2?	72°·4?	41°·4?	14°·6?	26°·8?	8
5·4	87·9	85·4	103·0	74·5	49·1	49·1	83·5	85·6	71·6	89·6	50·3	29·2	62°·7	70°·5	46°·8	15°·5	31°·3	8
9·2	86·0	79·7	107·9	69·0	53·2	51·0	85·5	84·1	—	86·9	39·5	41·4	63°·5	74°·5	42°·0	12°·0	30°·0	8
1·1	83·6	82·0	101·9	—	50·5	50·0	75·1	—	72·5?	102·0	45·9	41·5	68°·6?	73°·4?	38°·0?	12°·6?	25°·4?	7
3·7	79·9	74·3	107·5	61·8	54·6	54·8	81·2	86·2	80·8	82·6	52·7	46·3	67°·0	76°·4	36°·6	12°·6	24°·0	8
9·5	83·8	79·7	105·3	67·6	55·0	55·0	88·0	88·9	82·2	88·0	42·8	33·3	72°·6	65°·8	41°·6	14°·5	27°·1	
1·7	75·6	71·5	105·7	69·6?	55·5	53·5	74·2	73·9	84·9	82·6	60·0	61·2	65°·1?	72°·9?	42°·0?	13°·6?	28°·4?	
6·6	72·0	77·5	92·9	73·1?	50·9?	50·9?	90·2	85·5	79·2	75·7	—	—	65°·8?	73°·0?	41°·2?	13°·5?	27°·7?	
1·6	70·2	71·6	97·3	72·2	47·4	46·7	85·3	84·2	76·7	92·0	—	—	66°·9	73°·1	40°·0	11°·9	28°·1	
7·3	77·3	76·7	99·7	67·4	48·6	49·6	83·9	80·3	79·2	88·3	47·4	32·2	67°·0	75°·2	37°·8	6°·4?	31°·4?	
1·5	70·8	71·5	99·1	73·0	54·6	55·7	78·0	74·3	82·7	75·6	61·5	46·2	64°·9	71°·7	43°·4	15°·3	28°·1	
8·2	79·1	78·4	100·9	68·0	45·0?	45·0?	81·8	79·6	90·1	78·9	58·4	57·5	56°·0	86°·0	38°·0	3°·5	34°·5	
7·6	81·7	76·5	106·8	74·3	44·0	44·1	84·0	83·8	76·9	82·3	62·4	45·3	60°·5	75°·5	44°·0	13°·0	31°·0	
7·2	71·0	76·5	92·8	64·7?	—	—	—	79·2	—	86·5	51·2	47·4	62°·2	81°·6	36°·2	6°·6	29°·6	
3·2	74·5	73·6	101·2	69·4	48·6	47·4	81·2	84·4	85·7	80·8	86·5	49·4	62°·6	72°·8	44°·6	15°·2	29°·4	
8·5	87·3	89·1	98·0	71·7	49·3	49·3	82·5	81·0	89·2?	85·0	59·0	44·6	61°·9	77°·8	40°·3	10°·7	29°·6	
6·3	74·3	76·4	97·3	—	—	—	—	67·6	—	81·1	—	—	66°·1	78°·1	35°·8	8°·1	27°·7	
1·3	84·6	81·9	103·3	72·4	57·5	57·8	85·9	84·9	78·7	90·1	47·0	41·3	66°·2	69°·5	44°·3	15°·5	28°·8	
7·0	75·4	77·0	98·0	63·8	59·5	60·1	62·4	66·2	88·4	94·0	58·8	65·3	68°·3	75°·8	35°·9	4°·6	31°·3	
2·1	81·1	72·6	111·7	76·3?	50·6	51·0	80·4	79·5	—	81·6	55·4	21·5	64°·5?	71°·1?	44°·4?	15°·1?	29°·3?	
2·4?	76·9	72·8	105·5	—	—	—	—	—	—	85·6	—	—	—	—	—	—	—	
7·8	78·8	78·4	100·5	68·3	46·3	46·6	72·4	79·6	81·5	96·2	67·0	77·1	68°·7	71°·9	39°·4	14°·1	25°·3	
3·6	73·3	73·9	99·3	68·8	47·6	48·4	76·0	76·7	86·4	89·4	69·4	41·0	65°·0	78°·0	37°·0	12°·0	25°·0	
3·9	80·6?	74·1	108·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	

—	88·9	75·8	117·3	68·9	52·6	52·6	80·0	80·8	—	78·4	43·7	38·2	66°·9	69°·9	43°·2	13°·6?	29°·6?
76·6	91·9	77·2	118·9	66·0	53·3	52·7	88·0	88·5	73·7?	71·1	44·9	27·2	66°·8	76°·4	36°·8	14°·8	22°·0
84·6?	88·6	84·3	105·3	—	—	—	—	—	—	73·2	—	—	—	—	—	—	—
78·4	91·9	79·1	116·3	—	—	—	—	—	—	85·8	—	—	—	—	—	—	—
82·4	91·4	82·9	108·3	65·6?	51·2	50·9	84·0	81·2	—	84·8	48·0	25·0	67°·7?	71°·8?	40°·5?	16·2?°	24°·3?
83·8	100·0	84·2	118·8	67·0	51·5	51·5	96·0	97·6	93·2	86·1	37·6	35·2	67°·2	74°·1	38°·7	15°·7	23°·0
90·3	93·2	90·3	103·2	71·8	50·7	50·7	84·2	83·6	87·2	80·9	—	—	61°·6	77°·6	40°·8	16°·2	24°·6

							obelion; very marked pre-condyles nearly meeting to form bridge; syn
40·9	67°·8?	75°·2?	37°·0?	8°·8?	28°·2?	84°·0?	Old; sutures closed and partly obliterated; alveolar margin absorbed excep at lambda; malar marginal processes; R. tympanic plate perforated; sagittal suture depressed about vertex.
38·5	62°·9?	79°·5?	37°·6?	6°·3?	31°·3?	85°·8?	P.B. rounded; alveolar abscess near 1st R. molar; inflammatory conditio L. temporal; mutilation of R. nasal bone (due to syphilis?); foramen
33·8	68°·3	69°·7	42°·0	11°·6	30°·4	81°·3	P.B. rounded.
53·3	65°·0	69°·9	45°·1	12°·6	32°·5	82°·5	Old; sutures closing; teeth much worn; P.B. flat with sharp edge; sub-na
37·0	65°·8	69°·2	45°·0	10°·8	34°·2	80°·0	Adolescent; basilar synchondrosis; 3rd molars undeveloped; P.B. flat wi
28·0	66°·1?	71°·8?	42°·1?	11°·9?	30°·2?	83°·7	Metopic; teeth worn; P.B. flat with sharp edge; flattened obelion.
41·4	69°·0	68°·7	42°·3	12°·3	30°·0	81°·0	Teeth much worn giving surface an inward-upward slope; P.B. flat with
47·0	64°·6?	72°·7?	42°·7?	11°·8?	30°·9?	84°·5?	P.B. flat with sharp edge.
40·0	67°·2	67°·6	45°·2	16°·8	28°·4	84°·4	Young adult; P.B. flat with rounded edge; sub-nasal prognathism; wor L.; suggestion of same on R.
28·0	66°·2?	72°·4?	41°·4?	14°·6?	26°·8?	87°·0	Alveolus absorbed except about R. incisors; P.B. flat with blunt edge; plate perforated; glenoid cavity on L. flattened out by disease; long n in by a ridge.
29·2	62°·7	70°·5	46°·8	15°·5	31°·3	86°·0	Aged; sutures partly obliterated; teeth worn; P.B. flat with sharp edge; 3rd condyle formed by fusion of 2 pre-condyles.
41·4	63°·5	74°·5	42°·0	12°·0	30°·0	86°·5	R. 3rd molar diseased and much inflammation of maxilla above this; squamo-mastoid angles; very marked malar marginal processes; os ja
41·5	68°·6?	73°·4?	38°·0?	12°·6?	25°·4?	86°·0?	Senile; sutures becoming obliterated; alveolus almost all absorbed; P.B.
46·3	67°·0	76°·4	36°·6	12°·6	24°·0	89°·0	3rd molars coming through; P.B. flat with sharp edge; flattened obelio R. and L.; inner L. palate bridge.
33·3	72°·6	65°·8	41°·6	14°·5	27°·1	80°·3	P.B. flat with blunt edge; flattened obelion; wormians L. lambdoid; pro
61·2	65°·1?	72°·9?	42°·0?	13°·6?	28°·4?	86°·5	Senile; sutures closing; double parietal thinning and depression especiall maxillary on L. and of anterior part on R.; hole into maxillary omtr and upward slope and rounded edge.
—	65°·8?	73°·0?	41°·2?	13°·5?	27°·7?	86°·5?	Young adult; basilar synchondrosis; sutures open; marked R. occipital b sub-nasal prognathism; fronto-squamous process on R. and L.
—	66°·9	73°·1	40°·0	11°·9	28°·1	85°·0	Very bulging occiput; P.B. flat with blunt edge; sub-nasal prognathism asterion; upper part of right squamous divided off by transverse irregular suture; perforation of tympanic plate both sides; small L.
32·2	67°·0	75°·2	37°·8	6°·4?	31°·4?	81°·6?	Old; sagittal and part of coronal nearly obliterated; teeth much worn; during life to right side of frontal.
46·2	64°·9	71°·7	43°·4	15°·3	28°·1	87°·0	Sutures partly obliterated; teeth, especially incisors, much worn, surfa with sharp edge; ossicles R. and L. lambdoid; epipterics R. and L.;
57·5	56°·0	86°·0	38°·0	3°·5	34°·5	89°·5	Young adult; 3rd L. molar about to be cut; 3rd R. molar undeveloped;
45·3	60°·5	75°·5	44°·0	13°·0	31°·0	88°·5	P.B. flat with sharp edge; ossicle L. lambdoid; epipteric on R.; L. tymp
47·4	62°·2	81°·6	36°·2	6°·6	29°·6	88°·2	Bulging occiput; large wormians at lambda; P.B. flat with sharp edge;
49·4	62°·6	72°·8	44°·6	15°·2	29°·4	88°·0	Old; sagittal partly obliterated; surface of teeth much worn with inw pre-condyles R. and L.
44·6	61°·9	77°·8	40°·3	10°·7	29°·6	88°·5	P.B. flat with blunt edge; traces R. and L. of suture across occipital fr
—	66°·1	78°·1	35°·8	8°·1	27°·7	86°·2	Ageing; sagittal suture partly obliterated; 3rd molars uncut; P.B. flat
41·3	66°·2	69°·5	44°·3	15°·5	28°·8	85°·0	P.B. flat with rounded edge; ossicle in L. lambdoid.
65·3	68°·3	75°·8	35°·9	4°·6	31°·3	80°·4	P.B. flat with slightly rounded edge; inter-parietal os triangulare on R.
21·5	64°·5?	71°·1?	44°·4?	15°·1?	29°·3?	86°·2?	Old; sutures nearly obliterated; teeth much worn; P.B. flat with roun forated R. and L ; flattening at lambda.
—	—	—	—	—	—	—	Ageing; sutures becoming obliterated; flattened obelion.
77·1	68°·7	71°·9	39°·4	14°·1	25°·3	86°·0	3rd R. molar undeveloped; P.B. flat with sharp edge; ossification of R. and L. lambdoid.
41·0	65°·0	78°·0	37°·0	12°·0	25°·0	90°·0	R. parieto-temporal suture obliterated, other sutures have hardly begu teeth worn; 3rd molars single-fanged; P.B. with outward-upward sl behind L. post-condyloid foramen (due to disease?).
—	—	—	—	—	—	—	Depressed obelion; wormians R. and L. lambdoid; ossicle R. squamo-m

38·2	66°·9	69°·9	43°·2	13°·6?	29°·6?	83°·5?	First permanent molars coming through; basilar synchondrosis; sutur
27·2	66°·8	76°·4	36°·8	14°·8	22°·0	91°·2	2nd permanent L. incisor coming through; basilar synchondrosis; P.B asterion; sutures open.
—	—	—	—	—	—	—	Sutures open; basilar synchondrosis.
—	—	—	—	—	—	—	Sutures open; flattened lambdoid region, and obelion; large perforatio
25·0	67°·7?	71°·8?	40°·5?	16·2?°	24°·3?	88°·0?	Sutures open; basilar synchondrosis; 1st R. permanent incisor comi starting from R. and L. asterion; perforated tympanic plates R. a
35·2	67°·2	74°·1	38°·7	15°·7	23°·0	89°·8	Sutures open; basilar synchondrosis; 1st permanent molars coming th lambdoid; tympanic plates perforated R. and L.
—	61°·6	77°·6	40°·8	16°·2	24°·6	93°·8	Sutures open; basilar synchondrosis; cutting bi-cuspid; slig t torus p

* See Section

[illegible] palate bridge; sloping forehead; P.B. flattish with sharp edge; flattened
o form bridge; symmetrical anterior parietal flattening.
in absorbed except at L. canine; P.B. flat with blunt double edge; large wormian
late perforated; healed fracture of L. superciliary ridge and L. zygomatic arch;

mmatory condition about L. foramen ovale of sphenoid and petrous portion of
philis?); foramen magnum cut; asymmetric palate.

harp edge; sub-nasal prognathism.
ped; P.B. flat with blunt edge; wormian at R. squamo-mastoid angle.
d obelion.
e; P.B. flat with sharp edge; face asymmetrical; cranium very asymmetrical.

rognathism; wormians R. and L. lambdoid; epipteric on R.; os japonicum on

with blunt edge; epipteric on L.; fronto-squamous process on R.; R. tympanic
y disease; long narrow foramen magnum with deep depression at fore-end walled

with sharp edge; very asymmetrical; large ossicle in L. squamo-mastoid angle;

illa above this; metopic ridge; P.B. flat with sharp edge; ossicle in R. and L.
al processes; os japonicum on L.; horizontal malar suture nearly halfway on R.
ll absorbed; P.B. flat with blunt edge; perforated tympanic plate on L.
e; flattened obelion; ossicle R. lambdoid; beginning of horizontal malar suture

L. lambdoid; prognathous; perforated tympanic plate on L.

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epression especially on R.; arthritic condition of palate, of posterior part of upper
o maxillary omtrum on R. near 2nd molar, due to abscess; P.B. has outward

ked R. occipital bulge associated with R. os triangulare; P.B. flat with blunt edge;
R. and L.
nasal prognathism; flattened obelion; small wormians R. and L. lambdoid near
ff by transverse suture; frontal process to upper half, meeting frontal bone in
sides; small L. pre-condyle.
eeth much worn; arthritic condition of palate; P.B. flat with sharp edge; injury

much worn, surface having an inward-upward slant; receding forehead; P.B. flat
terics R. and L.; septic condition of teeth; enamel worn off outer surface.
olar undeveloped; P.B. flat with sharp edge; small epipteric on L.
ic on R.; L. tympanic plate perforated.
with sharp edge; flattened obelion.
h worn with inward-upward slant; P.B. flat with sharp edge; flattened obelion;

across occipital from asterion.
uncut; P.B. flat with sharp edge; epipterics R. and L.

triangulare on R.; ossicles in R. lambdoid.
B. flat with rounded edge; fronto-squamous process on R.; tympanic plate per-

on.
e; ossification of trochlea of superior oblique muscle R. and L. orbit; wormians

have hardly begun to disappear; sagittal depressed, and very complex throughout;
tward-upward slope and very sharp edge; R. tympanic plate perforated; opening
).
icle R. squamo-mastoid angle; R. and L. tympanic plates perforated.

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chondrosis; sutures open; P.B. rounded; small wormians in lambdoid R. and L.
nchondrosis; P.B. flat with sharp edge; ossicles R. and L. lambdoid and R. and L.

; large perforations in tympanic plates R. and L.
nent incisor coming through; P.B. rounded; beginning of sutures across occiput
panic plates R. and L.
molars coming through R. and L.; P.B. flat with slightly rounded edge; ossicle R.

d; slig t torus palatinus; P.B. flat with rounded edge; wormians R. lambdoid.

* See Section 9 of Text.

AN EXPERIMENTAL DETERMINATION OF THE PROBABLE ERROR OF DR SPEARMAN'S CORRELATION COEFFICIENTS.

[Being the paper read to the Society of Biometricians and Mathematical Statisticians, December 13th, 1920.]

By "STUDENT."

In the *British Journal of Psychology*, Vol. II. p. 96, 1906*, Dr Spearman suggested two methods of determining correlation, based on replacing actual measurements by ranks.

As an illustration we may take the following purely imaginary example:

TABLE I.

Individual	Height	Length of middle finger mm.	Rank in height	Rank in length of finger
A	6′ 0″	12·8	2	1
B	5′ 3″	11·5	4	3
C	5′ 7″	10·0	3	4
D	6′ 1″	12·4	1	2

Instead of correlating the figures in the second and third columns of the above table Dr Spearman proposed to use the figures in the fourth and fifth columns, and to determine one or other of two coefficients: of these the first (ρ) gives the ordinary correlation coefficient between the figures representing the ranks, and the second (R) was described as a 'footrule' for correlation, i.e. a rough instrument which could be used by the unskilled. Dr Spearman also proposed to use R in cases where it was thought advisable to weight mediocre observations more heavily than extremes.

The method of determining ρ and R was to take the difference D between the numbers representing the ranks, e.g. for A in Table I

$$D = 2 - 1 = 1.$$

* [Dr Spearman's results were first given in a paper entitled "The Proof and Measurement of Association between Two Things" in the *American Journal of Psychology*, Vol. XV. pp. 72—101, 1904. The dogmatic statements as to the accuracy of his methods in that paper are, I think, erroneous, and he does not lay adequate stress on the fact that correlation of ranks is not a correlation of variates and may differ very considerably from it. The suggestion of considering the correlation of ranks is due to A. Binet and V. Henri: see *La Fatigue Intellectuelle*, Paris, 1898, p. 252, also *L'Année Psychologique*, T. IV. p. 155, Paris, 1898. Their process is very obscure and they also do not appear to have realised that the correlation of variates is not that of ranks. Ed.]

Then $$\rho = 1 - \frac{S(D^2)}{\frac{n(n^2-1)}{6}} \quad \text{.....(i),}$$

and $$R = 1 - \frac{S(D)}{\frac{n^2-1}{6}} \quad \text{.....(ii),}$$

where n is the number in the sample: in the case of R, $S(D)$ denotes the summation of positive differences only.

Dr Spearman gave an empirical formula connecting R and ρ, viz. $\rho = \sin\left(\frac{\pi}{2}R\right)$, but I do not suppose that he attached any very great importance to this.

He further gave the probable errors of ρ and R for the case of no correlation as $\frac{\cdot 6745}{\sqrt{n}}$ and $\frac{\cdot 4266}{\sqrt{n}}$

In his memoir 'On further methods of determining correlation' Prof. Pearson investigated these coefficients for the case of the normal correlation surface and found the relations between ρ and R and r the ordinary correlation coefficient to be

$$r = 2\sin\left(\frac{\pi}{6}\rho\right) \quad \text{.....(iii),}$$

and $$r = 2\cos\frac{\pi}{3}(1-R) - 1 \quad \text{..... .....(iv)}$$

Pearson further found the standard error of ρ to be for large samples

$$\frac{1-\rho^2}{\sqrt{n}}\{1 + \cdot 086\rho^2 + \cdot 013\rho^4 + \cdot 002\rho^6 + \ldots\} \quad \text{.....(v),}$$

and of r_ρ, i.e. r determined from ρ by (iii), to be

$$1\cdot 0472\frac{1-r^2}{\sqrt{n}}\{1 + \cdot 042r^2 + \cdot 008r^4 + \cdot 002r^6 + \ldots\} \quad \text{.....(vi).}$$

He did not succeed in evaluating the error of R or of r_R (i.e. of r determined by (iv)), but pointed out that just as in the case of r the $\sqrt{n}$ in the denominator is really $\sqrt{n-1}$. He also pointed out that R can only take values between $+1$ and $-\cdot 5$ and that Spearman's $\frac{\cdot 4226}{\sqrt{n-1}}$ does not imply that R is more accurate than ρ or r with their probable error of $\frac{\cdot 6745}{\sqrt{n-1}}$ since R itself is smaller than ρ or r in about the same proportion.

Since that time the use of ρ and R has become general among psychologists, especially in America, where they are preferred to r on account of the ease and speed with which they can be determined for small samples.

For example in a note on correlation in *Employment Psychology*, by H. C. Link*, a book written to urge the claims of Psychology on the devotees of 'Scien-

* Macmillan, 1919.

tific Management,' the author mentioned three methods of determining correlation, ρ which is to be used for samples smaller than 30, R for samples over 30 and r which though acknowledged to be rather more accurate is not to be used at all since it takes four times as long to calculate as the others.

Now to save time at the expense of accuracy is justifiable when, and only when, the time saved can be devoted to increasing the number of observations so as to obtain greater accuracy on the whole series, otherwise it will take longer to get equally trustworthy conclusions, and it seems to be of interest to investigate the probable errors of ρ and R for samples of the size that the employment psychologist is contemplating. And here we may note that the saving of time only occurs when the sample is comparatively small; as it increases, the labour of grading becomes more and more severe till at some point in the neighbourhood of 40 it becomes quicker to use the ordinary product moment r if that be possible.

It should perhaps be pointed out that there are many cases where it is possible to grade a sample for some character which is not capable of being measured on a scale and it might be thought that in this case large samples could profitably be dealt with by the ρ or R method, but in fact it is just these scaleless characters which present the greatest difficulty in grading.

We have then to consider the variability of ρ and R and of the derivatives r_ρ and r_R, determined from small samples, and it seemed worth while to use the material of a former sampling experiment so as to get an idea of how small samples depart from the results obtained by Prof. Pearson for ideally large samples. The material in question consists of 750 samples of four drawn from a population of 3000 criminals whose height and left middle finger length give an approximately normal correlation surface with correlation ·66.

These are capable of being combined easily to give 375 samples of eight and in addition there are 100 samples of 30, which may be taken to be a size of sample which is no longer quite 'small.'

Accounts of the former results were given in *Biometrika*, VI. p. 1 and p. 302, since which time the frequency distributions of the correlation coefficients of small samples drawn from normally correlated populations have been very thoroughly investigated by Soper, Fisher and the authors of the cooperative paper in Vol. XI. p. 328 of *Biometrika*: it is hoped that some mathematician may be interested in the general solution of the problems raised in the present paper which may then afford material for checking his results.

When I came to apply the methods to my samples I found that owing to the rather coarse grouping, there were a large number of ties, so that it became necessary to find out the right correction for ties.

Prof. Pearson had discussed the question of ties and had suggested two ways of dealing with them. One way was to rank them all as if they were the highest number of the tie which he called the bracket-rank method and the other was to rank them all half-way down the tie which he called the mid-rank method. Thus

the first way would rank 1, 2, 2, 4 while the second would rank 1, $2\frac{1}{2}$, $2\frac{1}{2}$, 4 if the second and third of four individuals constituted a tie.

Now the first would give different results according as we read the scale forwards or backwards and also alter the mean of the set of numbers so I have only tried to use the mid-rank method, for which I have found the correction which follows.

Correction of ρ for ties.

If $D = x - y$, when x and y are any two variables measured from their means, then

$$D^2 = x^2 + y^2 - 2xy.$$

Summing for all n samples and dividing by n

$$\frac{\Sigma(D^2)}{n} = \sigma_x^2 + \sigma_y^2 + 2r_{xy}\sigma_x\sigma_y,$$

$$\therefore\ r_{xy} = \left(\sigma_x^2 + \sigma_y^2 - \frac{\Sigma(D^2)}{n}\right) \Big/ 2\sigma_x\sigma_y \quad \text{.....(vii).}$$

If now x and y are the first n numbers, then

$$\sigma_x^2 = \sigma_y^2 = \frac{1}{n} \times \text{sum of squares of 1st } n \text{ numbers} - \left(\frac{\text{sum of 1st } n \text{ numbers}}{n}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{n^2-1}{12} \quad \text{.....(viii).}$$

Substituting in (vii) we find

$$\rho = r_{xy} = \left(\frac{n^2-1}{6} - \frac{\Sigma(D^2)}{n}\right) \Big/ \frac{n^2-1}{6}$$

$$= \frac{\frac{n(n^2-1)}{6} - \Sigma(D^2)}{\frac{n(n^2-1)}{6}} \quad \text{.....(ix).}$$

Now suppose that there is on the x side a tie of t in number from q to $q+t-1$. Using the mid-rank method we substitute for each of the numbers

$$q,\ q+1,\ \ldots\ q+t-1 \text{ their mean } \frac{2q+t-1}{2}$$

Hence in finding σ_x^2 the mean is unaltered but in the sum of the squares $q^2 + (q+1)^2 + \ldots + (q+t-1)^2$ is replaced by $\frac{t(2q+t-1)^2}{4}$.

Hence σ_x^2 is smaller by

$$\frac{1}{n}\left\{q^2 + (q+1)^2 + \ldots + (q+t-1)^2 - \frac{t(2q+t-1)^2}{4}\right\}$$

This is equal to

$$\frac{1}{n}\left\{tq^2+2q\{1+2+\ldots+(t-1)\}+\{1^2+2^2+\ldots+(t-1)^2\}\right.$$
$$\left.-tq^2-qt(t-1)-\frac{t(t-1)^2}{4}\right\}$$
$$=\frac{1}{n}\left\{\frac{(t-1)t(2t-1)}{6}-\frac{t(t-1)^2}{4}\right\}$$
$$=\frac{t(t^2-1)}{12n},$$
$$\therefore\ \sigma_x^2=\frac{n^2-1}{12}-\frac{t(t^2-1)}{12n}$$

This is clearly additive for any number of ties so that if $T_x=\Sigma\left(\frac{t(t^2-1)}{12}\right)$ summing for all the ties on the x side and similarly T_y for the y side

$$\sigma_x^2=\frac{n^2-1}{12}-\frac{T_x}{n}\text{ and }\sigma_y^2=\frac{n^2-1}{12}-\frac{T_y}{n},$$

and substituting in (vii)

$$\rho=r_{xy}=\frac{\left\{\frac{n^2-1}{6}-\frac{1}{n}(T_x+T_y)-\frac{\Sigma(D^2)}{n}\right\}}{\sqrt{\left(\frac{n^2-1}{6}-\frac{2T_x}{n}\right)\left(\frac{n^2-1}{6}-\frac{2T_y}{n}\right)}}$$
$$=\frac{\frac{n(n^2-1)}{6}-(T_x+T_y)-\Sigma(D^2)}{\sqrt{\left(\frac{n(n^2-1)}{6}-2T_x\right)\left(\frac{n(n^2-1)}{6}-2T_y\right)}}\quad\ldots\ldots\ldots\ldots\ldots\ldots\text{(x)},$$
$$=\frac{\frac{n(n^2-1)}{6}-(T_x+T_y)-\Sigma(D^2)}{\left\{\frac{n(n^2-1)}{6}-(T_x+T_y)\right\}\sqrt{1-\frac{(T_x-T_y)^2}{\left\{\frac{n(n^2-1)}{6}-(T_x+T_y)\right\}^2}}}$$

So that if T_x and T_y do not differ appreciably

$$\rho=1-\frac{\Sigma(D^2)}{\frac{n(n^2-1)}{6}-(T_x+T_y)}\quad\ldots\ldots\ldots\ldots\ldots\ldots\text{(xi)}.$$

In estimating T_x or T_y each pair contributes $\frac{1}{2}$,

triplet „ 2,
quartet „ 5,
quintet „ 10,

and so on. For example, if the x ranks for a sample of 10 were

1, $2\frac{1}{2}$, $2\frac{1}{2}$, 5, 5, 5, $8\frac{1}{2}$, $8\frac{1}{2}$, $8\frac{1}{2}$, $8\frac{1}{2}$,

T_x would be $\frac{1}{2}+2+5=7\frac{1}{2}$ and if there were no ties in the y ranks ρ would be

$$\frac{165-7\frac{1}{2}-S(D^2)}{\sqrt{(165-15)\,165}}=\frac{157\frac{1}{2}-S(D^2)}{\sqrt{150\,.\,165}},$$

and if we were to take it as $1-\frac{S(D^2)}{157\frac{1}{2}}$ the error would come in the third significant place of decimals.

In determining ρ for my 375 samples of eight I found that much tied samples usually gave low values of ρ and it occurred to me that although undoubtedly equation (x) gives the true value of the correlation of ranks, yet it might be that the loss of precision due to ties would give low values for the correlation. To test this I doubled the width of my unit of grouping first for one variable and then for the other so that I got three values of ρ for each sample:

(i) Converting the original figures into ranks,

(ii) Using coarser grouping on one side and the original grouping on the other before converting into ranks,

(iii) Using coarser grouping on both sides.

An example will make my meaning clearer.

(1) Original figures		(2) x grouped coarsely		(3) Both grouped coarsely		Ranks					
						(1)		(2)		(3)	
x	y	Putting +1 and 0 as $\frac{1}{2}$, &c.	y	x	y	x	y	x	y	x	y
0	+3	$+\frac{1}{2}$	+3	$+\frac{1}{2}$	$+2\frac{1}{2}$	$5\frac{1}{2}$	3	$4\frac{1}{2}$	3	$4\frac{1}{2}$	$3\frac{1}{2}$
−2	0	$-1\frac{1}{2}$	0	$-1\frac{1}{2}$	$+\frac{1}{2}$	8	7	$7\frac{1}{2}$	7	$7\frac{1}{2}$	$6\frac{1}{2}$
+3	+3	$+1\frac{1}{2}$	+3	$+2\frac{1}{2}$	$+2\frac{1}{2}$	$1\frac{1}{2}$	3	$1\frac{1}{2}$	3	$1\frac{1}{2}$	$3\frac{1}{2}$
−1	−2	$-1\frac{1}{2}$	−2	$-1\frac{1}{2}$	$-1\frac{1}{2}$	7	8	$7\frac{1}{2}$	8	$7\frac{1}{2}$	8
+1	+3	$+\frac{1}{2}$	+3	$+\frac{1}{2}$	$+2\frac{1}{2}$	$3\frac{1}{2}$	3	$4\frac{1}{2}$	3	$4\frac{1}{2}$	$3\frac{1}{2}$
+1	+2	$+\frac{1}{2}$	+2	$+\frac{1}{2}$	$+2\frac{1}{2}$	$3\frac{1}{2}$	5	$4\frac{1}{2}$	5	$4\frac{1}{2}$	$3\frac{1}{2}$
+3	+4	$+1\frac{1}{2}$	+4	$+2\frac{1}{2}$	$+4\frac{1}{2}$	$1\frac{1}{2}$	1	$1\frac{1}{2}$	1	$1\frac{1}{2}$	1
0	+1	$+\frac{1}{2}$	+1	$+\frac{1}{2}$	$+\frac{1}{2}$	$5\frac{1}{2}$	6	$4\frac{1}{2}$	6	$4\frac{1}{2}$	$6\frac{1}{2}$
			Pairs ...	...		3	—	2	—	2	1
			Triplets	...		—	1	—	1	—	—
			Quartets	...		—	—	1	—	1	1

Here originally $T_x=1\frac{1}{2}$ and $T_y=2$ and $\frac{n(n^2-1)}{6}-(T_x+T_y)=80\frac{1}{2}$.

After grouping x coarsely $T_x=6$ and $T_y=2$ " $=76$.

After grouping both coarsely $T_x=6$ and $T_y=5\frac{1}{2}$ " $=72\frac{1}{2}$,

and ρ will be found to take the values ·833, ·869 and ·834 in succession. Working in this way I obtained three values of ρ for each of the 375 samples and determined the mean, standard deviation and mean (T_x+T_y) for each of the three series of 375. These results are given in Table II.

TABLE II.

	Mean ρ	σ_ρ	Mean T_x	Mean T_y	Mean (T_x+T_y)
Original series	·5798	·2887	1·92	1·90	3·82
x grouped coarsely ...	·5798	·2903	4·67	1·90	6·57
x and y grouped coarsely	·5696	·2874	4·67	4·37	9·04

Here an increase in the correction to be made for ties from 3·82 to 9·04 has made a difference of ·01 in the mean value of ρ, the probable error being about ·015, and a still less appreciable difference in the Standard Deviation. It is, I think, a fair inference that the correction is applicable to the series in question, and the reason for the observed low values of ρ in much tied samples is to be sought elsewhere*. But it will be asked 'what if no correction be made for ties?' The answer is that the mean value of ρ will rise as the ties become more numerous and the S.D. will fall. Thus Table II would become Table III if no corrections were made.

TABLE III.

	Mean ρ	σ_ρ	Mean (T_x+T_y)
Original series	·602	·2677	3·82
x grouped coarsely ...	·616	·2887	6·57
x and y grouped coarsely	·622	·2414	9·04

At first sight this may appear to be highly advantageous since the mean value approximates more nearly to the value which would be obtained from a large sample and the S.D. is smaller. A little reflection will show however that the means of the ρ's of all populations would be subject to the same rise and that in fact the ρ of one population is no more differentiated from the ρ of another population than it is when corrected, while the mean value when corrected is constant over a fairly wide range of ties. If the correction is not made ρ can be cooked up to any required value by increasing the ties.

The fact is that as soon as there is a single tie, uncorrected ρ can no longer take all values between $+1$ and -1 and if one of the scales be reversed the correlation instead of being $-\rho$ becomes $-\rho+\dfrac{(T_x+T_y)}{\dfrac{n(n^2-1)}{6}}$. We are therefore forced to use the correction which after all gives us the distribution of ρ that we should get from ideal material containing no ties.

* The low value of ρ for much tied samples is due to the fact that a much tied sample is as a rule one in which the S.D. of the original variables is low.

Now as a matter of experience I find that of samples drawn from a normally distributed population those with S.D. above the average tend to give high and stable values of the correlation coefficient, while those with S.D. below the average tend to give low and variable values.

The form of the correlation surface for variables σ_x and r_{xy} is of considerable interest to those who have to deal with small samples and merits the attention of mathematicians. I hope to deal with the experience obtained from my samples at some later time.

To see what happens when ties are carried to an extreme I determined ρ from the original table of 3000 entries (*Biometrika*, I. p. 216) and from the same table condensed to six groups each way by using a 4″ scale of height and ·8 mm. scale of finger lengths.

In the first case $\rho = ·637$ giving r_ρ ·655 and in the second ·557 with r_ρ ·575. There seems therefore in extreme cases to be a tendency for the correction to give too low a value of ρ.

Correction of R for grouping.

In Dr Spearman's original paper R is defined as $1 - \dfrac{S(D)}{\dfrac{n^2-1}{6}}$ when $\dfrac{n^2-1}{6}$ is taken as the average value which $S(D)$ assumes.

The simplest way to see that this is the average value is to write down all the possible D's thus:

1	1					
2	2	1				
3	3	2	1			
4	4	3	2	1		
⋮	⋮	⋮	⋮	⋮		
$(n-1)$	$(n-1)$	$(n-2)$	$(n-3)$	$(n-4)$	1	
n	n	$(n-1)$	$(n-2)$	$(n-3)$	... 2	1

Here the two columns on the left are composed of the first n numbers. The third column is formed by subtracting the top number of the first column from all the numbers in the second column in turn, the fourth by subtracting the second number from all numbers which give a positive remainder and so on.

Thus the numbers in the second column could be arranged opposite the numbers of the first column in $n!$ ways.

And in $(n-1)!$ of these arrangements any given pair will occur.

Hence the average value of $S(D)$ will be ·

$$\frac{(n-1)!}{n!}\{[1+2+\ldots+(n-1)]+[1+2+\ldots+(n-2)]+\ldots+(1+2)+1\},$$

$$\therefore \text{ average value of } S(D) = \frac{1}{n}\left\{\frac{n(n-1)}{2}+\frac{(n-1)(n-2)}{2}+\ldots+\frac{2.3}{2}+\frac{1.2}{2}\right\}$$

$$= \frac{1}{2n}S_1^n(n^2-n) = \frac{1}{2n}\left\{\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}\right\}$$

$$= \frac{(n+1)}{12}\{2n+1-3\} = \frac{n^2-1}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(xii)}.$$

If we now substitute in the second column ties instead of consecutive numbers we can find out what effect ties will have on the average value of $S(D)$. As I can

see no general way of proving the results I propose merely to state my results as follows:

(1) A tie of t on one side which is opposed by no ties on the other side will diminish $\frac{n^2-1}{6}$ by $\frac{t(t^2-1)}{24n}$ if t be odd and by $\frac{t(t^2-4)}{24n}$ if t be even.

(2) Overlapping ties on opposite sides interfere with the above simple rule, the total to be subtracted from $\frac{n^2-1}{6}$ being increased or decreased according to Table IV.

TABLE IV.

		Distance between centres of ties											
x tie	y tie	0	½	1	1½	2	2½	3	3½	4	4½	5	5½	6
2	2	+ 1		0→										
3	2		0→											
3	3	+ 2		−1		0→								
4	2	+ 2		0→										
4	3		+ 1		−1		0→							
4	4	+ 6		0		−1		0→						
5	2		0→											
5	3	+ 4		−1		−1		0→						
5	4		+ 3		−2		−1		0→					
5	5	+10		0		−4		− 1		0→				
6	2	+ 3		0→										
6	3		+ 2		−1		−1		0→					
6	4	+10		+1		−2		− 1		0→				
6	5		+ 7		−2		−4		−1		0→			
6	6	+19		+4		−4		− 4		−1		0→		
7	2		0→											
7	3	+ 6		−1		−1		− 1		0→				
7	4		+ 5		−2		−2		−1		0→			
7	5	+16		+2		−5		− 4		−1		0→		
7	6		+13		−1		−7		−4		−1		0→	
7	7	+28		+7		−6		−10		−4		−1		0→

etc., etc.

As an example of the use of Table IV, suppose a set of eight ranks to contain on the x side a tie of 5 centred at 3, i.e. let the x ranks be 3, 3, 3, 3, 3, 6, 7, 8, and let the y ranks have a tie of 4 centred at 2½, i.e. let the y ranks be 2½, 2½, 2½, 2½, 5, 6, 7, 8. Then the amount to be subtracted from $\frac{8^2-1}{6}$ is firstly $\frac{5}{8}$ (for the 5 tie) $+\frac{3}{8}$ (for the 4 tie) $+\frac{3}{8}$ (from Table IV) $= 1\frac{1}{4}$. Had the y ranks been 1, 3½, 3½, 3½, 3½, 6, 7, 8, the correction would be the same, but if the y ranks were 1, 2, 4½, 4½, 4½, 4½, 7, 8, the correction would be $\frac{5}{8}+\frac{3}{8}-\frac{3}{8}=\frac{5}{8}$,

and if 1, 2, 3, 5½, 5½, 5½, 5½, 8 $\quad \frac{5}{8}+\frac{3}{8}-\frac{1}{8}=\frac{3}{4}$,

and if 1, 2, 3, 4, 6½, 6½, 6½, 6½ $\quad \frac{5}{8}+\frac{3}{8}=\frac{7}{8}$.

It is only with very small and much tied samples that the correction is appreciable.

GRAPH FROM TABLE V.

Giving Frequency Distributions of various Correlation Coefficients from 375 samples of 8

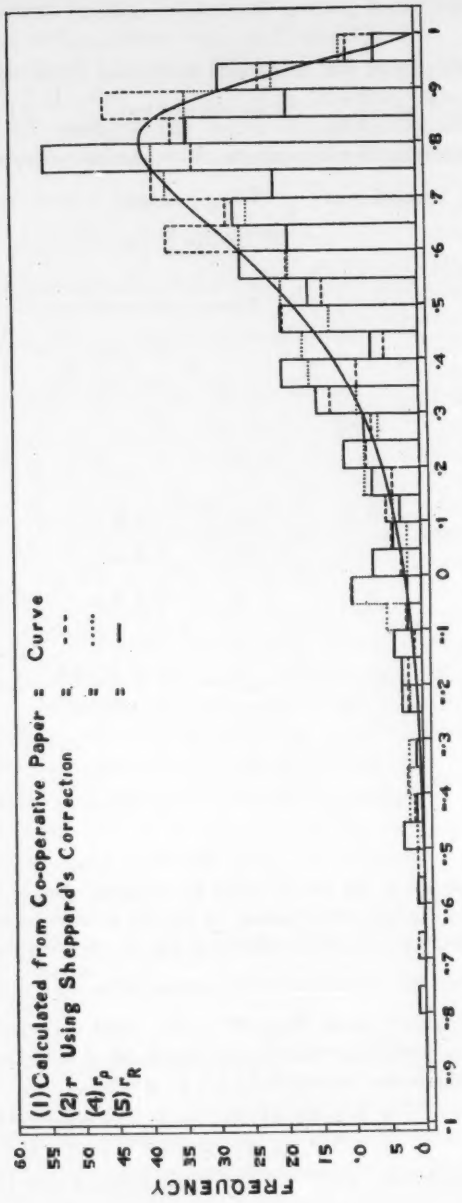
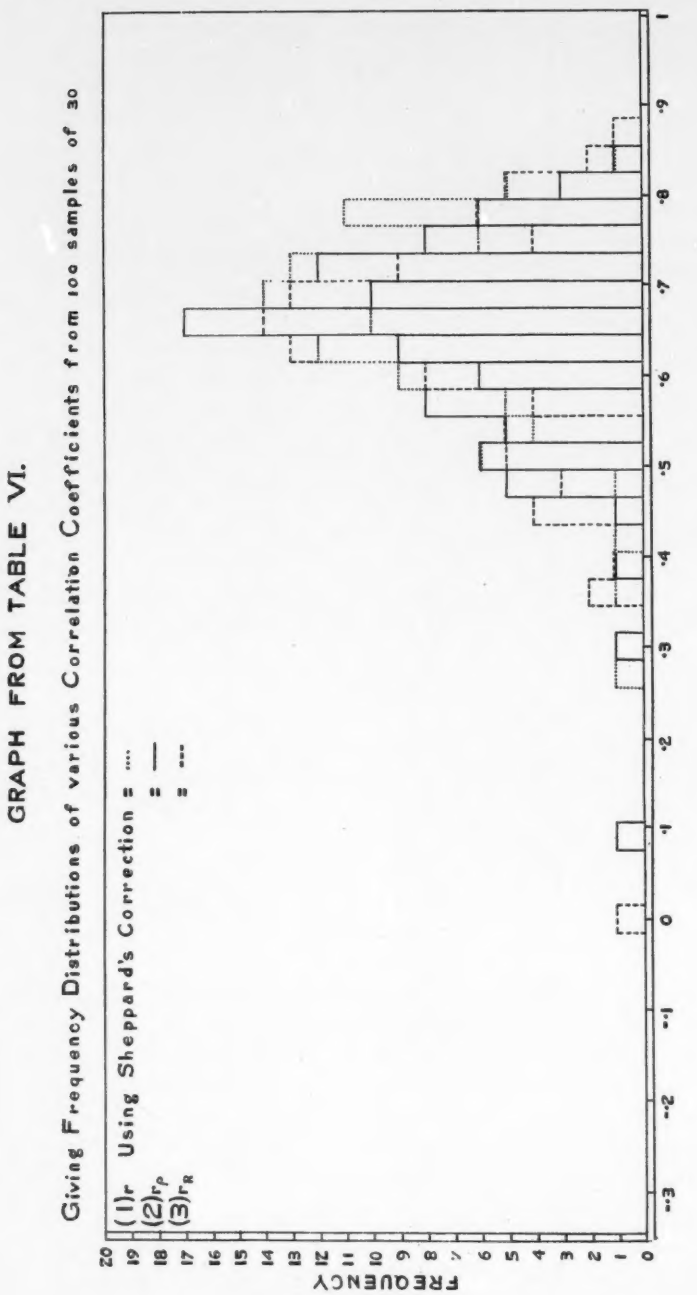


TABLE V. *Giving Frequency Distributions of Various Correlation Coefficients from* 375 *samples of* 8.

	−·8005 to −·7505	−·7505 to −·7005	−·7005 to −·6505	−·6505 to −·6005	−·6005 to −·5505	−·5505 to −·5005	−·5005 to −·4505	−·4505 to −·4005	−·4005 to −·3505	−·3505 to −·3005	−·3005 to −·2505	−·2505 to −·2005	−·2005 to −·1505	−·1505 to −·1005	−·1005 to −·0505	−·0505 to −·0005	−·0005 to ·0495	·0495 to ·0995	·0995 to ·1495	·1495 to ·1995	·1995 to ·2495	·2495 to ·2995	·2995 to ·3495	·3495 to ·3995	·3995 to ·4495	·4495 to ·4995	·4995 to ·5495	·5495 to ·5995	·5995 to ·6495	·6495 to ·6995	·6995 to ·7495	·7495 to ·7995	·7995 to ·8495	·8495 to ·8995	·8995 to ·9495	·9495 to ·9995
(1) Calculated from the cooperative paper	—	—	·1	·1	·1	·2	·3	·3	·4	·5	·7	·9	1·1	1·4	1·7	2·2	2·7	3·3	4·0	5·0	6·1	7·5	9·2	11·2	13·7	16·6	20·0	23·7	28·2	32·8	37·0	40·3	40·5	35·3	22·1	5·9
(2) r actual using Sheppard's correction	—	—	1	0	0	1	0	1	2	0	0	2	2	1	2	0	3	4	5	4	8	7	13	9	5	20	14	19	37	28	39	33	36	46	23	10
(3) r actual using no corrections	—	—	—	1	0	1	0	0	2	1	0	1	3	1	2	0	3	4	5	5	7	7	13	11	9	18	15	22	39	28	38	31	39	40	22	7
(4) r_ρ actual	—	—	—	—	—	—	1	2	2	2	0	2	0	0	5	10	2	2	4	8	8	6	9	16	17	13	20	19	26	25	37	39	34	34	21	11
(5) r_R „	1	0	0	0	1	0	3	1	0	1	0	3	3	4	2	2	7	4	3	7	11	8	15	20	7	20	16	26	19	27	21	55	34	19	29	6
(6) ρ „	—	—	—	—	—	—	—	3	1	3	0	1	1	0	5	10	2	2	5	8	8	6	11	16	18	15	22	21	25	24	39	36	35	29	21	8
(7) R „	—	—	—	—	—	—	—	—	1	0	1	4	1	4	6	4	11	9	12	22	21	27	16	28	24	29	43	31	27	19	19	10	2	3	1	—

TABLE VI. *Giving Frequency Distributions of Various Correlation Coefficients from* 100 *samples of* 30.

	−·165 to −·135	−·135 to −·105	−·105 to −·075	−·075 to −·045	−·045 to −·015	−·015 to ·015	·015 to ·045	·045 to ·075	·075 to ·105	·105 to ·135	·135 to ·165	·165 to ·195	·195 to ·225	·225 to ·255	·255 to ·285	·285 to ·315	·315 to ·345	·345 to ·375	·375 to ·405	·405 to ·435	·435 to ·465	·465 to ·495	·495 to ·525	·525 to ·555	·555 to ·585	·585 to ·615	·615 to ·645	·645 to ·675	·675 to ·705	·705 to ·735	·735 to ·765	·765 to ·795	·795 to ·825	·825 to ·855	·855 to ·885	·885 to ·915
(1) r actual using Sheppard's correction	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	0	0	1	0	1	0	1	6	4	5	9	12	10	14	13	6	11	5	1	—	—
(2) r_ρ actual	—	—	—	—	—	—	—	—	1	0	0	0	0	0	0	1	0	0	1	0	1	5	6	5	8	6	9	17	10	12	8	6	3	1	—	—
(3) r_R „	—	—	—	—	—	1	0	0	0	0	0	0	0	0	0	0	0	2	1	0	4	3	5	5	4	8	13	14	13	9	4	6	5	2	1	—
(4) r actual by median fourfold division $=\cos\frac{\pi B}{A+b}$	1	0	0	0	0	0	0	0	1	1	0	0	1	1	0	1	3	0	0	3	7	0	10	5	5	5	9	8	6	9	5	1	9	6	1	2
(5) ρ actual	—	—	—	—	—	—	—	1	0	0	0	0	0	0	0	1	0	1	0	1	2	7	4	8	5	10	13	13	8	12	6	5	2	1	—	—
(6) R „	—	—	—	—	—	1	0	0	0	0	0	0	2	1	3	6	5	7	9	15	17	12	8	4	6	3	0	0	1	—	—	—	—	—	—	—


GRAPH FROM TABLE VI.
Giving Frequency Distributions of various Correlation Coefficients from 100 samples of 30
(1)r Using Sheppard's Correction = ...
(2)r_P = —
(3)r_R = ---
FREQUENCY
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-3
-2
-1
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1

Discussion of the Frequency Distributions obtained.

Tables V and VI give the frequency distributions of r, determined with Sheppard's corrections for grouping, of ρ and of R and their derivatives (from equations (iii) and (iv)) r_ρ and r_R.

In addition we have, in Table V, r determined without Sheppard's corrections and the theoretical distribution of r calculated from the table (*Biometrika*, XI. p. 384), of the cooperative paper by interpolating between $\rho = \cdot 65$ and $\rho = \cdot 70$, drawing the frequency curve and estimating the areas by counting the squares: it is probably not very accurate, but fairly close to the truth.

In Table VI is included r calculated from the fourfold table taken through the medians by Sheppard's formula $r = \cos \frac{\pi B}{A+B}$ where B is the frequency in the 'small' cells. This probably suffers a good deal from the coarse grouping which makes it necessary to divide the centre groups in an arbitrary manner.

The most remarkable thing about these tables is the very wide spread of all the distributions. There is of course nothing new in this, but I cannot help thinking that an examination of these tables may be beneficial for all who try to work with very small samples.

Besides this there is not very much to be found in these tables which is not seen to greater advantage in Tables VIII and IX of the means and S.D.'s, but as a matter of interest I have compared lines 2—4 of Table V with line 1 by the χ^2 test with the following results:

TABLE VII.

	25 groups		16 groups	
	χ^2	P	χ^2	P
r with Sheppard's corrections ...	30·10	·18	20·49	·17
r without Sheppard's corrections	20·43	·67	12·39	·66
r_ρ actual	60·25	·000,064	30·63	·01
r_R „	74·10	·000,002	55·65	(say) ·000,002

The 25 groups were the 24 groups on the right of the table and the tail, which includes all groups which are less than 1·0 in line 1: the 16 groups were taken so that no group in line 1 was less than 10.

With such a small sample as 375 the χ^2 test is only decisive for considerable departures from the theoretical and the regular excess over the theoretical for all groups less than ·40 avoids detection.

At the same time it is interesting to note that judged by the χ^2 test Sheppard's corrections do not seem to have improved the calculation of r.

TABLE VIII.

Certain Constants of the Frequency Distributions of Various Correlation Coefficients derived from 375 *Samples of* 8.

	Mean	S.D.	Coefficient of Variation	Number of samples required to give as great accuracy as 100 samples of (1)	Number of samples required to give as great accuracy as 100 samples of (2)
(1) r calculated from co-operative paper	·631	·250	39·6	100	—
(2) r actual using Sheppard's corrections	·624 ± ·010	·274 ± ·007	43·9 ± 1·3	120 ± 5·9	100
(3) r actual using no correction for grouping	·614 ± ·010	·271 ± ·007	44·1 ± 1·3	117 ± 5·8	98
(4) r_ρ actual	·586 ± ·010	·291 ± ·007	49·7 ± 1·5	135 ± 6·7	113
(5) r_R „	·566 ± ·011	·309 ± ·008	54·6 ± 1·7	153 ± 7·5	127
(6) ρ „	·580 ± ·011	·289 ± ·007	49·8 ± 1·5	—	—
(7) R „	·407 ± ·008	·237 ± ·006	58·2 ± 1·9	—	—

TABLE IX.

Certain Constants of the Frequency Distributions of Various Correlation Coefficients derived from 100 *Samples of* 30.

	Mean	S.D.	Coefficient of Variation	Number of samples required to give as great accuracy as 100 samples of (1)	Number of samples required to give as great accuracy as 100 samples of (2)
(1) r calculated from co-operative paper	·653	·109	16·7	100	—
(2) r actual using Sheppard's corrections	·661 ± ·007	·101 ± ·005	15·3 ± ·7	86 ± 8·2	100
(3) r_ρ actual	·639 ± ·008	·113 ± ·005	17·7 ± ·9	108 ± 10·3	125
(4) r_R „	·638 ± ·008	·122 ± ·006	19·1 ± ·9	125 ± 11·9	146
(5) r actual from median 4-fold division $= \cos \frac{\pi B}{A+B}$	·609 ± ·012	·183 ± ·009	30·1 ± 1·6	282 ± 25·1	328
(6) ρ actual	·624 ± ·008	·116 ± ·006	18·6 ± ·9	—	—
(7) R „	·428 ± ·007	·100 ± ·005	23·4 ± 1·2	—	—

Tables VIII and IX give the means, S.D.'s and coefficients of variation of the frequency distributions in Tables V and VI and in addition the calculated constants for the samples of 30.

As well as this I have calculated the number of samples which would be required to give as great accuracy by the less accurate methods as 100 samples determined (1) on the theoretical basis of normal correlation, and (2) on the actual samples by the product moment method using Sheppard's corrections.

The object of this is to get an idea of how much time must be saved in order to gain by using the rank methods. First however we may note in Table VIII the marked difference between the theoretical S.D. and that actually obtained by the product moment method.

I attribute this almost entirely to the grouping which was unfortunately rather coarse and which cannot be corrected by Sheppard's corrections in small samples. The slight divergency of the population from normal correlation may have helped to a very small extent but for the most part the excess in the lower values of r which cause the mean to be low and the S.D. to be high is due to those samples which have low S.D.'s and I incline to believe that if the grouping can be chosen so that the S.D.'s are not less than 3 the actual distribution of r will be found to be very close to the calculated for samples drawn from normally correlated material.

In Table IX on the other hand the actual has a higher mean and lower S.D. than the calculated, but as the differences are in each case less than twice the probable error, I think we may put them down to the error of random sampling which is of course large in such a small sample as 100.

Next we may note that Prof. Pearson's formulae, no doubt because they are correct for grades, do not enable us to correct rank correlations for small samples. The means of both r_ρ and r_R are too low for samples of eight, and for samples of 30 probably so.

As for the S.D.'s of ρ and r_ρ the values found are in the case of samples of eight much higher than those calculated from equations (v) and (vi) which are ·258 and ·243 respectively. The samples of 30 however give values which agree sufficiently well, for the calculated S.D. is in each case ·114, well within the P.E.

Line 5 in Table IX shows that as determined in this investigation Sheppard's median division formula gives a mean value of r well below the population value and a very high standard deviation *. While this is not unlikely to be the case for small samples the arbitrary division of the central groups makes it impossible to say that this is not due to the fact that we have only used an approximation to median division in this case.

The chief point of interest however in Tables VIII and IX lies in column 4 showing the number of samples which we must have to get the same accuracy by the various methods as that given by 100 samples in which r is determined with sufficiently fine grouping by the product moment method.

Column 5 is put in in case there are any who do not accept my explanation of the difference between the calculated and actual distribution of r namely that it is due to the coarse grouping. I have not been able to estimate the P.E.'s of the figures in column 5 as they are complicated by correlation between the numerator and denominator of the fractions from which the figures are calculated. They must however be larger than the P.E.'s in column 4.

* The S.D. calculated from the formula $\sigma_r = \frac{2\pi\sqrt{1-r^2}}{\sqrt{N}} \left\{\frac{ab}{N^2}\right\}^{\frac{1}{4}}$ is however rather higher, being ·191 if r be taken as ·66 and ·207 if r be taken as ·609. Miss Elderton kindly looked up this formula for me, but I cannot find that it has been published. [See, however, *Biometrika*, Vol. IX. p. 23. It is also involved in the early paper by Sheppard, *Phil. Trans.* Vol. 192, A, p. 147 etc. Ed.]

In any case there is a strong indication that with samples of eight the loss of accuracy due to the use of r_ρ instead of r will practically always more than counterbalance the gain of time in calculation. Either method is however so little to be depended upon for a single sample of very small size, except as the merest indication, that very little is lost by the use of r_ρ. If however a number of small samples can be averaged so as to obtain a coefficient of some value, the product moment method should be used when possible.

With samples of 30 the 8 °/₀ more samples required compares fairly with Prof. Pearson's 10 °/₀ more for large samples, but seeing that the particular sample of 100 gave too low a value for σ_r, the value of σ_{r_ρ} which must be correlated with it is likely to be low also and the 8 °/₀ may easily be 18 °/₀ or more.

In any case it would very seldom pay to have to collect 8 °/₀ more samples of 30 even if one could save 8 °/₀ of the time on samples of that size.

In both tables there is a considerable loss from the use of r_R instead of r_ρ, since from 13 °/₀ to 16 °/₀ more samples would be required of the former to give the same accuracy as the latter. The gain in calculation is not very appreciable since most of the time is spent in ranking the samples. Dr Spearman prefers R to ρ at times because less importance attaches to outlying samples, but as the extremes of small samples tend to be outliers even in normally correlated material owing to the phenomenon to which attention was drawn in Galton's Difference problem*, it seems to me that as much weight as possible should be given to them.

To Combine two Methods of Determination.

At an early stage in the investigation I hoped to be able to combine r and r_ρ to get a value less subject to error than either. Curiously enough Prof. Pearson in his editorial in the last number of this Journal gives the equations which I proposed to use for the purpose (p. 7 (29)).

As they are perfectly general I will state them in a slightly more general form.

If x and y be two estimates of any quantity obtained in different ways then a quantity z can always be found which will have a lower error than either of them, unless x and y are perfectly correlated.

Thus
$$z = \frac{\sigma_y^2 - r_{xy}\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2 - 2r_{xy}\sigma_x\sigma_y} . x + \frac{\sigma_x^2 - r_{xy}\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2 - 2r_{xy}\sigma_x\sigma_y} . y \quad \text{.....(xiii),}$$

and
$$\sigma_z^2 = \frac{\sigma_x^2\sigma_y^2(1 - r_{xy}^2)}{\sigma_x^2 + \sigma_y^2 - 2r_{xy}\sigma_x\sigma_y} \quad \text{.....(xiv).}$$

* *Biometrika*, Vol. I. pp. 385—399. In this connection it is of interest to note that the correlation surface of ranks is not an elliptical hill as is the normal correlation surface but two comparatively steep ridges joined by a saddle, the ridges having a skew section.

In the case of the samples of eight x may be taken as r without Sheppard's corrections and y as r_ρ, when we have

$$\sigma_x^2 = (\cdot 271)^2 = \cdot 073{,}441, \quad \sigma_x\sigma_y = \cdot 078{,}861,$$
$$\sigma_y^2 = (\cdot 291)^2 = \cdot 084{,}681,$$
$$r_{xy} = \cdot 885,$$

and hence from (xiii)

$$z = \cdot 804r + \cdot 196r_\rho,$$

and

$$\sigma_z = \cdot 270,$$

i.e. there is no appreciable gain in our case since σ_r is ·271. It may be that with a lower value of the population correlation the gain would be greater, but on the other hand if r had been determined for very fine grouping σ_r^2 would have been ·0625, the contribution of r_ρ to z would have been practically negligible, and the gain in accuracy by the use of z less than that found. There is however another case where the above formulae might be applied, namely to the values of ρ obtained from the original grouping and those from coarse grouping.

These are given in Table II from the first and third lines of which it appears that σ_{ρ_1} and σ_{ρ_3} may both be taken as ·288.

In this case σ_z^2 reduces to $\dfrac{\sigma_\rho^2(1 + r_{\rho_1\rho_3})}{2}$,

and as $r_{\rho_1\rho_3} = \cdot 903, \quad \sigma_z = \cdot 281.$

This is somewhat more encouraging, but the process is rather troublesome and could only be applied to cases where there is a proper scale. If however there is a proper scale greater accuracy could be obtained by the product moment method with very little more trouble (since we have now to make two calculations to find ρ).

We may therefore conclude that as far as this sampling experiment may be taken as typical:

(1) Where the unit of grouping is small (say $< \frac{1}{3}$ the standard deviation) the product moment method should be used if the most is to be made of the time and statistics at our disposal, however small the sample.

(2) Where a coarse grouping has to be used, the mean value of r will fall below that calculated from the cooperative paper (*Biometrika*, XI. p. 328 et seq.) and the S.D will rise. For small samples Sheppard's corrections will approximately correct the former but will increase the latter still further. Indeed it is possible that for very coarse grouping ρ might vary less than r.

(3) For this, or any other, purpose ties should be dealt with by one or other of the formulae in equations (x) and (xi) of this paper.

(4) Where one or both variables can be ranked but not scaled, as frequently happens in some kinds of work, or for what Professor Pearson has called "purposes of assay," ρ can be determined with advantage and may be considered the natural method to adopt.

TABLE X.

Giving Correlation (·885) between r (no corrections) and r_s for samples of 8.

r	r_s																														
	−·5005 to −·4505	−·4505 to −·4005	−·4005 to −·3505	−·3505 to −·3005	−·3005 to −·2505	−·2505 to −·2005	−·2005 to −·1505	−·1505 to −·1005	−·1005 to −·0505	−·0505 to −·0005	−·0005 to ·0495	·0495 to ·0995	·0995 to ·1495	·1495 to ·1995	·1995 to ·2495	·2495 to ·2995	·2995 to ·3495	·3495 to ·3995	·3995 to ·4495	·4495 to ·4995	·4995 to ·5495	·5495 to ·5995	·5995 to ·6495	·6495 to ·6995	·6995 to ·7495	·7495 to ·7995	·7995 to ·8495	·8495 to ·8995	·8995 to ·9495	·9495 to ·9995	Totals
−·6505 to −·6005	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
−·6005 to −·5505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
−·5505 to −·5005	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
−·5005 to −·4505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
−·4505 to −·4005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
−·4005 to −·3505	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
−·3505 to −·3005	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
−·3005 to −·2505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
−·2505 to −·2005	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
−·2005 to −·1505	—	—	1	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
−·1505 to −·1005	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
−·1005 to −·0505	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
−·0505 to −·0005	—	—	—	—	—	—	—	—	—	—	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
−·0005 to ·0495	—	—	1	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
·0495 to ·0995	—	—	—	—	—	1	—	—	—	2	—	—	1	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	6
·0995 to ·1495	—	—	—	—	—	—	—	—	1	1	—	1	—	1	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	6
·1495 to ·1995	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	2	2	1	1	—	—	—	—	—	—	—	—	—	—	—	8
·1995 to ·2495	—	—	—	—	—	—	—	—	—	2	—	—	—	2	—	2	1	4	1	1	—	—	—	—	—	—	—	—	—	—	—	8
·2495 to ·2995	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2	—	3	2	4	1	—	—	—	—	—	—	—	—	—	—	—	6
·2995 to ·3495	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	4	2	1	1	—	—	—	—	—	—	—	—	—	—	—	9
·3495 to ·3995	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	4	2	1	—	—	—	—	—	—	—	—	—	—	16
·3995 to ·4495	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	1	—	1	1	3	—	—	—	—	—	—	—	—	—	—	17
·4495 to ·4995	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	3	3	1	2	—	—	—	—	—	—	—	—	—	13
·4995 to ·5495	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	4	1	2	—	—	—	—	—	—	—	—	20
·5495 to ·5995	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	4	1	1	—	—	—	—	—	—	—	19
·5995 to ·6495	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	3	5	4	2	1	—	—	—	—	—	—	26
·6495 to ·6995	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	4	5	2	1	—	—	—	—	—	25
·6995 to ·7495	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	—	—	1	—	2	3	2	—	—	—	—	—	37
·7495 to ·7995	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	2	4	2	—	—	—	—	39
·7995 to ·8495	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	2	7	5	—	—	—	34
·8495 to ·8995	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	2	7	5	—	—	34
·8995 to ·9495	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5	5	2	—	21
·9495 to ·9995	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5	15	4	2	11
Totals	1	2	2	2	0	2	0	0	5	10	2	2	4	8	8	6	9	16	17	13	20	19	26	25	37	39	34	34	21	11	375

TABLE XI.

TABLE XI.

Giving Correlation (·903) between ρ Calculated in Original Grouping (ρ_1) and ρ Calculated in Groups twice as coarse (ρ_3).*

ρ_3 \ ρ_1	− ·4995 to − ·4495	− ·4495 to − ·3995	− ·3995 to − ·3495	− ·3495 to − ·2995	− ·2995 to − ·2495	− ·2495 to − ·1995	− ·1995 to − ·1495	− ·1495 to − ·0995	− ·0995 to − ·0495	− ·0495 to + ·0005	+ ·0005 to ·0505	·0505 to ·1005	·1005 to ·1505	·1505 to ·2005	·2005 to ·2505	·2505 to ·3005	·3005 to ·3505	·3505 to ·4005	·4005 to ·4505	·4505 to ·5005	·5005 to ·5505	·5505 to ·6005	·6005 to ·6505	·6505 to ·7005	·7005 to ·7505	·7505 to ·8005	·8005 to ·8505	·8505 to ·9005	·9005 to ·9505	·9505 to 1·0005	Totals
− ·4995 to − ·4495	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
− ·4495 to − ·3995	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
− ·3995 to − ·3495	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
− ·3495 to − ·2995	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
− ·2995 to − ·2495	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
− ·2495 to − ·1995	—	—	—	2	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
− ·1995 to − ·1495	—	1	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
− ·1495 to − ·0995	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
− ·0995 to − ·0495	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
− ·0495 to + ·0005	—	—	—	—	—	—	—	—	1	1	—	1	2	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	8
+ ·0005 to ·0505	—	—	1	—	—	—	—	—	1	1	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	5
·0505 to ·1005	—	—	—	—	—	—	—	—	—	2	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
·1005 to ·1505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
·1505 to ·2005	—	—	—	—	—	—	—	—	1	1	—	1	1	—	1	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	9
·2005 to ·2505	—	—	—	—	—	—	—	—	1	—	—	—	1	5	—	—	2	3	—	—	—	—	—	—	—	—	—	—	—	—	—	12
·2505 to ·3005	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	2	—	2	1	1	1	—	—	—	—	—	—	—	—	—	—	10
·3005 to ·3505	—	—	—	—	—	1	—	—	—	1	—	—	2	2	—	1	—	—	2	—	—	2	—	—	—	—	—	—	—	—	—	11
·3505 to ·4005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	1	4	2	1	2	1	—	—	—	—	—	—	—	—	14
·4005 to ·4505	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	—	2	4	3	2	3	—	1	—	—	—	—	—	—	—	—	18
·4505 to ·5005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2	3	4	3	2	—	2	1	—	—	—	—	—	—	19
·5005 to ·5505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	1	5	6	6	2	—	—	—	1	—	—	—	—	24
·5505 to ·6005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	5	2	8	5	5	2	1	—	—	—	—	29
·6005 to ·6505	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	1	3	5	9	4	1	2	—	—	—	—	27
·6505 to ·7005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	6	4	2	8	6	2	2	—	—	—	33
·7005 to ·7505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	3	9	6	6	1	1	—	—	29
·7505 to ·8005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	4	3	3	10	6	—	4	—	—	31
·8005 to ·8505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	6	6	4	3	1	—	23
·8505 to ·9005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	2	3	9	14	4	—	—	35
·9005 to ·9505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	4	2	1	—	11
·9505 to 1·0005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	5	1	—	8
Totals	1	2	1	3	0	2	0	0	6	10	1	3	6	10	9	5	12	16	18	15	23	24	26	27	35	35	36	27	19	3	375

* The reader is requested to note that the subranges here are not the same as in Tables V, VI, and X. This is the only table which contains a perfect correlation coefficient which occurred in the ρ_3 series.

(5) In such cases it should be borne in mind that for small samples the distribution of ρ is similar to the distribution of r, but that the mean, even of r_ρ, is lower than that of r and the S.D. greater, by amounts which doubtless depend on the population correlation.

(6) R and r_R are not worth determining in serious work; their use should therefore be confined to the elementary statistics for which its author intended R.

(7) It is interesting to observe that Sheppard's median division fourfold table has given for small samples a mean value very much below the population value. While this is only what one might have expected, it may in this case be due to the coarse grouping which prevented me from making an accurate median division.

(8) The following problems might be of interest to mathematicians:

(*a*) The determination of the form of the Rank Correlation Surface.

(*b*) The determination of the frequency distribution of ρ for small samples drawn from a normally distributed population.

(*c*) The determination of the nature of the correlation surface when a standard deviation is taken as one variable and the correlation coefficient as the other, both being determined from small samples drawn from a normally distributed population.

ON THE MATHEMATICAL EXPECTATION OF THE MOMENTS OF FREQUENCY DISTRIBUTIONS.

PART II.

By PROFESSOR AL. A. TCHOUPROFF of Petrograd*

CHAPTER I

I

LET $X_1, X_2, \ldots X_N$ be N variates *each following its own law of frequency distribution* and let N experiments be performed,—the first on the variate X_1, the second on the variate X_2, and so forth, the last on the variate X_N, where the N experiments are mutually independent. Put:

$$EX_i = m_1^{(i)}\,;\quad EX_i^2 = m_2^{(i)}\,;\quad EX_i^r = m_r^{(i)}\,;$$

$$E[X_i - m_1^{(i)}]^2 = m_2^{(i)} - [m_1^{(i)}]^2 = \mu_2^{(i)}\,;\quad E[X_i - m_1^{(i)}]^r = \mu_r^{(i)}\,;$$

$$\frac{1}{N}\sum_{i=1}^{N} m_1^{(i)} = m_{[1,N]};\quad \frac{1}{N}\sum_{i=1}^{N} m_r^{(i)} = m_{[r,N]};$$

$$\frac{1}{N}\sum_{i=1}^{N} \mu_2^{(i)} = \mu_{[2,N]};\quad \frac{1}{N}\sum_{i=1}^{N} \mu_r^{(i)} = \mu_{[r,N]}.$$

Let X_i' be the chance value obtained by the experiment on the variate X_i, and put further

$$\frac{1}{N}\sum_{i=1}^{N} X_i' = X_{(N)},$$

$$EX_{(N)}^r = m_{r,(N)};\quad E[X_{(N)} - m_{1,(N)}]^r = \mu_{r,(N)}.$$

Noting, that $$m_{1,(N)} = \frac{1}{N}\sum_{i=1}^{N} EX_i' = m_{[1,N]},$$

we see that

$$\mu_{r,(N)} = E[X_{(N)} - m_{[1,N]}]^r = \sum_{h=0}^{r} (-1)^h C_r^h\, m_{[1,N]}^h\, m_{r-h,(N)}.$$

Conversely,

$$m_{r,(N)} = E[X_{(N)} - m_{[1,N]} + m_{[1,N]}]^r = \sum_{h=0}^{r} C_r^h\, m_{[1,N]}^h\, \mu_{r-h,(N)}.$$

* This paper was most kindly translated by Dr L. Isserlis from the Russian and read by him with Professor Tchouproff's permission to the Society of Biometricians and Mathematical Statisticians, June 7, 1921.

On the other hand,

$$\left.\begin{aligned}\mu_{[2,\,N]} &= \frac{1}{N}\sum_{i=1}^{N}\left[m_2^{(i)} - (m_1^{(i)})^2\right] = m_{[2,\,N]} - \frac{1}{N}\sum_{i=1}^{N}\left[m_1^{(i)}\right]^2 \\ &= m_{[2,\,N]} - m^2_{[1,\,N]} - \frac{1}{N}\sum_{i=1}^{N}\left[m_1^{(i)} - m_{[1,\,N]}\right]^2 \\ \mu_{[r,\,N]} &= \frac{1}{N}\sum_{i=1}^{N}\sum_{h=0}^{r}(-1)^h C_r^h (m_1^{(i)})^h m_{r-h}^{(i)} \\ &= m_{[r,\,N]} + \sum_{h=1}^{r-2}(-1)^h C_r^h \frac{1}{N}\sum_{i=1}^{N}\left[m_1^{(i)}\right]^h m_{r-h}^{(i)} \\ &\quad + (-1)^{r-1}(r-1)\frac{1}{N}\sum_{i=1}^{N}\left[m_1^{(i)}\right]^r\end{aligned}\right\} \ldots(1).$$

II

Now, $$\left[\sum_{i=1}^{N} X_i\right]^r = \sum_j \sum_{i_1}\sum_{i_2}\ldots\sum_{ij}\sum \frac{r!}{r_1!\,r_2!\ldots r_j!} X_{i_1}^{r_1} X_{i_2}^{r_2}\ldots X_{ij}^{rj},$$

where the summation for j is for all integer values from 1 to either r or N whichever is the smaller, the summation with regard to $i_1, i_2, i_3, \ldots i_j$ extends to all integer and mutually unequal values of $i_1, i_2, \ldots i_j$ from 1 to N, and the last summation to all integer values of $r_1, r_2, \ldots r_j$ satisfying the equation

$$r_1 + r_2 + \ldots + r_j = r.$$

Hence,

$$\left.\begin{aligned}m_{r,\,(N)} &= EX_{(N)}^r = \frac{1}{N^r} E\left[\sum_{i=1}^{N} X_i\right]^r \\ &= \frac{1}{N^r}\sum_j\sum_{i_1}\sum_{i_2}\ldots\sum_{ij}\sum \frac{r!}{r_1!\,r_2!\ldots r_j!} m_{r_1}^{(i_1)} m_{r_2}^{(i_2)}\ldots m_{rj}^{(ij)}\end{aligned}\right\} \ldots\ldots\ldots(2),$$

or, when $r \leqq N$,

$$\left.\begin{aligned}m_{r,\,(N)} &= \frac{1}{N^r}\left\{\sum_{i=1}^{N} m_r^{(i)} + \sum_{i_1=1}^{N-1}\sum_{i_2=i_1+1}^{N}\sum_{r_1=1}^{r-1}\frac{r!}{r_1!\,(r-r_1)!} m_{r_1}^{(i_1)} m_{r-r_1}^{(i_2)}\right. \\ &+ \sum_{i_1=1}^{N-2}\sum_{i_2=i_1+1}^{N-1}\sum_{i_3=i_2+1}^{N}\sum_{r_1=1}^{r-2}\sum_{r_2=1}^{r-r_1-1}\frac{r!}{r_1!\,r_2!\,(r-r_1-r_2)!} m_{r_1}^{(i_1)} m_{r_2}^{(i_2)} m_{r-r_1-r_2}^{(i_3)} + \ldots \\ &+ \sum_{i_1=1}^{N-r+2}\sum_{i_2=i_1+1}^{N-r+3}\ldots\sum_{i_{r-1}=i_{r-2}+1}^{N}\sum_{r_1=1}^{2}\sum_{r_2=1}^{3-r_1}\ldots\sum_{r_{r-2}=1}^{r-r_1-r_2-\ldots-r_{r-3}-1}\frac{r!}{2} m_{r_1}^{(i_1)} m_{r_2}^{(i_2)}\ldots \\ &\qquad \ldots m_{r_{r-2}}^{(i_{r-2})} m_{r-r_1-r_2-\ldots-r_{r-2}}^{(i_{r-1})} \\ &\left. + \sum_{i_1=1}^{N-r+1}\sum_{i_2=i_1+1}^{N-r+2}\ldots\sum_{i_r=i_{r-1}+1}^{N} r!\, m_1^{(i_1)} m_1^{(i_2)}\ldots m_1^{(i_r)}\right\} \\ &= m^r_{[1,\,N]} + \frac{1}{N} C_r^2 m^{r-2}_{[1,\,N]}\mu_{[2,\,N]} \\ &\quad + \frac{1}{N^2}\left\{1\,.\,3\,C_r^4 m^{r-4}_{[1,\,N]}\mu^2_{[2,\,N]} + C_r^3 m^{r-3}_{[1,\,N]}\mu_{[3,\,N]}\right\} + \ldots\end{aligned}\right\} (3)^*.$$

* Cf. "On the Mathematical Expectation of the Moments of Frequency Distributions" (*Biometrika*, Vol. XII, Chap. 1, § II, equation (10), p. 151).

When $r=2$ we find

$$m_{2,(N)} = m^{2}_{[1,N]} + \frac{1}{N}\mu_{[2,N]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Similarly we find*

$$\left.\begin{aligned} m_{3,(N)} &= m^{3}_{[1,N]} + \frac{3}{N} m_{[1,N]}\mu_{[2,N]} + \frac{1}{N^2}\mu_{[3,N]} \\ m_{4,(N)} &= m^{4}_{[1,N]} + \frac{6}{N} m^{2}_{[1,N]}\mu_{[2,N]} + \frac{1}{N^2}\left\{4 m_{[1,N]}\mu_{[3,N]} + 3\mu^{2}_{[2,N]}\right\} \\ &\quad + \frac{1}{N^3}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_4^{(i)} - 3(\mu_2^{(i)})^2\right]\right\} \end{aligned}\right\} \quad (5).$$

Comparing these relations with the corresponding relations established in (11) of Chapter I of the first Part of my paper on the "Mathematical Expectation of the Moments of Frequency Distributions†," we see that when each variate follows its own law of distribution we may obtain $m_{2,(N)}$ and $m_{3,(N)}$ by replacing m_1, μ_2 and μ_3 in equations (11) by the corresponding arithmetical averages for all the N variates. But in the case of $m_{4,(N)}$ this method would only produce the first three terms; the term of order $\frac{1}{N^3}$ is not equal to $\mu_{[4,N]} - 3\mu^{2}_{[2,N]}$ but to the arithmetic mean of the differences $\mu_4^{(i)} - 3[\mu_2^{(i)}]^2$.

III

If in (2) we put $m_1^{(i)} = 0$ and replace the m's by the corresponding μ's, we find

$$\mu_{r,(N)} = \frac{1}{N^r}\sum_{j}\sum_{i_1}\sum_{i_2}\ldots\sum_{i_j}\sum\frac{r!}{r_1!\,r_2!\ldots r_j!}\mu_{r_1}^{(i_1)}\mu_{r_2}^{(i_2)}\ldots\mu_{r_j}^{(i_j)},$$

where the summation with regard to j extends to all integer values from 1 to r or N whichever is the smaller, the summation with regard to $i_1, i_2, \ldots i_j$ extends to all mutually unequal integer values of $i_1, i_2, \ldots i_j$ from 1 to N, and the last summation to all integer values, not less than 2, of $r_1, r_2, \ldots r_j$ satisfying the relation $r_1 + r_2 + \ldots + r_j = r$.

Hence,

$$\left.\begin{aligned} \mu_{2r,(N)} &= \frac{1}{N^{2r}}\left\{\sum_{i=1}^{N}\mu_{2r}^{(i)} + \sum_{i_1=1}^{N-1}\sum_{i_2=i_1+1}^{N}\sum_{r_1=2}^{2r-2}\frac{[2r]!}{r_1![2r-r_1]!}\mu_{r_1}^{(i_1)}\mu_{2r-r_1}^{(i_2)}\right. \\ &\quad + \sum_{i_1=1}^{N-2}\sum_{i_2=i_1+1}^{N-1}\sum_{i_3=i_2+1}^{N}\sum_{r_1=2}^{2r-4}\sum_{r_2=2}^{2r-r_1-2}\frac{[2r]!}{r_1!\,r_2![2r-r_1-r_2]!}\mu_{r_1}^{(i_1)}\mu_{r_2}^{(i_2)}\mu_{2r-r_1-r_2}^{(i_3)} \\ &\quad \left. + \ldots + 1.3.5\ldots(2r-1)\,r!\sum_{i_1=1}^{N-r+1}\sum_{i_2=i_1+1}^{N-r+2}\ldots\sum_{i_r=i_{r-1}+1}^{N}\mu_2^{(i_1)}\mu_2^{(i_2)}\ldots\mu_2^{(i_r)}\right\} \\ &= 1.3.5\ldots(2r-1)\left\{\frac{1}{N^r}\mu^{r}_{[2,N]} + \frac{1}{N^{r+1}}\left[\tfrac{1}{6}r^{[-2]}\mu^{r-2}_{[2,N]}\mu_{[4,N]}\right.\right. \\ &\quad \left.\left. + \tfrac{1}{9}r^{[-3]}\mu^{r-3}_{[2,N]}\mu^{2}_{[3,N]} - \tfrac{1}{2}r^{[-2]}\mu^{r-2}_{[2,N]}\left(\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^2\right)\right] + \ldots\right\} \end{aligned}\right\} \quad (6)\ddagger,$$

* Cf. Tschuproff, "Zur Theorie der Stabilität statistischer Reihen" (*Skandinavisk Aktuarietidskrift*, 1919, p. 82).

† *Biometrika*, Vol. XII, p. 151.

‡ Cf. *Biometrika*, Vol. XII, p. 154, equation (22).

$$\left.\begin{aligned}
\mu_{2r+1,\,(N)} &= \frac{1}{N^{2r+1}}\left\{\sum_{i=1}^{N}\mu_{2r+1}^{(i)} + \sum_{i=1}^{N-1}\sum_{i_2=i_1+1}^{N}\sum_{r_1=2}^{2r-1}\frac{[2r+1]!}{r_1!\,[2r+1-r_1]!}\mu_{r_1}^{(i_1)}\,\mu_{2r+1-r_1}^{(i_2)}\right.\\
&\quad + \ldots + \frac{1.3.5\ldots(2r+1)\,r!}{3}\sum_{i_1=1}^{N-r+1}\sum_{i_2=i_1+1}^{N-r+2}\cdots\sum_{i_r=i_{r-1}+1}^{N}\sum_{r_1=2}^{3}\sum_{r_2=2}^{5-r_1}\cdots\\
&\quad \left.\cdots\sum_{r_{r-1}=2}^{2r-1-r_1-r_2-\ldots-r_{r-2}}\mu_{r_1}^{(i_1)}\,\mu_{r_2}^{(i_2)}\cdots\mu_{r_{r-1}}^{(i_{r-1})}\,\mu_{2r+1-r_1-\ldots-r_{r-1}}^{(i_r)}\right\}\\
&= 1.3.5\ldots(2r+1)\left\{\frac{1}{N^{r+1}}\tfrac{1}{3}r\,\mu_{[2,\,N]}^{r-1}\,\mu_{[3,\,N]}\right.\\
&\quad + \frac{1}{N^{r+2}}\left[\tfrac{1}{30}r^{[-2]}\,\mu_{[2,\,N]}^{r-2}\,\mu_{[5,\,N]} + \tfrac{1}{18}r^{[-3]}\,\mu_{[2,\,N]}^{r-3}\,\mu_{[3,\,N]}\,\mu_{[4,\,N]}\right.\\
&\quad + \tfrac{1}{81}r^{[-4]}\,\mu_{[2,\,N]}^{r-4}\,\mu_{[3,\,N]}^{3} - \tfrac{1}{3}r^{[-2]}\,\mu_{[2,\,N]}^{r-2}\left(\frac{1}{N}\sum_{i=1}^{N}\mu_2^{(i)}\,\mu_3^{(i)}\right)\\
&\quad \left.\left. - \tfrac{1}{6}r^{[-3]}\,\mu_{[2,\,N]}^{r-3}\left(\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^2\right)\left(\frac{1}{N}\sum_{i=1}^{N}\mu_3^{(i)}\right)\right] + \ldots\right\}
\end{aligned}\right\}\quad\ldots(7)^{*}.$$

Putting $r = 1, 2, \ldots$ we find †

$$\left.\begin{aligned}
\mu_{2,\,(N)} &= \frac{1}{N}\,\mu_{[2,\,N]}\\
\mu_{3,\,(N)} &= \frac{1}{N^2}\,\mu_{[3,\,N]}\\
\mu_{4,\,(N)} &= \frac{3}{N^2}\,\mu_{[2,\,N]}^{2} + \frac{1}{N^3}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_4^{(i)} - 3\,(\mu_2^{(i)})^2\right]\right\}\\
\mu_{5,\,(N)} &= \frac{10}{N^3}\,\mu_{[3,\,N]}\,\mu_{[2,\,N]} + \frac{1}{N^4}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_5^{(i)} - 10\mu_3^{(i)}\,\mu_2^{(i)}\right]\right\}
\end{aligned}\right\}\quad\ldots\ldots(8).$$

IV

Relations (6) and (7) show that as N increases $\dfrac{\mu_{2r,\,(N)}}{\mu_{2,\,(N)}^{r}}$ tends to the limit $1.3.5\ldots(2r-1)$ and $\dfrac{\mu_{2r-1,\,(N)}^{2}}{\mu_{2,\,(N)}^{2r-1}}$ tends to the limit zero provided that

$$\frac{\sum_{i=1}^{N}\mu_{2h}^{(i)}}{\left[\sum_{i=1}^{N}\mu_2^{(i)}\right]^{h}} = \frac{N\mu_{[2h,\,N]}}{[N\mu_{[2,\,N]}]^{h}} \quad\text{and}\quad \frac{\left[\sum_{i=1}^{N}\mu_{2h-1}^{(i)}\right]^{2}}{\left[\sum_{i=1}^{N}\mu_2^{(i)}\right]^{2h-1}} = \frac{N^2\mu_{[2h-1,\,N]}^{2}}{[N\mu_{[2,\,N]}]^{2h-1}}$$

tend, as N increases, to the limit zero, for $h = 2, 3, \ldots r$. Hence, if

$$\frac{\left[\sum_{i=1}^{N}\mu_r^{(i)}\right]^{s}}{\left[\sum_{i=1}^{N}\mu_2^{(i)}\right]^{r}} \quad\text{or}\quad \frac{N\mu_{[r,\,N]}}{[N\mu_{[2,\,N]}]^{r/2}}$$

* Cf. *Biometrika*, Vol. XII, p. 154, equation (23).

† *Vide* Tschuproff, "Zur Theorie der Stabilität statistischer Reihen" (*Skandinavisk Aktuarietidskrift*, 1919, p. 82) and *Biometrika*, Vol. XII, p. 155, equation (26).

tends, as N increases, to the limit zero, for $r = 3, 4, 5, \ldots \infty$, then the law of distribution of the values of $X_{(N)}$ tends to become the Laplace-Gauss law as N becomes infinitely great, whatever may be the law of distribution of the individual variates*.

V

If all the variates follow the Laplace-Gauss law then for all positive integral values of r,

$$\mu_{2r+1,\,(N)} = 0,$$

$$\begin{aligned}
\mu_{2r,\,(N)} = \frac{1}{N^{2r}} \Big\{ & 1.3.5 \ldots (2r-1) \sum_{i=1}^{N} [\mu_2^{(i)}]^r + \sum_{i_1=1}^{N-1} \sum_{i_2=i_1+1}^{N} \sum_{r_1=1}^{r-1} \frac{[2r!]}{[2r_1]![2r-2r_1]!} \\
& \times 1.3.5 \ldots (2r_1-1).1.3.5 \ldots (2r-2r_1-1)[\mu_2^{(i_1)}]^{r_1}[\mu_2^{(i_2)}]^{r-r_1} \\
& + \ldots + 1.3.5 \ldots (2r-1).1.2.3 \ldots r \sum_{i_1=1}^{N-r+1} \sum_{i_2=i_1+1}^{N-r+2} \ldots \sum_{i_r=i_{r-1}+1}^{N} \mu_2^{(i_1)} \mu_2^{(i_2)} \ldots \mu_2^{(i_r)} \Big\} \\
= \frac{1.3.5 \ldots (2r-1)}{N^{2r}} \Big\{ & \sum_{i=1}^{N} [\mu_2^{(i)}]^r + \sum_{i_1=1}^{N-1} \sum_{i_2=i_1+1}^{N} \sum_{r_1=1}^{r-1} \frac{r!}{r_1!(r-r_1)!} [\mu_2^{(i_1)}]^{r_1}[\mu_2^{(i_2)}]^{r-r_1} \\
& + \ldots + \sum_{i_1=1}^{N-r+1} \sum_{i_2=i_1+1}^{N-r+2} \ldots \sum_{i_r=i_{r-1}+1}^{N} r!\, \mu_2^{(i_1)} \mu_2^{(i_2)} \ldots \mu_2^{(i_r)} \Big\} \\
= \frac{1.3.5 \ldots (2r-1)}{N^{2r}} & \left[\sum_{i=1}^{N} \mu_2^{(i)} \right]^r = \frac{1.3.5 \ldots (2r-1)}{N^r} \mu_{[2,\,N]}^r,
\end{aligned}$$

and consequently $$\frac{\mu_{2r,(N)}}{[\mu_{2,(N)}]^r} = 1.3.5 \ldots (2r-1).$$

Thus in this case, the arithmetic mean of the random values taken by the variates follows the Laplace-Gauss law whatever may be the constants of the laws of distribution of the individual variates†.

CHAPTER II

I

Putting‡
$$\frac{1}{N} \sum_{i=1}^{N} [X_i' - m_1^{(i)}]^r = \mu'_{[r,\,N]},$$
$$\frac{1}{N} \sum_{i=1}^{N} [X_i' - m_{[1,\,N]}]^r = \mu''_{[r,\,N]},$$

we find

$$\left.\begin{aligned}
E\mu'_{[r,\,N]} &= \frac{1}{N} \sum_{i=1}^{N} \mu_r^{(i)} = \mu_{[r,\,N]} \\
E\mu''_{[r,\,N]} &= \frac{1}{N} \sum_{i=1}^{N} \sum_{h=0}^{r} C_r^h \mu_{r-h}^{(i)} [m_1^{(i)} - m_{[1,\,N]}]^h \\
&= \mu_{[r,\,N]} + r \frac{1}{N} \sum_{i=1}^{N} \mu_{r-1}^{(i)} [m_1^{(i)} - m_{[1,\,N]}] \\
&+ \ldots + \frac{r(r-1)}{2} \frac{1}{N} \sum_{j=1}^{N} \mu_2^{(i)} [m_1^{(i)} - m_{[1,\,N]}]^{r-2} + \frac{1}{N} \sum_{i=1}^{N} [m_1^{(i)} - m_{[1,\,N]}]^r
\end{aligned}\right\} \ldots (1).$$

* Cf. *Biometrika*, Vol. XII, pp. 156—157.

† Cf. *Biometrika*, Vol. XII, p. 158.

‡ *Vide* Tschuproff, "Zur Theorie der Stabilität statistischer Reihen" (*Skandinavisk Aktuarietidskrift*, 1919, pp. 82—83).

It follows that if all the quantities $m_1^{(i)}$ are equal to one another, then $\mu''_{[r,N]} = \mu'_{[r,N]}$, whatever the frequency distributions of the separate variates may be. If however the quantities $m_1^{(i)}$ are unequal then $\mu''_{[r,N]} \neq \mu'_{[r,N]}$ and, in general, $E\mu''_{[r,N]}$ is also not equal to $E\mu'_{[r,N]}$.

On the other hand

$$E\mu''_{[r,N]} = \frac{1}{N}\sum_{i=1}^{N}\sum_{h=0}^{r}(-1)^h C_r^h m_{[1,N]}^h m_{r-h}^{(i)} = \sum_{h=0}^{r}(-1)^h C_r^h m_{[1,N]}^h m_{[r-h,N]}$$

$$= m_{[r,N]} + \sum_{h=1}^{r-2}(-1)^h C_r^h m_{[1,N]}^h m_{[r-h,N]} + (-1)^{r-1}(r-1)\, m_{[1,N]}^r.$$

When $r = 1$ we get

$$\frac{1}{N}\sum_{i=1}^{N}[X_i' - m_{[1,N]}] = \frac{1}{N}\sum_{i=1}^{N}[X_i' - m_1^{(i)}] = X_{(N)} - m_{[1,N]};$$

so that the quantities $\mu'_{[1,N]}$ and $\mu''_{[1,N]}$ are identically equal. If in the relation (1) we put r equal to 2, 3, 4 successively, we find,

$$\left.\begin{aligned}
E\mu''_{[2,N]} &= \mu_{[2,N]} + \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2 \\
E\mu''_{[3,N]} &= \mu_{[3,N]} + \frac{3}{N}\sum_{i=1}^{N}\mu_2^{(i)}[m_1^{(i)} - m_{[1,N]}] + \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^3 \\
E\mu''_{[4,N]} &= \mu_{[4,N]} + \frac{4}{N}\sum_{i=1}^{N}\mu_3^{(i)}[m_1^{(i)} - m_{[1,N]}] + \frac{6}{N}\sum_{i=1}^{N}\mu_2^{(i)}[m_1^{(i)} - m_{[1,N]}]^2 \\
&\quad + \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^4.
\end{aligned}\right\} \ldots(2).$$

II

(1) Noting that $\mu'_{[r,N]}$ is the arithmetic mean of the mutually independent quantities $[X_1' - m_1^{(1)}]^r$, $[X_2' - m_1^{(2)}]^r$, ... $[X'_N - m_1^{(N)}]^r$ and that $E[X_i' - m_1^{(i)}]^r = \mu_r^{(i)}$, while $E[X_i' - m_1^{(i)}]^{kr} = \mu_{kr}^{(i)}$, we find from relations (3) of Chapter I, on replacing $m_{[1,N]}$ by $\frac{1}{N}\sum_{i=1}^{N}\mu_r^{(i)} = \mu_{[r,N]}$, $\mu_{[2,N]}$ by $\frac{1}{N}\sum_{i=1}^{N}[\mu_{2r}^{(i)} - (\mu_r^{(i)})^2]$ and so forth,

$$\left.\begin{aligned}
E[\mu'_{[r,N]}]^m &= \mu_{[r,N]}^m + \frac{1}{N}\frac{m(m-1)}{2}\mu_{[r,N]}^{m-2}\left(\frac{1}{N}\sum_{i=1}^{N}[\mu_{2r}^{(i)} - (\mu_r^{(i)})^2]\right) \\
&\quad + \frac{1}{N^2}\left\{\frac{m^{[-4]}}{8}\mu_{[r,N]}^{m-4}\left(\frac{1}{N}\sum_{i=1}^{N}[\mu_{2r}^{(i)} - (\mu_r^{(i)})^2]\right)^2\right. \\
&\quad \left. + \frac{m^{[-3]}}{6}\mu_{[r,N]}^{m-3}\left(\frac{1}{N}\sum_{i=1}^{N}[\mu_{3r}^{(i)} - 3\mu_{2r}^{(i)}\mu_r^{(i)} + 2(\mu_r^{(i)})^3]\right)\right\} + \ldots
\end{aligned}\right\} \ldots(3)^*.$$

* Cf. *Biometrika*, Vol. XII, p. 160, equation (6).

Similarly we find

$$\left.\begin{aligned}
E[\mu'_{[r,N]}]^2 &= \mu^2_{[r,N]} + \frac{1}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{2r} - (\mu^{(i)}_r)^2]\right\} \\
E[\mu'_{[r,N]}]^3 &= \mu^3_{[r,N]} + \frac{3}{N}\mu_{[r,N]}\left\{\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{2r} - (\mu^{(i)}_r)^2]\right\} \\
&\qquad + \frac{1}{N^2}\left\{\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{3r} - 3\mu^{(i)}_{2r}\,\mu^{(i)}_r + 2(\mu^{(i)}_r)^3]\right\} \\
E[\mu'_{[r,N]}]^4 &= \mu^4_{[r,N]} + \frac{6}{N}\mu^2_{[r,N]}\left\{\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{2r} - (\mu^{(i)}_r)^2]\right\} \\
&\quad + \frac{1}{N^2}\left\{4\mu_{[r,N]}\left(\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{3r} - 3\mu^{(i)}_{2r}\,\mu^{(i)}_r + 2(\mu^{(i)}_r)^3]\right)\right. \\
&\qquad \left. + 3\left(\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{2r} - (\mu^{(i)}_r)^2]\right)^2\right\} \\
&\quad + \frac{1}{N^3}\left\{\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{4r} - 4\mu^{(i)}_{3r}\,\mu^{(i)}_r + 6\mu^{(i)}_{2r}(\mu^{(i)}_r)^2 - 3(\mu^{(i)}_r)^4]\right. \\
&\qquad \left. - 3\left(\frac{1}{N}\sum_{i=1}^{N}[\mu^{(i)}_{2r} - (\mu^{(i)}_r)^2]\right)^2\right\}
\end{aligned}\right\} \quad \ldots(4).$$

(2) Introduce the notation,

$$\left.\begin{aligned}
Z^{(i)}_{(kr,N)} &= E[X_i' - m_{[1,N]}]^{kr} = \sum_{h=0}^{kr}(-1)^h C^h_{kr}\, m^h_{[1,N]}\, m^{(i)}_{rk-h} \\
&= \sum_{h=0}^{kr} C^h_{kr}[m^{(i)}_1 - m_{[1,N]}]^h\, \mu^{(i)}_{rk-h} \\
Z_{(kr,N)} &= \frac{1}{N}\sum_{i=1}^{N} Z^{(i)}_{(kr,N)} = \sum_{h=0}^{kr}(-1)^h C^h_{kr}\, m^h_{[1,N]}\, m_{[kr-h,N]}
\end{aligned}\right\} \quad (5).$$

Noting that $\mu''_{(r,N)}$ is the arithmetic mean of the mutually independent quantities

$$[X_1' - m_{[1,N]}]^r,\ [X_2' - m_{[1,N]}]^r, \ldots [X'_N - m_{[1,N]}]^r,$$

we find

$$\left.\begin{aligned}
E[\mu''_{[r,N]}]^m &= [Z_{(r,N)}]^m + \frac{1}{N}\frac{m(m-1)}{2}[Z_{[r,N]}]^{m-2}\left[Z_{(2r,N)} - \frac{1}{N}\sum_{i=1}^{N}(Z^{(i)}_{r,N})^2\right] \\
&\quad + \frac{1}{N^2}\left\{\frac{m^{[-4]}}{8}[Z_{(r,N)}]^{m-4}\left[Z_{(2r,N)} - \frac{1}{N}\sum_{i=1}^{N}(Z^{(i)}_{(r,N)})^2\right]^2\right. \\
&\quad \left. + \frac{m^{[-3]}}{6}[Z_{(r,N)}]^{m-3}\left[Z_{(3r,N)} - 3\frac{1}{N}\sum_{i=1}^{N} Z^{(i)}_{(2r,N)}\, Z^{(i)}_{(r,N)}\right.\right. \\
&\qquad \left.\left. + 2\frac{1}{N}\sum_{i=1}^{N}(Z^{(i)}_{(r,N)})^3\right]\right\} + \ldots
\end{aligned}\right\} \quad (6).$$

III

(1) Put

$$\sum_{h=0}^{k}(-1)^h C^h_k [\mu_r{}^{(i)}]^h\, \mu^{(i)}_{(k-h)r} = \chi_k{}^{(i)}, \quad \frac{1}{N}\sum_{i=1}^{N}\chi_k{}^{(i)} = \chi_{[k,N]}.$$

If in equations (6) and (7) of Chapter I we replace $\mu_k^{(i)}$ by $\chi_k^{(i)}$ and $\mu_{[k,N]}$ by $\chi_{[k,N]}$ we find

$$\left.\begin{aligned} E\left[\mu'_{[r,N]}-\mu_{[r,N]}\right]^{2m} = 1.3.5\ldots(2m-1)\left\{\frac{1}{N^m}(\chi_{[2,N]})^m\right. \\ +\frac{1}{N^{m+1}}\left[\tfrac{1}{8}m^{[-2]}(\chi_{[2,N]})^{m-2}\chi_{[4,N]}+\tfrac{1}{8}m^{[-2]}(\chi_{[2,N]})^{m-2}(\chi_{[3,N]})^2\right. \\ \left.\left.-\tfrac{1}{2}m^{[-2]}(\chi_{[2,N]})^{m-2}\left(\frac{1}{N}\sum_{i=1}^{N}(\chi_2^{(i)})^2\right)\right]+\ldots\right\} \end{aligned}\right\} \quad\ldots\ldots(7),$$

$$\left.\begin{aligned} &E\left[\mu'_{[r,N]}-\mu_{[r,N]}\right]^{2m+1} \\ &=1.3.5\ldots(2m+1)\left\{\frac{1}{N^{m+1}}\tfrac{1}{8}m(\chi_{[2,N]})^{m-1}\chi_{[3,N]}\right. \\ &\quad+\frac{1}{N^{m+2}}\left[\tfrac{1}{30}m^{[-2]}(\chi_{[2,N]})^{m-2}\chi_{[5,N]}+\tfrac{1}{18}m^{[-2]}(\chi_{[2,N]})^{m-2}\chi_{[3,N]}\chi_{[4,N]}\right. \\ &\quad+\tfrac{1}{81}m^{[-4]}(\chi_{[2,N]})^{m-4}(\chi_{[3,N]})^3-\tfrac{1}{8}m^{[-2]}(\chi_{[2,N]})^{m-2}\left(\frac{1}{N}\sum_{i=1}^{N}\chi_2^{(i)}\chi_3^{(i)}\right) \\ &\quad\left.\left.-\tfrac{1}{6}m^{[-3]}(\chi_{[2,N]})^{m-3}\left(\frac{1}{N}\sum_{i=1}^{N}(\chi_2^{(i)})^2\right)\left(\frac{1}{N}\sum_{i=1}^{N}\chi_3^{(i)}\right)\right]+\ldots\right\} \end{aligned}\right\} \quad(8).$$

Hence*

$$\left.\begin{aligned} E\left[\mu'_{[r,N]}-\mu_{[r,N]}\right]^2&=\frac{1}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{2r}^{(i)}-(\mu_r^{(i)})^2\right]\right\} \\ E\left[\mu'_{[r,N]}-\mu_{[r,N]}\right]^3&=\frac{1}{N^2}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{3r}^{(i)}-3\mu_{2r}^{(i)}\mu_r^{(i)}+2(\mu_r^{(i)})^3\right]\right\} \\ E\left[\mu'_{[r,N]}-\mu_{[r,N]}\right]^4&=\frac{3}{N^2}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{2r}^{(i)}-(\mu_r^{(i)})^2\right]\right\}^2 \\ &\quad+\frac{1}{N^3}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{4r}^{(i)}-4\mu_{3r}^{(i)}\mu_r^{(i)}+6\mu_{2r}^{(i)}(\mu_r^{(i)})^2-3(\mu_r^{(i)})^4\right]\right. \\ &\quad\left.-3\frac{1}{N}\sum_{i=1}^{N}\left[\mu_{2r}^{(i)}-(\mu_r^{(i)})^2\right]^2\right\} \end{aligned}\right\} \quad(9).$$

When $r=2$, for the case in which the frequency distributions of all the variates follow the Laplace-Gauss law, we have

$$\left.\begin{aligned} E\left[\mu'_{[2,N]}-\mu_{[2,N]}\right]^2&=\frac{2}{N}\left(\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^2\right) \\ E\left[\mu'_{[2,N]}-\mu_{[2,N]}\right]^3&=\frac{8}{N^2}\left(\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^3\right) \\ E\left[\mu'_{[2,N]}-\mu_{[2,N]}\right]^4&=\frac{12}{N^2}\left(\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^2\right)^2+\frac{48}{N^3}\left(\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^4\right) \end{aligned}\right\} \quad\ldots(10).$$

* Cf. *Biometrika*, Vol. XII, pp. 162—163, equations (14), (15) and (16).

(2) Putting

$$\sum_{j=0}^{s} (-1)^j C_s^j (Z^{(i)}_{(r,N)})^j Z^{(i)}_{([s-j]r,N)} = \phi_s^{(i)}, \quad \frac{1}{N}\sum_{i=1}^{N} \phi_s^{(i)} = \phi_{[s,N]},$$

we find

$$E[\mu''_{[r,N]} - Z_{[r,N]}]^{2m} = 1.3.5\ldots(2m-1)\left\{\frac{1}{N^m}(\phi_{[2,N]})^m + \frac{1}{N^{m+1}}\left[\tfrac{1}{6}m^{[-2]}(\phi_{[2,N]})^{m-2}\phi_{[4,N]} + \tfrac{1}{9}m^{[-3]}(\phi_{[2,N]})^{m-3}(\phi_{[3,N]})^2 - \tfrac{1}{2}m^{[-2]}(\phi_{[2,N]})^{m-2}\left(\frac{1}{N}\sum_{i=1}^{N}(\phi_2^{(i)})^2\right)\right] + \ldots\right\} \quad (11).$$

$$E[\mu''_{[r,N]} - Z_{[r,N]}]^{2m+1} = 1.3.5\ldots(2m+1)\left\{\frac{1}{N^{m+1}}\tfrac{1}{3}m(\phi_{[2,N]})^{m-1}\phi_{[3,N]} + \frac{1}{N^{m+2}}\left[\tfrac{1}{30}m^{[-2]}(\phi_{[2,N]})^{m-2}\phi_{[5,N]} + \tfrac{1}{18}m^{[-3]}(\phi_{[2,N]})^{m-3}\phi_{[3,N]}\phi_{[4,N]} + \tfrac{1}{81}m^{[-4]}(\phi_{[2,N]})^{m-4}(\phi_{[3,N]})^3 - \tfrac{1}{3}m^{[-2]}(\phi_{[2,N]})^{m-2}\left(\frac{1}{N}\sum_{i=1}^{N}\phi_2^{(i)}\phi_3^{(i)}\right) - \tfrac{1}{6}m^{[-3]}(\phi_{[2,N]})^{m-3}\left(\frac{1}{N}\sum_{i=1}^{N}(\phi_2^{(i)})^2\right)\left(\frac{1}{N}\sum_{i=1}^{N}\phi_3^{(i)}\right)\right] + \ldots\right\} \quad (12).$$

Hence

$$E[\mu''_{[r,N]} - Z_{[r,N]}]^2 = \sigma^2\mu''_{[r,N]} = \frac{1}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}\left([\mu_{2r}^{(i)} - (\mu_r^{(i)})^2] + [m_1^{(i)} - m_{[1,N]}][2r\mu_{2r-1}^{(i)} - 2r\mu_r^{(i)}\mu_{r-1}^{(i)}] + \sum_{h=2}^{r-2}[m_1^{(i)} - m_{[1,N]}]^h\left[C_{2r}^h\mu_{2r-h}^{(i)} - \sum_{j=0}^{h} C_r^j C_r^{h-j}\mu_{r-j}^{(i)}\mu_{r-h+j}^{(i)}\right] + (m_1^{(i)} - m_{[1,N]})^{r-1}\left[C_{2r}^{r-1}\mu_{r+1}^{(i)} - \sum_{j=1}^{r-2} C_r^j C_r^{r-1-j}\mu_{r-j}^{(i)}\mu_{1+j}^{(i)}\right] + (m_1^{(i)} - m_{[1,N]})^r\left[C_{2r}^r\mu_r^{(i)} - 2\mu_r^{(i)} - \sum_{j=2}^{r-2} C_r^j C_r^{r-j}\mu_{r-j}^{(i)}\mu_j^{(i)}\right] + \sum_{h=1}^{r-2}[m_1^{(i)} - m_{[1,N]}]^{r+h}\left[C_{2r}^{r+h}\mu_{r-h}^{(i)} - \sum_{j=0}^{r-h} C_r^{r-j}C_r^{h+j}\mu_j^{(i)}\mu_{r-h-j}^{(i)}\right]\right)\right\} \quad (13).$$

When $r = 2$, we find

$$\sigma^2\mu''_{[2,N]} = \frac{1}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}\left([\mu_4^{(i)} - (\mu_2^{(i)})^2] + 4[m_1^{(i)} - m_{[1,N]}]\mu_3^{(i)} + 4[m_1^{(i)} - m_{[1,N]}]^2\mu_2^{(i)}\right)\right\} \quad (14).$$

In the case for which all the variates are distributed according to the Gauss-Laplace law,

$$\sigma^2\mu''_{[2,N]} = \frac{2}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}\left(\mu_2^{(i)}[\mu_2^{(i)} + 2(m_1^{(i)} - m_{[1,N]})^2]\right)\right\} \quad \ldots\ldots\ldots(15).$$

(3) Noting that

$$E[\mu''_{[r,N]} - \mu_{[r,N]}]^2 = E[\mu''_{[r,N]} - Z_{[r,N]}]^2 + [Z_{(r,N)} - \mu_{(r,N)}]^2,$$

we find (cf. supra equation (5) and Chapter I, equation (1))

$$\left.\begin{aligned} E[\mu''_{[2,N]} - \mu_{[2,N]}]^2 &= \left\{\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right\}^2 \\ &+ \frac{1}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}\left([\mu_4^{(i)} - (\mu_2^{(i)})^2] + 4[m_1^{(i)} - m_{[1,N]}]\mu_3^{(i)}\right.\right. \\ &\left.\left. + 4[m_1^{(i)} - m_{[1,N]}]^2\mu_2^{(i)}\right)\right\} \end{aligned}\right\} \ldots(16).$$

But (see equation (2) above)

$$E[\mu''_{[2,N]} - \mu_{[2,N]}] = \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2.$$

We thus see that

$$\left.\begin{aligned} \sigma^2{}_{\mu''_{[2,N]} - \mu_{[2,N]}} &= E\left\{[\mu''_{[2,N]} - \mu_{[2,N]}] - \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right\}^2 \\ &= \frac{1}{N}\left\{\frac{1}{N}\sum_{i=1}^{N}\left([\mu_4^{(i)} - (\mu_2^{(i)})^2] + 4[m_1^{(i)} - m_{[1,N]}]\mu_3^{(i)}\right.\right. \\ &\left.\left. + 4[m_1^{(i)} - m_{[1,N]}]^2\mu_2^{(i)}\right)\right\} \end{aligned}\right\} \ldots(17).$$

(4) From the relations (2) and (1) we get

$$E[\mu''_{[2,N]} - \mu'_{[2,N]}] = \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2.$$

Noting, that

$$\begin{aligned} \mu''_{[2,N]} - \mu'_{[2,N]} &= \frac{1}{N}\sum_{i=1}^{N}[X_i' - m_{[1,N]}]^2 - \frac{1}{N}\sum_{i=1}^{N}[X_i' - m_1^{(i)}]^2 \\ &= \frac{1}{N}\left\{2\sum_{i=1}^{N}X_i'[m_1^{(i)} - m_{[1,N]}] - \sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right\}, \end{aligned}$$

we find

$$E[\mu''_{[2,N]} - \mu'_{[2,N]}]^2 = \frac{4}{N^2}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\mu_2^{(i)} + \left(\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right)^2 \quad (18),$$

and

$$\left.\begin{aligned} \sigma^2{}_{\mu''_{[2,N]} - \mu'_{[2,N]}} &= E\left\{[\mu''_{[2,N]} - \mu'_{[2,N]}] - \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right\}^2 \\ &= \frac{4}{N^2}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\mu_2^{(i)} \end{aligned}\right\} \quad (19).$$

CHAPTER III

I

Let us put

$$\frac{1}{N}\sum_{i=1}^{N}[X_i' - X_{(N)}]^r = \nu'_{r,(N)},$$

$$E\nu'_{r,(N)} = \nu_{r,(N)}.$$

When $r=1$, we have the identities

$$\nu'_{i,(N)}=0 \text{ and } \nu_{1,(N)}=0.$$

Noting that

$$X_i'-X_{(N)}=[X_i'-m_1^{(i)}]+[m_1^{(i)}-m_{[1,N]}]-\frac{1}{N}\sum_{j=1}^{N}[X_j'-m_1^{(j)}]$$

$$=\frac{N-1}{N}[X_i'-m_1^{(i)}]+[m_1^{(i)}-m_{[1,N]}]-\frac{N-1}{N}\frac{1}{N-1}\sum_{j\neq i}[X_j'-m_1^{(j)}],$$

and putting

$$E\left\{\frac{1}{N-1}\sum_{j\neq i}[X_j'-m_1^{(j)}]\right\}^g=\mu_{g,(N-1)/i},$$

we find

$$\nu_{r,(N)}=\frac{1}{N}\sum_{i=1}^{N}\sum_{g=0}^{r}\sum_{h=0}^{r-g}(-1)^g C_r^g C_{r-g}^h\left[\frac{N-1}{N}\right]^{r-h}[m_1^{(i)}-m_{[1,N]}]^h\mu_{r-g-h}^{(i)}\mu_{g,(N-1)/i}$$

$$=\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^r+\frac{1}{N}\sum_{i=1}^{N}C_r^{r-2}\left[\frac{N-1}{N}\right]^2[m_1^{(i)}-m_{[1,N]}]^{r-2}[\mu_2^{(i)}+\mu_{2,(N-1)/i}]$$

$$+\frac{1}{N}\sum_{i=1}^{N}C_r^{r-3}\left[\frac{N-1}{N}\right]^3[m_1^{(i)}-m_{[1,N]}]^{r-3}[\mu_3^{(i)}-\mu_{3,(N-1)/i}]$$

$$+\frac{1}{N}\sum_{i=1}^{N}\sum_{h=1}^{r-4}C_r^h\left(\frac{N-1}{N}\right)^{r-h}[m_1^{(i)}-m_{[1,N]}]^h$$

$$\times\left[\mu_{r-h}^{(i)}+\sum_{g=2}^{r-h-2}(-1)^g C_{r-h}^g\mu_{r-h-g}^{(i)}\mu_{g,(N-1)/i}+(-1)^{r-h}\mu_{r-h,(N-1)/i}\right]$$

$$+\frac{1}{N}\sum_{i=1}^{N}\left[\frac{N-1}{N}\right]^r\left[\mu_r^{(i)}+\sum_{g=2}^{r-2}(-1)^g C_r^g\mu_{r-g}^{(i)}\mu_{g,(N-1)/i}+(-1)^r\mu_{r,(N-1)/i}\right]$$

Hence, noting that

$$\mu_{2,(N-1)/i}=\frac{1}{(N-1)^2}\sum_{j\neq i}\mu_2^{(j)}=\frac{N}{(N-1)^2}\mu_{[2,N]}-\frac{1}{(N-1)^2}\mu_2^{(i)},$$

$$\mu_{3,(N-1)/i}=\frac{N}{(N-1)^3}\mu_{[3,N]}-\frac{1}{(N-1)^3}\mu_3^{(i)} \text{ and so forth,}$$

we find*

$$\left.\begin{aligned}\nu_{r,(N)}=&\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^r+C_r^2\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^{r-2}\mu_2^{(i)}\\&+C_r^3\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^{r-3}\mu_3^{(i)}+\sum_{h=0}^{r-4}C_r^h\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^h\mu_{r-h}^{(i)}\\&+\frac{1}{N}\left\{C_r^2\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^{r-2}[\mu_{[2,N]}-2\mu_2^{(i)}]\right.\\&\qquad\left.-3C_r^3\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^{r-3}\mu_3^{(i)}\right.\\&+\sum_{h=0}^{r-4}C_r^h(r-h)\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)}-m_{[1,N]}]^h\\&\qquad\left.\times\left[\frac{r-h-1}{2}\mu_{[2,N]}\mu_{r-2-h}^{(i)}-\mu_{r-h}^{(i)}\right]\right\}+\cdots\end{aligned}\right\}\quad(1)$$

* Cf. *Biometrika*, Vol. XII, p. 186, equation (6)

When $r = 2, 3, 4$ we find similarly*

$$\left.\begin{aligned}
\nu_{2,(N)} &= \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2 + \frac{N-1}{N}\mu_{[2,N]} \\
\nu_{3,(N)} &= \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^3 \\
&\qquad + 3\frac{N-2}{N}\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]\mu_2^{(i)} + \frac{N^2-3N+2}{N^2}\mu_{[3,N]} \\
\nu_{4,(N)} &= \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^4 + \frac{6}{N}\mu_{[2,N]}\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2 \\
&\quad + 6\frac{N-2}{N}\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\mu_2^{(i)} \\
&\qquad + 4\frac{N^2-3N+3}{N^2}\frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]\mu_3^{(i)} \\
&\quad + \frac{(N-1)(N^2-3N+3)}{N^3}\mu_{[4,N]} + 3\frac{2N-3}{N^3}\mu_{[2,N]}^2 \\
&\qquad - 3\frac{(2N-3)}{N^3}\frac{1}{N}\sum_{i=1}^{N}(\mu_2^{(i)})^2
\end{aligned}\right\} \quad (2).$$

II

Noting that

$$\begin{aligned}
\left\{\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right\}^2 &= \left\{\sum_{i=1}^{N}(m_1^{(i)})^2 - \frac{1}{N}\left[\sum_{i=1}^{N}m_1^{(i)}\right]^2\right\}^2 \\
&= \frac{1}{N^2}\left[\sum_{i=1}^{N}m_1^{(i)}\right]^4 - \frac{2}{N}\left[\sum_{i=1}^{N}m_1^{(i)}\right]^2\sum_{i=1}^{N}(m_1^{(i)})^2 + \left[\sum_{i=1}^{N}(m_1^{(i)})^2\right]^2 \\
&= \frac{1}{N^2}\left[\sum_{i=1}^{N}m_1^{(i)}\right]^4 + \left[1 - \frac{2}{N}\right]\left[\sum_{i=1}^{N}(m_1^{(i)})^4 + \sum_{i_1=1}^{N}\sum_{i_2\neq i_1}(m_1^{(i_1)})^2(m_1^{(i_2)})^2\right] \\
&\quad - \frac{2}{N}\left[2\sum_{i_1=1}^{N}\sum_{i_2\neq i_1}(m_1^{(i_1)})^3(m_1^{(i_2)}) + \sum_{i_3=1}^{N}\sum_{i_2\neq i_3}\sum_{i_1\neq i_2\neq i_3}(m_1^{(i_1)})^2(m_1^{(i_2)})(m_1^{(i_3)})\right],
\end{aligned}$$

we find, after some fairly tedious but quite straightforward transformations†,

$$\begin{aligned}
E\left[\sum_{i=1}^{N}(X_i' - X_{(N)})^2\right]^2 &= \left\{\Sigma[m_1^{(i)} - m_{[1,N]}]^2\right\}^2 + \frac{(N-1)^2}{N}\mu_{[4,N]} \\
&\quad + \frac{N^2-2N+3}{N^2}\left\{\left[\sum_{i=1}^{N}\mu_2^{(i)}\right]^2 - \sum_{i=1}^{N}[\mu_2^{(i)}]^2\right\} \\
&\quad + 4(N-1)\left\{\frac{1}{N}\sum_{i=1}^{N}\mu_3^{(i)}m_1^{(i)} - \mu_{[3,N]}m_{[1,N]}\right\} \\
&\quad + \frac{2N-1}{N}\left(\sum_{i=1}^{N}\mu_2^{(i)}\right)\left(\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right) \\
&\quad + 4\sum_{i=1}^{N}\mu_2^{(i)}[m_1^{(i)} - m_{[1,N]}]^2,
\end{aligned}$$

* *Vide* Tschuproff, "Zur Theorie der Stabilität statistischer Reihen" (*Skandinavisk Aktuarietidskrift*, 1919, p. 84, equations (1) and (2)) and *Biometrika*, Vol. XII, p. 186, equation (7).

† Cf. *Biometrika*, Vol. XII, p. 192, equation (19).

and hence

$$\sigma^2_{\nu'_{2,(N)}} = E[\nu'_{2,(N)} - \nu_{2,(N)}]^2$$

$$= E\left\{\frac{1}{N}\sum_{i=1}^{N}[X_i' - X_{(N)}]^2\right\}^2 - \left(\frac{N-1}{N}\mu_{[2,N]} + \frac{1}{N}\sum_{i=1}^{N}[m_1^{(i)} - m_{[1,N]}]^2\right)^2$$

$$= \frac{(N-1)^2}{N^3}\frac{1}{N}\sum_{i=1}^{N}[\mu_4^{(i)} - 3(\mu_2^{(i)})^2] + 2\frac{N-1}{N^2}\frac{1}{N}\sum_{i=1}^{N}[\mu_2^{(i)}]^2$$

$$- \frac{2}{N^2}\frac{1}{N}\sum_{i=1}^{N}[\mu_2^{(i)} - \mu_{[2,N]}]^2 + \frac{4}{N}\frac{1}{N}\sum_{i=1}^{N}\mu_2^{(i)}[m_1^{(i)} - m_{[1,N]}]^2$$

$$+ 4\frac{N-1}{N^2}\left\{\frac{1}{N}\sum_{i=1}^{N}\mu_3^{(i)} m_1^{(i)} - \mu_{[3,N]} m_{[1,N]}\right\}^*\dagger.$$

* *Vide* Tschuproff, "Zur Theorie der Stabilität statistischer Reihen" (*Skandinavisk Aktuarietidskrift*, 1919, p. 85, equation (4)) and *Biometrika*, Vol. XII, p. 193, equation (23).

† The problems discussed above can also, as regards most of them, be attacked by the methods employed in the fourth chapter of the first part of my paper "On the Mathematical Expectation of the Moments of Frequency Distributions" (*Biometrika*, Vol. XII, pp. 194—210). Suppose that the aggregate of all the possible values that may be assumed by $X_1, X_2, \ldots X_N$ reduces to $\xi_1, \xi_2, \ldots \xi_k$. Denote by $p_j^{(i)}$ the probability that the variate X_i may take the value ξ_j, where $p_j^{(i)} = 0$, if ξ_j does not occur among the possible values of X_i; denote by n_j the number of times, that among the N experiments performed, one or other of the variates takes the value ξ_j.

Putting
$$p_j = \frac{1}{N}\sum_{i=1}^{N} p_j^{(i)},$$

we find

$$m_r^{(i)} = \sum_{j=1}^{k} p_j^{(i)} \xi_j^r; \quad m_{[r,N]} = \sum_{j=1}^{k} p_j \xi_j^r;$$

$$\mu_r^{(i)} = \sum_{j=1}^{k} p_j^{(i)} [\xi_j - m_1^{(i)}]^r; \quad \mu_{[r,N]} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{k} p_j^{(i)} [\xi_j - m_1^{(i)}]^r;$$

$$X_{(N)} = \frac{1}{N}\sum_{j=1}^{k} n_j \xi_j = m'_{[1,N]}, \text{ where } m'_{[r,N]} = \frac{1}{N}\sum_{j=1}^{k} n_j \xi_j^r;$$

$$\mu''_{[r,N]} = \frac{1}{N}\sum_{j=1}^{k} n_j [\xi_j - m_{[1,N]}]^r;$$

$$\nu'_{r,(N)} = \frac{1}{N}\sum_{j=1}^{k} n_j [\xi_j - X_{(N)}]^r = \sum_{h=0}^{r} (-1)^h C_r^h (\mu''_{[r,N]})^{r-h} \omega^h, \text{ where } \omega = X_{(N)} - m_{[1,N]}.$$

Of the various quantities discussed above, $\mu'_{[r,N]}$ is the only one that cannot be expressed in terms of n_j and ξ_j. The others can be studied by means of relations analogous to those numbered (1) to (6) in the fourth chapter of the first part of my paper (*Biometrika*, Vol. XII, pp. 195—6). This method is however of little practical use, as the resulting formulae are too cumbrous. Thus with $En_j = Np_j$, we get for the mathematical expectation of the square of n_j,

$$En_j^2 = N^2 p_j^2 + Np_j - \sum_{i=1}^{N} (p_j^{(i)})^2$$

$$= N^2 p_j^2 + Np_j(1 - p_j) - \sum_{i=1}^{N} [p_j^{(i)} - p_j]^2.$$

For the mathematical expectation of the cube of n_j we find

$$En_j^3 = N^3 p_j^3 + 3N^2 p_j^2 (1 - p_j) + Np_j[1 - 3p_j + 2p_j^2] - 3[Np_j - 2p_j + 1]\sum_{i=1}^{N}[p_j^{(i)} - p_j]^2 + 2\sum_{i=1}^{N}[p_j^{(i)} - p_j]^3$$

and so forth.

MISCELLANEA.

I. On a General Method of determining the successive terms in a Skew Regression Line*.

By KARL PEARSON, F.R.S.

Let x and y be the two variates. Suppose the x-range divided up into any not necessarily equal intervals h_x giving arrays of y of which the mean of those which centre at x (lying somewhere in h_x) is $\bar{y}_x$ and the array total n_x. Let $\bar{x}$, $\bar{y}$, σ_x, σ_y be the means and standard deviations of the total population N of the two variates. Assume the form of the regression line of y on x to be:

$$\frac{\bar{y}_x-\bar{y}}{\sigma_y}=a_0\psi_0+a_1\psi_1+\ldots+a_n\psi_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)},$$

where $a_0, a_1 \ldots a_n$ are absolute constants to be determined and ψ_s is an orthogonal function of $\frac{x-\bar{x}}{\sigma_x}$, i.e. is subject to the condition that:

$$S(n_x\psi_s\psi_{s'})=0, \quad s \text{ and } s' \text{ unequal} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)},$$

if the summation S be taken for all values of x corresponding to the arbitrary system of arrays.

Further let us suppose† that

$$q_{ij}=\frac{S\{n_x(\bar{y}_x-\bar{y})(x-\bar{x})^i\}}{N\sigma_y\sigma_x{}^i}=\frac{\Sigma\{n_{xy}(y-\bar{y})(x-\bar{x})^i\}}{N\sigma_y\sigma_x{}^i}=p_{ij}/(\sigma_y\sigma_x{}^i)$$

in the usual product-moment notation.

Clearly if the ψ_s's can be determined we must have by virtue of (ii) from (i)

$$S\left(\frac{n_x(\bar{y}_x-\bar{y})}{N\sigma_y}\psi_n\right)=a_n\frac{S(n_x\psi_n{}^2)}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}.$$

Now if ψ_n be determined as an integral function of $(x-\bar{x})/\sigma_x$ of the nth degree, the left-hand side of (iii) is expressible in terms of the q_{ij}'s, or the product-moments of the correlation distribution. Thus a_n will be determined. Let us write:

$$\left.\begin{aligned}\kappa_n&=\frac{S\{n_x(\bar{y}_x-\bar{y})\psi_n\}}{N\sigma_y}\\ \text{and}\quad \lambda_n&=\frac{S(n_x\psi_n{}^2)}{N}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)}.$$

Then

$$a_n=\kappa_n/\lambda_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iii) bis}$$

is known, if the ψ_s's have once been determined, from the product-moments of the system of the correlated variates, and the moment coefficients of the x-variate.

Square (i), multiply by n_x/N and sum for all arrays, we have:

$$S\left(\frac{n_x(\bar{y}_x-\bar{y})^2}{N\sigma_y{}^2}\right)=\eta^2_{y,x}=a_0{}^2\lambda_0+a_1{}^2\lambda_1+\ldots+a_n{}^2\lambda_n \quad \ldots\ldots\ldots\ldots\text{(v)}.$$

Here $\eta_{y,x}$ is the well-known correlation ratio of y-variate on x-variate, and must always lie between 0 and 1. The series on the right-hand side of (v) consists of a system of squares, and accordingly every additional term we take in our series for the regression line must carry us nearer this value of $\eta^2_{y,x}$. Unless λ_n tends to zero the a_n's must grow smaller and smaller, or we have considerable anticipation of the convergency of the series, but this does not amount of course to definite proof.

* Reproduced from lecture notes.

† S denotes summation of all arrays, Σ denotes summation for each individual point.

Now suppose we had to determine our a_s's from (i) by the method of least squares, each array being weighted by its frequency. We should have to make

$$u^2 = S\left\{\frac{n_x}{N}\left(\frac{\bar{y}_x - \bar{y}}{\sigma_y} - a_0\psi_0 - a_1\psi_1 - \ldots - a_n\psi_n\right)^2\right\}$$

a minimum

But
$$\frac{du^2}{da_s} = 0 = 2S\left(\frac{n_x(\bar{y}_x - \bar{y})}{N\sigma_y}\psi_s\right) - 2a_s\frac{S(n_x\psi_s^2)}{N},$$

or
$$a_s = \kappa_s/\lambda_s,$$

in agreement with the result obtained in (iii) bis above as a result of the ψ_s's being orthogonal functions. Now the fit by least squares to the means of the arrays is precisely the same as the fit by least squares to the whole swarm of points. In other words if we fit our regression line as above indicated to the whole population it will be "the best" fitting curve, if we make the assumption that least squares does give the best fit*. Anyhow it is likely to be a good fit, and that suffices for our present purposes.

We will now write $X = (x - \bar{x})/\sigma_x$ and $Y_x = (\bar{y}_x - \bar{y})/\sigma_y$, whence it follows that $q_{1t} = S(n_x Y_x X^t)/N$. We shall further write:

$$\mu_s = S(n_x X^s)\,\sigma_x^s/N,$$

and
$$\beta_{2s} = \mu_{2s+2}/\sigma_x^{2s+2} = \frac{S\{n_x(x-\bar{x})^{2s+2}\}}{N\sigma_x^{2s+2}} = \frac{S(n_x X^{2s+2})}{N},$$

$$\beta_{2s-1} = \frac{\mu_{2s+1}\mu_3}{\sigma^{2s+4}}.$$

We may rewrite (i) $Y = a_0\psi_0 + a_1\psi_1 + \ldots + a_n\psi_n$ .....(i) bis.

Multiply both sides by $n_x\psi_0/N$ and sum:

$$\frac{S(n_x Y\psi_0)}{N} = a_0\frac{S(n_x)}{N}\psi_0^2 = a_0,$$

if we assume $\psi_0 = 1$ as we are at liberty to do. But the left-hand side is zero. Therefore $a_0 = 0$.

Now assume $\psi_1 = X - c_{10}\psi_0$ and multiply both sides by $n_x\psi_0/N$ and sum:

$$\frac{S(n_x\psi_0\psi_1)}{N} = 0 \text{ by hypothesis} = \frac{S(n_x X\psi_0)}{N} - c_{10}\frac{S(n_x\psi_0^2)}{N}.$$

But
$$\frac{S(n_x X\psi_0)}{N} = 0, \quad \therefore\ c_{10} = 0, \text{ and } \psi_1 = X.$$

Multiply both sides by $n_x X/N$ and sum:

$$\frac{S(n_x YX)}{N} = r = a_1\frac{S(n_x X\psi_1)}{N} = a_1\frac{S(n_x X^2)}{N} = a_1.$$

Hence $a_1 = r$

Now assume $\psi_2 = X^2 - c_{21}\psi_1 - c_{20}\psi_0$. Multiply by $n_x\psi_0/N$ and sum:

$$0 = \frac{S(n_x X^2)}{N}\psi_0 - 0 - c_{20} = 1 - c_{20}.$$

Thus $c_{20} = 1$.

Multiply by $n_x\psi_1/N$ and sum:

$$0 = \frac{S(n_x\psi_1 X^2)}{N} - c_{21}\frac{S(n_x\psi_1^2)}{N}$$

$$\frac{S(n_x X^3)}{N} = \sqrt{\beta_1} = c_{21}\frac{S(n_x X^3)}{N} = c_{21}.$$

Thus $c_{21} = \sqrt{\beta_1}$.

* In order that this statement should be absolutely true our arrays would have to follow the normal or Gaussian distribution. This produces, however, an unnecessary limitation. It is known, but possibly not well known, that expansions in orthogonal functions as defined by (ii) give least square fits. I am rather inclined to think that this is an argument in favour of least square fits, rather than a justification of the expansion, i.e. that the method of least squares has a wider validity than Gauss' proof provides.

$\sqrt{\beta_1}$ must of course be given the same sign as μ_3. Accordingly we have:

$$\psi_2 = X^2 - \sqrt{\beta_1} X - 1.$$

Square, multiply by n_x/N and sum and we have:

$$\frac{S(n_x \psi_2^2)}{N} = \frac{S(n_x \psi_2 X^2)}{N} = \beta_2 - \beta_1 - 1.$$

It remains to find:

$$\frac{S(n_x Y \psi_2)}{N} = S\{n_x Y (X^2 - \sqrt{\beta_1} X - 1)/N\} = q_{12} - r\sqrt{\beta_1}.$$

Thus it follows from (iii) that $\quad a_2 = (q_{12} - r\sqrt{\beta_1})/(\beta_2 - \beta_1 - 1).$

We now proceed to find ψ_3:

$$\psi_3 = X^3 - c_{32}\psi_2 - c_{31}\psi_1 - c_{30}\psi_0.$$

Multiply by $n_x \psi_0/N$ and sum:

$$0 = \frac{S(n_x X^3)}{N} - c_{30}, \quad \text{or} \quad c_{30} = \sqrt{\beta_1}.$$

Multiply by $n_x \psi_1/N$ and sum:

$$0 = \frac{S(n_x X^4)}{N} - c_{31}\frac{S(n_x X^2)}{N}, \quad \text{or} \quad c_{31} = \beta_2.$$

Multiply by $n_x \psi_2/N$ and sum:

$$0 = \frac{S\{n_x X^3 (X^2 - \sqrt{\beta_1} X - 1)\}}{N} - c_{32}\frac{S(n_x \psi_2^2)}{N},$$

$$\frac{\mu_5}{\sigma_x^5} - \sqrt{\beta_1}\beta_2 - \sqrt{\beta_1} = c_{32}(\beta_2 - \beta_1 - 1),$$

$$\frac{\beta_3 - \beta_1\beta_2 - \beta_1}{\sqrt{\beta_1}} = c_{32}(\beta_2 - \beta_1 - 1),$$

$$c_{32} = \frac{\beta_3 - \beta_1\beta_2 - \beta_1}{\sqrt{\beta_1}(\beta_2 - \beta_1 - 1)}.$$

Accordingly: $\quad \psi_3 = X^3 - \frac{\beta_3 - \beta_1\beta_2 - \beta_1}{\sqrt{\beta_1}(\beta_2 - \beta_1 - 1)}\psi_2 - \beta_2\psi_1 - \sqrt{\beta_1}\psi_0,$

or $\quad \psi_3 = X^3 - \frac{\beta_3 - \beta_1\beta_2 - \beta_1}{\sqrt{\beta_1}(\beta_2 - \beta_1 - 1)} X^2 + \frac{\beta_3 - \beta_2^2 + \beta_2 - \beta_1}{\beta_2 - \beta_1 - 1} X + \frac{\beta_3 - 2\beta_1\beta_2 + \beta_1^2}{\sqrt{\beta_1}(\beta_2 - \beta_1 - 1}.$

We have next to determine κ_3:

$$\kappa_3 = \frac{S(n_x Y \psi_3)}{N}$$

$$= q_{13} - \frac{\beta_3 - \beta_1\beta_2 - \beta_1}{\sqrt{\beta_1}(\beta_2 - \beta_1 - 1)} q_{12} + \frac{\beta_3 - \beta_2^2 + \beta_2 - \beta_1}{\beta_2 - \beta_1 - 1} r$$

$$= q_{13} - \beta_2 r - \frac{\beta_3 - \beta_1\beta_2 - \beta_1}{\sqrt{\beta_1}(\beta_2 - \beta_1 - 1)}(q_{12} - r\sqrt{\beta_1}).$$

Let us write: $\quad \epsilon_{12} = q_{12} - r\sqrt{\beta_1}, \quad \epsilon_{13} = q_{13} - \beta_2 r,$

$$\phi_2 = \beta_2 - \beta_1 - 1, \quad \phi_3 = (\beta_3 - \beta_1\beta_2 - \beta_1)/\sqrt{\beta_1}.$$

Thus $\quad \kappa_3 = \epsilon_{13} - \frac{\phi_3}{\phi_2}\epsilon_{12}.$

We have next to find λ_3:

$$\lambda_3 = \frac{S(n_x \psi_3^2)}{N} = S\left\{\frac{n_x X^3}{N}\psi_3\right\}$$

$$= \beta_4 - \frac{(\beta_3 - \beta_1\beta_2 - \beta_1)\beta_3}{\beta_1(\beta_2 - \beta_1 - 1)} + \frac{(\beta_3 - \beta_2^2 + \beta_2 - \beta_1)\beta_2}{\beta_2 - \beta_1 - 1} + \frac{\beta_3 - 2\beta_1\beta_2 + \beta_1^2}{\beta_2 - \beta_1 - 1}$$

$$= \beta_4 - \beta_2^2 - \beta_1 - \frac{(\beta_3 - \beta_1\beta_2 - \beta_1)^2}{\beta_1(\beta_2 - \beta_1 - 1)}$$

$$= \phi_4 - \phi_3^2/\phi_2 \quad \text{f} \ \phi_4 = \beta_4 - \beta_2^2 - \beta_1.$$

Accordingly: $$a_3=\frac{\kappa_3}{\lambda_3}=\frac{\epsilon_{13}\phi_2-\epsilon_{12}\phi_3}{\phi_2\phi_4-\phi_3^2}.$$

We now pass to the fourth order regression line function:

$$\psi_4=X^4-c_{43}\psi_3-c_{42}\psi_2-c_{41}\psi_1-c_{40}\psi_0,$$

and we find:

$$\frac{S(n_x X^4\psi_3)}{N}=c_{43}\frac{S(n_x\psi_3^2)}{N}=c_{43}\lambda_3$$

$$=\frac{\beta_5}{\sqrt{\beta_1}}-\frac{(\beta_3-\beta_1\beta_2-\beta_1)\beta_4}{\sqrt{\beta_1}(\beta_2-\beta_1-1)}+\frac{(\beta_3-\beta_2^2+\beta_2-\beta_1)\beta_3}{\sqrt{\beta_1}(\beta_2-\beta_1-1)}+\frac{(\beta_3-2\beta_1\beta_2+\beta_1^2)\beta_2}{\sqrt{\beta_1}(\beta_2-\beta_1-1)}$$

$$=\{\beta_5(\beta_2-\beta_1-1)-\beta_4(\beta_3-\beta_1\beta_2-\beta_1)+\beta_3(\beta_3-\beta_2^2+2\beta_2-\beta_1)+\beta_1\beta_2(\beta_1-2\beta_2)\}\div\sqrt{\beta_1}(\beta_2-\beta_1-1).$$

Let $$\phi_5=\frac{\beta_5(\beta_2-\beta_1-1)-\beta_4(\beta_3-\beta_1\beta_2-\beta_1)+\beta_3(\beta_3-\beta_2^2+2\beta_2-\beta_1)+\beta_1\beta_2(\beta_1-2\beta_2)}{\sqrt{\beta_1}}.$$

Thus: $$c_{43}=\frac{\phi_5}{\phi_4\phi_2-\phi_3^2}.$$

Again: $$\frac{S(n_x X^4\psi_2)}{N}=c_{42}\lambda_2,$$

$$\beta_4-\beta_3-\beta_2=c_{42}(\beta_2-\beta_2-1),$$

or $$\phi_4-\phi_3\sqrt{\beta_1}+\phi_2\beta_2=\phi_4',\ \text{say},\ =c_{42}\phi_2.$$

Thus: $$c_{42}=\frac{\phi_4'}{\phi_2}$$

Further: $$\frac{\beta_3}{\sqrt{\beta_1}}=c_{41},$$

and $$\beta_2=c_{40}.$$

Thus: $$\psi_4=X^4-\frac{\phi_5}{\phi_4\phi_2-\phi_3^2}\psi_3-\frac{\phi_4'}{\phi_2}\psi_2-\frac{\beta_3}{\sqrt{\beta_1}}\psi_1-\beta_2\psi_0.$$

To find a_4 we must determine κ_4 and λ_4:

$$\lambda_4=\frac{S(n_x X^4\psi_4)}{N}$$

$$=\beta_6-\frac{\phi_5^2}{(\phi_4\phi_2-\phi_3^2)\phi_2}-\frac{\phi_4'}{\phi_2}(\beta_4-\beta_3-\beta_2)-\frac{\beta_3^2}{\beta_1}-\beta_2^2$$

$$=\phi_6-\frac{\phi_5^2}{\phi_2(\phi_4\phi_2-\phi_3^2)}-\frac{\phi_4'^2}{\phi_2},$$

where $$\phi_6=\beta_6-\frac{\beta_3^2}{\beta_1}-\beta_2^2,$$

$$\kappa_4=\frac{S(n_x Y\psi_4)}{N}=q_{14}-\frac{\phi_5}{\phi_4\phi_2-\phi_3^2}\left(\epsilon_{13}-\frac{\phi_3}{\phi_2}\epsilon_{12}\right)-\frac{\phi_4'}{\phi_2}(q_{13}-r\sqrt{\beta_1})-\frac{\beta_3}{\sqrt{\beta_1}}r.$$

Let us write $q_{14}-\frac{\beta_3}{\sqrt{\beta_1}}r=\epsilon_{14}$, then

$$a_4=\frac{\epsilon_{14}-\frac{\phi_5}{\phi_4\phi_2-\phi_3^2}\left(\epsilon_{13}-\frac{\phi_3}{\phi_2}\epsilon_{12}\right)-\frac{\phi_4'}{\phi_2}\epsilon_{12}}{\phi_6-\frac{\phi_5^2}{\phi_2(\phi_4\phi_2-\phi_3^2)}-\frac{\phi_4'^2}{\phi_2}}.$$

We have thus obtained the regression orthogonal functions up to the fourth order. Higher order terms can also be found, but their expressions become very complicated and such expressions involving fifth product-moments and eighth marginal total moments will be subject to very large probable errors.

We can now by aid of (v) express $\eta_{y.x}$. We have:

$$\eta^2_{y.x}=r^2+\epsilon_{12}^2/\phi_2+\left(\epsilon_{13}-\frac{\phi_3}{\phi_2}\epsilon_{12}\right)^2\Big/(\phi_4-\phi_3^2/\phi_2)$$
$$+\frac{\left\{\epsilon_{14}-\frac{\phi_4'}{\phi_2}\epsilon_{12}-\frac{\phi_5}{\phi_2^2}\left(\epsilon_{13}-\frac{\phi_3}{\phi_2}\epsilon_{12}\right)\Big/(\phi_4-\phi_3^2/\phi_2)\right\}^2}{\phi_6-\phi_4'^2/\phi_2-\frac{\phi_5^2}{\phi_2^2}\Big/(\phi_4-\phi_3^2/\phi_2)}+\text{etc.}.....$$

The conditions therefore for linear regression, or $\eta_{rx}=r$, are

$$\epsilon_{12}=\epsilon_{13}=\epsilon_{14}=\text{etc.}=0.....$$

That is: $$q_{12}=r\sqrt{\beta_1},\quad q_{13}=r\beta_2,\quad q_{14}=r\frac{\beta_3}{\sqrt{\beta_1}},\ \text{etc.}.....$$

For parabolic regression: $$\epsilon_{13}=\frac{\phi_3}{\phi_2}\epsilon_{12},\quad \epsilon_{14}=\frac{\phi_4'}{\phi_2}\epsilon_{12},\ \text{etc.}.....,$$

or
$$q_{13}-\frac{\phi_3}{\phi_2}q_{12}=r\left(\beta_2-\frac{\phi_3}{\phi_2}\sqrt{\beta_1}\right),$$
$$q_{14}-\frac{\phi_4'}{\phi_2}q_{12}=r\left(\frac{\beta_3}{\sqrt{\beta_1}}-\frac{\phi_4'}{\phi_2}\sqrt{\beta_1}\right),\ \text{etc.}.....$$

And lastly for cubical regression:

$$\epsilon_{14}-\frac{\phi_4'}{\phi_2}\epsilon_{12}-\frac{\phi_5}{\phi_2^2}\left(\epsilon_{13}-\frac{\phi_3}{\phi_2}\epsilon_{12}\right)\Big/\left(\phi_4-\frac{\phi_3^2}{\phi_2}\right)=0,\ \text{etc.}.....$$

Such conditions, especially with regard to their probable errors, become less and less manageable as we proceed.

The general principle involved in the present paper has been discussed by Tchebycheff*, and more adequately by J. P. Gram†, but the former had in view the fitting or graduating of curves. He calculated quantities which correspond to our μ_s's on the assumption that $n_x=1$, i.e. that the weight of the $\bar{y}_x$'s are all the same or that the marginal total is a *rectangle*. He was thinking of fitting a curve to a curve and not fitting a curve to a swarm of points. In his case each μ_s and accordingly each β and each ϕ is expressible in terms of the total number m of subranges which he takes of equal length. There are I think simpler methods of calculating the equation to a higher order parabola in such cases‡. As far as I am aware these orthogonal regression functions have not hitherto been dealt with and they throw a good deal of light on the original equations I provided in 1905 for skew regression. I had not recognised at that time that my expressions of each order were true orthogonal functions. It will be seen that my solution does not involve equality of subranges and is not limited to any special frequency distribution.

II. Note on the "Fundamental Problem of Practical Statistics."

(*Biometrika*, Vol. XIII, p. 1.)

Some misunderstanding has arisen with regard to my paper under the above title in the last issue of this *Journal*. I believe it to be due to the critics not having read Bayes' original theorem as given by Price in the *Phil. Trans.*, Vol. LIII. Bayes takes a ball and places it at random on a table, say of breadth unity, and its distance from one side being x, its chance of falling between x and $x+\delta x$ is δx. x is thus not a chance, but a *variate*. He now calls a "success," the chance that any other ball placed at random on the table will be nearer to the same side than the first

* *Mémoires de l'Académie de Saint-Pétersbourg*. Memoirs in 1854 and 1859. A *résumé* by R. Radau: *Bulletin Astronomique*, T. VIII, Paris, 1891, pp. 350, 376 *et seq.* See also Liouville's *Journal*, 2e Série, T. III (1858), p. 289 *et seq.*

† *Thesis*: "Om Rækkeudviklinger bestemte ved Hjaelp af de mindste Kradvaters Methode." Kjøbenhavn, 1879.

‡ *Biometrika*, Vol. II, pp. 12—16.

placed ball and a "failure" that it should be greater. The chance therefore of p successes and q failures now happening is

$$x^p(1-x)^q \times dx \times \frac{\lfloor p+q}{\lfloor p \lfloor q}$$

It is solely the fact that all possible values of the variate x are made *a priori* equally likely that makes the chance of a success x, equal to the variate itself. Those who criticise Bayes after reading his actual paper, say that he ought not to have made the chances of a ball being placed anywhere on the table equally likely. He makes in fact his distribution of the variate x a straight line—a somewhat unusual form of frequency distribution*. My answer to that objection to Bayes' work was that you can make the distribution of that variate—i.e. position on the table—any continuous curve you please as Bayes' Theorem with Bayes' results will flow from it equally well. Against this position my critics raise the cry that the chance is no longer x of a success and $1-x$ of a failure. Of course not, because that depends on horizontality of frequency distribution and it was merely fortuitous that for that case Bayes' variate x corresponded to a chance. In other cases the chance is a function of the variate x and not x itself. But if the critics say: Then this is not what we mean by Bayes' Theorem, I would reply: Quite so, but it is what Bayes meant by his own Theorem, and it probably fits much better the type of cases to which we are accustomed to apply it than what you mean by Bayes' Theorem.

Let me illustrate this point. An event has happened p times and failed q times; what is the chance that in $r+s$ further trials it will occur r times and fail s? The critics say: this is *our* Bayes' Theorem, not your Bayes' Theorem. I reply that it is both, but that your way of solving it is not Bayes'. Perhaps Bayes saw further than some of the critics who have not troubled to read his original paper. What Bayes said was this, the event will happen when there is an excess (or it may be defect) of a certain variate, but I do not know what is the limiting value of this variate *a priori*. Look at Bayes' billiard table from a more modern standpoint. Men will sicken from a disease when their resistance falls below x an *a priori* unknown value of the variate. Bayes took the chance of this limiting value lying between x and $x+\delta x$ to be dx, if the total range be taken as unity. He ought to have taken it $\phi(x)\,dx$, where $\phi(x)$ is arbitrary. He took the chance of occurrence of the disease to be x, when he ought to have taken it $\int_0^x \phi(x)\,dx = P_x$; and of failure $1-x$ instead of $1-P_x$.

But had he taken these better values he would have reached finally precisely the same result as he did by his equal distribution of ignorance. Bayes made every value of his variate x equally likely. He ought to have given them a perfectly arbitrary frequency distribution. *A priori* all degrees of immunity are not equally likely to be the limiting value in the case of a disease. The generalised Bayes as thus envisaged has a very wide application to vital statistics; in fact it seems to me to entirely replace the other sort of Bayes' Theorem suggested by his critics.

Nay I would go further, and say that it is Bayes' Theorem in Bayes' sense that we need in most questions of prediction of the future from the past. If, for example, two men play a set of games and A has won p and B q games and we consider the chance in the following $r+s$ games of A's winning r and B's winning s, then I believe that A's winning may be accurately considered as depending on the excess of a certain variate x which is a function of A's skill relative to B's. *A priori* we do not know what the value of x is for which A will win. All we can say is that when relative to B he exhibits a certain excess of skill he will win, but that we must not assume with Bayes that all limit-values of x are equally likely. We must take any frequency curve for the possible distribution of x.

I believe that in most cases such a variate may be hypothecated and if it can the objection to Bayes that he made all positions of his balls on the table "equally likely" can be removed, and if removed one fundamental objection to his theorem as he stated it, i.e. in terms of excess or defect of a variate, disappears. K. P.

* It is in fact the "rectangle point" $\beta_1=0$, $\beta_2=1{\cdot}8$, just as limited a distribution as the Gaussian $\beta_1=0$, $\beta_2=3$.

III. Second Note on the Coefficient of Correlation as determined from the Quantitative Measurement of one Variate and the Ranking of a Second Variate (see *Biometrika*, Vol. x, p. 416).

By KARL PEARSON, F.R.S.

In a previous note I pointed out that if $\rho_{g_x y}$ be the correlation of the grade of a variate x with the quantitative value of a second variate y, then the *variate* correlation between x and y would be

$$r_{xy} = \sqrt{\frac{\pi}{3}}\,\rho_{g_x y} = 1{\cdot}023{,}278\rho_{g_x y}{}^{*}$$

If we use ranks instead of grades, i.e. σ_{v_1} instead of σ_{g_1}, we have

$$r_{xy} = \sqrt{\frac{\pi}{3}}\,\frac{\sqrt{N^2-1}}{N}\,\rho_{v_x y},$$

which gives a slight further correcting factor depending on N, but as a rule of very little importance.

It follows from the above that on the assumption of a normal distribution the correlation of variate and grade for the same character will be ·9773. Accordingly when we are testing correlations of rank and variate we are actually sampling a population of which the correlation of the two characters is ·9773. But we know from a paper in *Biometrika*, Vol. xi, p. 401, that even for $\rho = {\cdot}9$ in the sampled population the curve of distribution of r in samples is markedly skew and accordingly deviations in defect can occur, which are not possible in excess.

If we take samples of 25 the mean value of r found from (xxv), p. 336 of the above mentioned memoir is

$$\bar{r} = {\cdot}9763,$$

and for samples of 35:

$$\bar{r} = {\cdot}9766.$$

We should thus anticipate that the mean of small samples of this order would come out slightly less than ·9773.

Accordingly with the help of my colleagues Miss Elderton, Miss Karn and Miss Moul, the correlations between rank and marks were worked out for 14 Civil Service Examinations and for 16 examinations in a Technical School most kindly provided by my friend Dr Ritchie Scott. These examinations furnished the correlation coefficients given in the following table.

It will be seen that the English results of the Technical School are somewhat erratic. This is an experience similar to that which I had some years ago, when drawing up a report on the Matriculation Examination of the University of London, the language marking in that case being peculiarly subject to personal equation of an erratic kind.

Taking the 14 Civil Service Examinations we find for the weighted mean,

$$\text{Correlation} = -{\cdot}9666 \pm {\cdot}0086.$$

The result to be anticipated, using $(xx)^{bis}$ to obtain the probable error of the mean is

$$\text{Correlation} = -{\cdot}9760 \pm {\cdot}0093.$$

The difference is therefore slightly less than once the probable error of the last result and on the basis of this alone could not be considered significant.

If we take the eight Second Year Examinations of the Technical Schools *including English* the result is

$$\text{Correlation} = -{\cdot}9601 \pm {\cdot}0086,$$

while the theoretical result is $\text{Correlation} = -{\cdot}9766 \pm {\cdot}0123$.

The difference is about 1·3 times the probable error and is not in itself significant.

* Actually the sign may be positive or negative according as the variate in one case rises or falls with the rank of the other variate.

Correlation of Rank and Examination Marks.

Examination	Numbers	Correlation and P.E.
Civil Service Junior Examinations:		
Liverpool Messengers, 1901	27	$-\cdot9739\pm\cdot0067$
Glasgow Messengers, 1901	24	$-\cdot9831\pm\cdot0046$
Manchester Messengers, 1901	21	$-\cdot9323\pm\cdot0196$
Liverpool Postmen, 1901	19	$-\cdot9861\pm\cdot0043$
Glasgow Postmen, 1901	21	$-\cdot9359\pm\cdot0183$
Manchester Postmen, 1901	18	$-\cdot9647\pm\cdot0110$
Female Learners, Post Office, Nottingham, 1900	26	$-\cdot9416\pm\cdot0150$
" " " Leeds, 1900 ...	46	$-\cdot9694\pm\cdot0060$
" " " Bristol, 1900 ...	33	$-\cdot9795\pm\cdot0048$
" " " Manchester, 1899	21	$-\cdot9784\pm\cdot0064$
" " " Leeds, 1899 ...	20	$-\cdot9736\pm\cdot0079$
Female Skilled Telegraphists, December, 1899 ...	18	$-\cdot9684\pm\cdot0099$
" " " May, 1899 ...	17	$-\cdot9238\pm\cdot0240$
Female Learners, Post Office, Birmingham, 1899	29	$-\cdot9951\pm\cdot0012$
Technical School:		
English, Second Year	37	$-\cdot8690\pm\cdot0271$
Mechanics "	35	$-\cdot9286\pm\cdot0157$
Metalwork "	38	$-\cdot9676\pm\cdot0070$
Gymnastics "	36	$-\cdot9728\pm\cdot0060$
Machine Drawing, Second Year	36	$-\cdot9831\pm\cdot0038$
Art, Second Year	36	$-\cdot9845\pm\cdot0035$
Science "	33	$-\cdot9876\pm\cdot0030$
Mathematics, Second Year...	38	$-\cdot9896\pm\cdot0023$
English, First Year	33	$-\cdot9957\pm\cdot0010$
Mechanics "	31	$-\cdot9725\pm\cdot0066$
Metalwork "	33	$-\cdot9877\pm\cdot0029$
Gymnastics "	31	$-\cdot9694\pm\cdot0073$
Machine Drawing, First Year	33	$-\cdot9508\pm\cdot0113$
Art, First Year	32	$-\cdot9838\pm\cdot0038$
Science "	30	$-\cdot9905\pm\cdot0023$
Mathematics, First Year	33	$-\cdot9742\pm\cdot0060$

Now taking the eight First Year examinations we have

$$\text{Correlation} = -\cdot9780\pm\cdot0032,$$

while the theoretical result is $-\cdot9766\pm\cdot0123$.

The difference is well under the probable error.

If we omitted the erratic English markings we obtain for the two cases $-\cdot9735$ and $-\cdot9754$ both fairly good representations of $-\cdot9766$.

If we include the English results and pool both series we obtain

$$\text{Correlation} = -\cdot9685\pm\cdot0070$$

against the theoretical result $\text{Correlation} = -\cdot9766\pm\cdot0087$,

the difference being just below the probable error of the latter result.

The impression formed upon my mind by these results is that the result of correlating rank and marks of the candidates in an examination will give a result slightly less than that to be anticipated by the theory of a normal distribution; but so little less that an effective control for detecting erratic markings such as that of the English in the technical school returns (or possibly the Female Skilled Telegraphists, May 1899 of the Civil Service Examinations) may be obtained by this correlation.

As a second illustration I will take the following data from a Civil Service Examination for Messengers. There were 10 posts to be filled and 27 candidates. The subjects were Arithmetic,

Orthography, Handwriting, Geography and English Composition. The total marks were 300 in the first subject and 200 in each of the others—grand total 1100. I give the results below as far as I need them.

Rank in Total	Total Marks	Rank in Arithmetic	Marks in Arithmetic
1	907	1	230
2	764	9	158
3	748	2	228
4	746	10	154
5	724	8	162
6	718	5	182
7	710	14	129*
8	703	7	164
9	677	3	187
10	665	4	186
11	645	11	151
12	643	6	167
13	634	20	103
14	628	12	146
15	580	13	131
16	561	15	128
17	560	18	116
18	532	22	82
19	529	16	125
20	526	17	122
21	515	19	114
22	484	21	93
23	463	25	61
24	444	26	38
25	386	27	37
26	369	23	63
27	288	24	62
a	*b*	*c*	*d*

I first correlated ranks a with ranks c. This leads to

$$\rho_{ac} = \cdot 8834$$

by the formula (xx) of my memoir "On Further Methods of Determining Correlation" (*Drapers Company Research Memoirs*, IV, p. 13, Cambridge University Press).

Using formula (xviii) of the same memoir, we obtain for the correlation of variates, i.e. general capacity and arithmetical capacity:

$$r'_{bd} = \cdot 893.$$

I worked out directly by the product-moment method the same correlation and found

$$r_{bd} = \cdot 896 \pm \cdot 026.$$

The agreement between r_{bd} and r'_{bd} in this case is excellent.

I next investigated the correlation of total marks and ranks in arithmetic and found

$$\rho_{bc} = -\cdot 8712\dagger,$$

whence we deduce by the fundamental equation of the present paper,

$$r''_{bd} = \cdot 892.$$

* I have altered this figure by one mark to avoid the trouble of dealing with a single bracket, the only one involved.

† See footnote on p. 302 as to change of sign between ρ_{bc} or ρ_{ad} and r''_{bd} or r'''_{bd}.

The next step was to find the correlation of rank in the total examination with marks in arithmetic. There resulted

$$\rho_{ad} = -\cdot 8805^*,$$

whence we deduce $r'''_{bd} = \cdot 901$.

All four correlations r_{bd}, r'_{bd}, r''_{bd} and r'''_{bd} differ by less than ·01, or less than half the probable error of r_{bd}. The differences therefore have no significance, or the values are interchangeable.

Equally clearly r''_{bd} and r'''_{bd} are as stated above substantially equal which involves the equality of ρ_{bc} and ρ_{ad} to the same degree as stated above.

Lastly I correlated total marks with rank in total examination and found

$$\rho_{ab} = -\cdot 9739.$$

Multiplying this by 1·023,278 we have $r_{bb} = \cdot 997$,

which is as near the desired result of perfect correlation as we can expect to get on the assumption of a normal distribution of the variate.

As far therefore as this isolated example is concerned the method appears to be satisfactory.

The results reached in both our illustrations suggest that the correlation of rank and variate for the same character may be of considerable value for control purposes in dealing with examination results.

IV. A Study of Women Delinquents in New York State by Mabel R. Fernald, Mary H. S. Hayes and Almena Dawley with a Statistical Chapter by Beardsley Ruml and a preface by Katherine Bement Davis. Published by The Century Co., New York City, 1920.

By ETHEL M. ELDERTON.

At the present time there seems to be an increasing demand for statistics in connection with social problems and a desire to collect data sometimes without a very clear idea as to how the material when collected can be used but with the pious hope that it will be useful in some way. The interesting memoir before us if it had done nothing else would be extremely valuable as showing what is required before any social problem can be adequately studied from the statistical aspect, how investigators are hampered on all sides by a lack of comparative data and how after years of work and a most careful compilation of facts the main problem may be still unsolved. The authors of this book on women delinquents in New York State fully realize the necessity of such a study and the difficulties in the way. They start by pointing out that though there is abundant literature on the subject of the criminal population much of it is useless because it rests "at its best, upon the most casual and superficial observations, and at its worst upon what the writer thinks he would find on observation." The writers point out that when the object of imprisonment was simply the two-fold one of punishment for the crime and protection of the state from further crimes an exact study of the causes of criminal acts was of less importance than it is to-day when the object of the imprisonment of criminals is of a more humanitarian nature, namely not only to protect the community but to attempt to readjust the criminal to society. As far as we know this is the earliest extensive study of women delinquents using modern statistical methods. The first efforts of the investigators were directed towards obtaining information as to the distinguishing characteristics of women convicted in New York State, of their mental capacity and of the main facts of their personal and environmental histories. Physical and medical facts had to be disregarded almost entirely since adequate medical data for most of the groups of women studied could not be obtained. The first comparison which the authors wished to make was between the women delinquents and women in general: Are the delinquent women a special selection out of the female population or are they a random sample of the total differentiated only by their criminal career? Do they differ significantly in their mentality, in their environment, in their education? Here the difficulty is, that as in so much literature dealing with medical and social questions only the special group is studied and not the general population, and though the United States census gives some information it is too general to be of much use, and the authors have to fall back on data collected for special groups of women other than criminals who may not be representative of women as a whole. A second comparison which might be made is with men criminals, but the study of men is almost entirely confined to men convicted of relatively serious offences and does not deal with those committed to workhouses, county penitentiaries and reformatories. Dr Goring's study of English criminals would cover practically none of this group; the criminals he considered were felons while the

* See footnote on p. 302 as to change of sign between ρ_{bc} or ρ_{ad} and r''_{bd} or r'''_{bd}.

bulk of these women delinquents are misdemeanants. A third comparison can be made within the delinquent group dividing them according to class of offence, nationality, mental capacity, etc.

The women considered are those who have a legal age of 16 years and over and were convicted in the courts of New York State. The investigators would have preferred to secure the women at the time of conviction as they came through the various courts but this was not possible and the plan followed, though necessary, is open to objections. To avoid selection consecutive commitments to the different institutions were taken whenever possible but even this could not always be carried out, as for example in the workhouse. The groups studied were (*a*) New York State Reformatory for Women at Bedford Hills; this includes women convicted of felonies and misdemeanors. Women convicted of murder in the first or second degree or who had previously committed a felony are not committed to Bedford Hills and very few women over 30 are found there. There were 102 examined for the main part of the enquiry. (*b*) Auburn, for felons only; 88 were examined and the enquiry was carried on for over two years. (*c*) The New York Magdalen Home under private management; includes women between the ages of 16 and 35; felons are not generally admitted. (*d*) The New York County Penitentiary; there is no organized training provided and one would expect unpromising cases, indeterminate sentences and older women to be sent here. (*e*) The New York City Workhouse which has only misdemeanants. The types of women who are committed here are (1) general misdemeanants who seem unpromising or have failed before, (2) new and relatively hopeful cases who have a sentence of only a few days, (3) intoxication cases. Only the first group could be studied since the second group were not in the workhouse long enough and the group of inebriates seemed to the investigators so senile and untruthful and their dwelling places were so shifting that information could not be obtained*. (*f*) Probation Cases from the Women's Night Court of Manhattan and the Bronx; these women were all misdemeanants and generally first offenders. It is quite true that all offenders are represented except inebriates, but the author's material is not a true sample of all women delinquents even when the results are pooled and this seems to me to invalidate much of the work on age for instance. The period over which the women were examined varies considerably in the different institutions, as for example in Auburn, where the enquiry was carried out for over two years and we shall therefore get a larger proportion of the type of women committed to Auburn than we should have in the general delinquent population; in this case we shall have more felons and probably more older felons among these delinquent women than really exist. We are inclined to think that consecutive cases should have been examined over the same length of time in each institution or that the institutions should never have been pooled. Generally each institution is kept separate but not always.

Correlation coefficients and ratios have been largely employed and in all cases an "index of reservation is given"; this index is the probable error of more general use without the constant ·67449. We are not sure that an η from three groups is very satisfactory but the only alternative is to use a bi-serial "r" but we believe that this method is never used throughout the book.

The mental condition of these women has been studied very fully. Four tests were used, Binet-Simon Scale 1911, Yerkes-Bridges Point Scale, Stanford-Binet Scale and Woolley Series, and finally educational tests were given to the Bedford women. Great care was taken in scoring and all the answers were taken down verbatim so that a second examiner could check the marks given. Also the women were judged by Professor Pearson's scale and generally speaking those who judged by this scale did not know the result of the test examination. The final tests selected were those which gave the highest correlation between intelligence as judged by estimation and the result of tests applied to the Bedford group. The social investigation was carried out by six field workers. Information from the offender was followed by visits to the home, to other relatives, to the employers and to social agencies who had been in touch with the offender. This investigation and that of the mental capacity of these women must have entailed much work and seems to us an excellent model for future investigations which one would like to see carried out on an unselected female population. The relationship between age and number of previous convictions is discussed and compared with Dr Goring's results for English convicts but one doubts whether such comparisons are very useful. Dr Goring was dealing only with felons and he omitted first offenders while nearly half these women delinquents are first offenders and more than half are not felons, and as Dr Goring pointed out in England men and women differ considerably in the annual number of convictions for crime and in the type of crime committed. The larger number of previous convictions among the users of alcohol is probably significant but we think a correction for age should be applied, since use of alcohol is correlated with age in the data as here presented and there is a significant though small correlation between the number of convictions and the age of the women. One would also like to know whether, if

* To eliminate the inebriates, however, is to eliminate a considerable element of mental defectives, for to judge by English experience it is largely the mentally defective who take to alcohol.

convictions for intoxication were omitted the user of alcohol is more often guilty of misdemeanors and felonies than the non-user. It is interesting to note that foreign white women have a smaller percentage in each institutional group than they have in the population as a whole in view of the prevailing opinion that foreigners in New York are largely responsible for crime in that city. The foreign-born are however more prone to commit serious offences than the native-born. We do not consider that much can be gained by discussing the numbers age, etc. of native coloured delinquents in the institutions since some of them, such as the Magdalen, take no negro women, and as we said before, a combination of institutions does not seem to be justifiable.

A very careful record was made of the home conditions, combining the economic condition, moral standard and adequacy of parental supervision, and it was found that there was a significant correlation between the condition of the home and the age at first conviction (·31), the women from the poorest homes being first convicted at an earlier age, but there seems to be no correlation between the number of previous convictions and the home conditions though here also we should like to see a correction for age. It is very unfortunate, as the authors point out, that there is no control series, so that no comparison with the normal population can be made. Neither, for the same reason, is the hereditary aspect considered, and we feel that we must protest against the form in which the data available are given. On p. 241 it is stated that in 15·9 per cent. of families, which means here parents and siblings, some member had been convicted of crime, but there is no statement as to the number of siblings, and one criminal in a family of eleven for instance is a different matter from one criminal in a family of four. This information about the number of the family is available and we should have thought that by taking estimates of criminality in New York State the hereditary side could have been dealt with. A study of the educational opportunities of these women is practically limited to a study of their age at leaving school and the years they were at school, and here again only a comparison within the group can be carried out. There seems to be no clear relationship between age at leaving school and age at first conviction. As far as educational attainments are concerned, school attainment does not seem to have kept pace with length of time at school but we do not know how far the two are connected in the non-delinquent population. The older women were found to have reached a lower standard and to have been fewer years at school than the younger women, but probably this would be the same in the general population. There is no correlation between number of convictions and the grade reached even when age is made constant.

As far as the occupation of these women is concerned there seem to be more domestic workers among the delinquent women than in the general population not necessarily as the investigators state that domestic work causes delinquency but it may be that those who go into domestic service represent the unskilled untrained women from whom the delinquents are drawn. The domestic workers appear to have reached a lower grade than the women employed in other occupations. There was no evidence of a connection between earning capacity and number of convictions. The women seem to be earning a low and irregular wage but they belong largely to unskilled workers and judging by the low grade reached at school they were probably for the most part incapable of better work. The authors do not think that the delinquency is the result of the low and irregular wage; an elaborate investigation would be needed to enable us to discuss this point; judging by the accounts of the individuals one wonders that some of the women were ever employed at all and in the worst cases work and character were so unsatisfactory that dismissal occurred soon after employment.

In connection with intelligence there is a good deal of comparison with Dr Goring's work but we are doubtful as to how far such a comparison is useful considering the differences in sex, type of offence, etc. The women delinquents' intelligence is judged by the test aggregate and no figures are given showing the percentages found in the grades which correspond to those used by Dr Goring. As judged by the test aggregate, delinquent women are 1·6 to 2·0 years behind the army group. If we take Table 165 and consider army men and delinquent women as samples of the population and work out the correlation coefficient we find it is $\cdot 38 \pm \cdot 02$ (where ·02 is the probable error not the index of reservation) between delinquency and a low intelligence. The actual figures are not given so that I have had to restore the table to its original form by multiplying the columns by 447 and 653 respectively, never a very satisfactory business.

In a work of this kind it would be an enormous advantage if the summarized data for each individual were given as Dr Goring gave it in Part II of his work for the criminals he studied; this would enable those interested to work out other correlations and test fresh points for themselves. More than half of these delinquent women are guilty of sex offences and since the correlation between mental defect and sexual crime was found by Dr Goring to be ·46 we do not feel that there is any great disagreement in these conclusions though we know that we are not comparing data that are strictly comparable. There are certainly some differences between English male criminals and these delinquent women but these may be due to a different defini-

tion of degree of criminality. Dr Goring uses the number of convictions per year of freedom and number of months imprisoned per year of freedom but since in New York the indeterminate sentence is given for hopeful cases instead of a short sentence "number of convictions" was used among these delinquent women as a measure of criminality. Dr Goring found frequency of conviction associated with the relatively weak-minded but shorter periods of imprisonment, while though delinquent women tended in the same direction the values were barely significant. Property offenders seem to be more intelligent than offenders against chastity which, if property offenders includes burglars and thieves, agrees with Dr Goring's conclusions. In discussing wage earned and mental capacity those who "live in," chiefly domestic servants, have had to be separated from the other groups of employment. Among domestic servants the authors find no correlation between mental capacity and wage but they do find a correlation between wage and mental capacity among workers other than domestic servants. We can hardly assume however that domestic service differs from other occupations in that mental capacity does not vary with wage in this occupation while it does in others, since no comparison within one group of occupations has been undertaken and except in factory workers the numbers would not be large enough for this to be worth doing. There is certainly a correlation between mental capacity and type of occupation and between wage and type of occupation and between mental capacity and wage when different occupations are considered, but the wage within any one occupation need not necessarily vary with intelligence any more than it does with domestic service though we may expect it to do so.

We have briefly indicated the lines on which this enquiry has proceeded and throughout the reading we have been much impressed by the caution with which opinions have been expressed and it is therefore the more astonishing to find in the last paragraph that these guarded expressions of opinion have gone to the winds. We find the sentence "We disagree (i.e. with Dr Goring) however in the preëminence attached to such a constitutional factor as defective intelligence in contrast with economic factors." We fail to find this disagreement in the text of the book. Dr Goring finds that 10 to 20 per cent. of the criminals were feeble minded, the authors of this book find 17 per cent. of the delinquent women mentally poorer than the 296 working girls who "should make a fairly good average for the whole community." If these working girls are a sample of the whole community probably one or two would be feeble minded and all those below the worst of these girls in intelligence would certainly be feeble minded and the agreement with Dr Goring's estimate is very striking. On p. 244 the authors write "How much of a factor the element of poor home conditions is, as causative of delinquency, it is difficult to judge, since we have no similar estimates of the general population." This sentence and the number of feeble minded among the delinquent women do not seem to agree with the sentence from the last paragraph of the book quoted above. It is true that within the group crime begins at an earlier age in the poorer homes but there is no correlation between poor homes and the number of convictions. In this connection we might mention that in 1902 when Dr Goring started collecting his data the work of Binet and Simon had not been published and also that in estimating the amount of feeble mindedness in the general population Dr Goring was guided not only by the report of the Royal Commission, as stated in this work, but also by a knowledge of the number of feeble minded children in the schools of London and Liverpool. Personally we prefer to draw our conclusions from a study of the book as a whole rather than from the summary and that study seems to us to emphasize the necessity of an investigation into the economic conditions, home environment and mental capacity of a sample of the non-criminal women of New York State without which it seems to us no comparison as to the relative importance of constitutional and environmental factors as determinants of crime can be adequately discussed. It is more difficult among these women to obtain an estimate of the amount of criminality owing to the existence of the indeterminate sentence than it was among the criminals in England and many apparently contradictory facts emerge when the two estimates of criminality, age at first conviction and number of convictions, are taken. In every country we need a very full control series with which we can compare our special groups and we wish that such an investigation could be carried out on the lines followed in this interesting study of women delinquents.